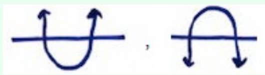
Functions Prep *[134 marks]*

1a. *[2 marks]*

Markscheme

METHOD 1 (using symmetry to find *p*)

valid approach ***(M1)***

eg $\frac{-1+3}{2}$, 

$p = 1$ ***A1 N2***

Note: Award no marks if they work backwards by substituting $a = 2$ into $-\frac{b}{2a}$ to find *p*.

Do not accept $p = \frac{2}{a}$.

METHOD 2 (calculating *a* first)
(i) & (ii) valid approach to calculate *a* ***M1***

eg $a + 4 - c = a(3^2) - 4(3) - c$, $f(-1) = f(3)$

correct working ***A1***

eg $8a = 16$

$a = 2$ ***AG N0***

valid approach to find *p* ***(M1)***

eg $-\frac{b}{2a}$, $\frac{4}{2(2)}$

$p = 1$ ***A1 N2***

[2 marks]

1b. *[2 marks]*

Markscheme

METHOD 1

valid approach ***M1***

eg $-\frac{b}{2a}$, $\frac{4}{2a}$ (might be seen in (i)), $f'(1) = 0$

correct equation ***A1***

eg $\frac{4}{2a} = 1$, $2a(1) - 4 = 0$

$a = 2$ ***AG N0***

METHOD 2 (calculating *a* first)
(i) & (ii) valid approach to calculate *a* ***M1***

eg $a + 4 - c = a(3^2) - 4(3) - c$, $f(-1) = f(3)$

correct working ***A1***

eg $8a = 16$

$a = 2$ ***AG N0***

[2 marks]

1c. *[3 marks]*

Markscheme

valid approach ***(M1)***

eg $f(-1) = 5$, $f(3) = 5$

correct working ***(A1)***

eg $2 + 4 - c = 5$, $18 - 12 - c = 5$

$c = 1$ ***A1 N2***

[3 marks]

2a. *[1 mark]*

Markscheme

correct range (do not accept $0 \leqslant x \leqslant 7$) ***A1 N1***

eg $[0,\ 7]$, $0 \leqslant y \leqslant 7$

[1 mark]

2b. *[1 mark]*

Markscheme

$f(2) = 3$ ***A1 N1***

[1 mark]

2c. *[1 mark]*

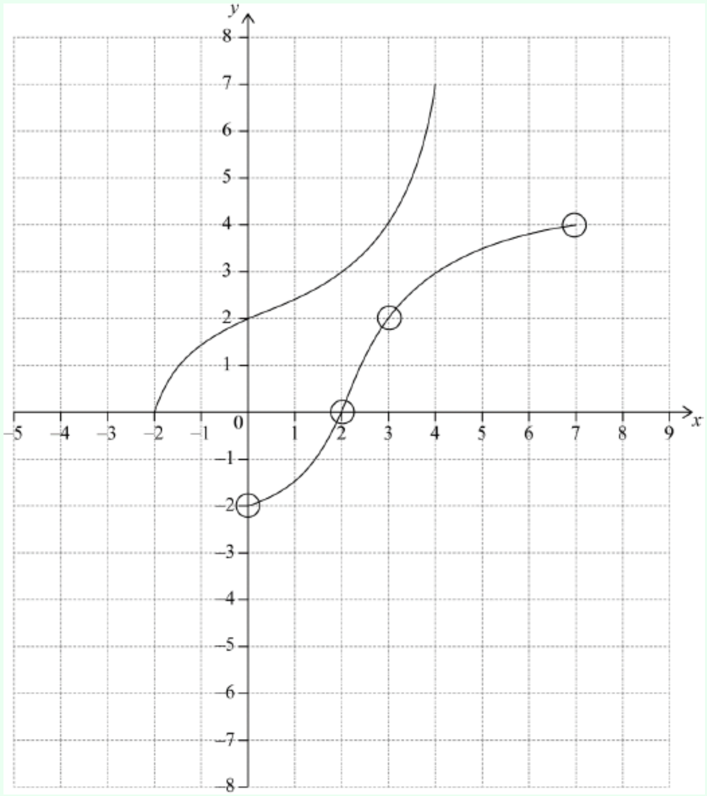
Markscheme

$f^{-1}(2) = 0$ ***A1 N1***

[1 mark]

2d. *[3 marks]*

Markscheme



A1A1A1 ***N3***

Notes: Award ***A1*** for both end points within circles,

A1 for images of $(2,\ 3)$ and $(0,\ 2)$ within circles,

A1 for approximately correct reflection in $y = x$, concave up then concave down shape (do not accept line segments).

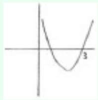
[3 marks]

3a. *[3 marks]*

Markscheme

METHOD 1 (using *x*-intercept)

determining that 3 is an x-intercept ***(M1)***

eg $x-3=0,$ 

valid approach ***(M1)***

eg $3-2.5,\ \frac{p+3}{2}=2.5$

$p=2$ ***A1 N2***

METHOD 2 (expanding *f* (*x*))

correct expansion (accept absence of a) ***(A1)***

eg $ax^2-a(3+p)x+3ap,\ x^2-(3+p)x+3p$

valid approach involving equation of axis of symmetry ***(M1)***

eg $\frac{-b}{2a}=2.5,\ \frac{a(3+p)}{2a}=\frac{5}{2},\ \frac{3+p}{2}=\frac{5}{2}$

$p=2$ ***A1 N2***

METHOD 3 (using derivative)

correct derivative (accept absence of a) ***(A1)***

eg $a(2x-3-p),\ 2x-3-p$

valid approach ***(M1)***

eg $f'(2.5)=0$

$p=2$ ***A1 N2***

[3 marks]

3b. *[3 marks]*

Markscheme

attempt to substitute $(0,\ -6)$ ***(M1)***

eg $-6=a(0-2)(0-3),\ 0=a(-8)(-9),\ a(0)^2-5a(0)+6a=-6$

correct working ***(A1)***

eg $-6=6a$

$a=-1$ ***A1 N2***

[3 marks]

3c. *[8 marks]*

Markscheme

METHOD 1 (using discriminant)

recognizing tangent intersects curve once ***(M1)***

recognizing one solution when discriminant = 0 ***M1***

attempt to set up equation ***(M1)***

eg $g = f,\ kx - 5 = -x^2 + 5x - 6$

rearranging their equation to equal zero ***(M1)***

eg $x^2 - 5x + kx + 1 = 0$

correct discriminant (if seen explicitly, not just in quadratic formula) ***A1***

eg $(k-5)^2 - 4,\ 25 - 10k + k^2 - 4$

correct working ***(A1)***

eg $k - 5 = \pm 2,\ (k-3)(k-7) = 0,\ \frac{10 \pm \sqrt{100 - 4 \times 21}}{2}$

$k = 3,\ 7$ ***A1A1 N0***

METHOD 2 (using derivatives)

attempt to set up equation ***(M1)***

eg $g = f,\ kx - 5 = -x^2 + 5x - 6$

recognizing derivative/slope are equal ***(M1)***

eg $f' = m_T,\ f' = k$

correct derivative of f ***(A1)***

eg $-2x + 5$

attempt to set up equation in terms of either x or k ***M1***

eg $(-2x+5)x - 5 = -x^2 + 5x - 6,\ k\left(\frac{5-k}{2}\right) - 5 = -\left(\frac{5-k}{2}\right)^2 + 5\left(\frac{5-k}{2}\right) - 6$

rearranging their equation to equal zero ***(M1)***

eg $x^2 - 1 = 0,\ k^2 - 10k + 21 = 0$

correct working ***(A1)***

eg $x = \pm 1,\ (k-3)(k-7) = 0,\ \frac{10 \pm \sqrt{100 - 4 \times 21}}{2}$

$k = 3,\ 7$ ***A1A1 N0***

[8 marks]

4a. *[2 marks]*

Markscheme

correct approach ***(A1)***

eg $\frac{-(-4)}{2},\ f'(x) = 2x - 4 = 0,\ (x^2 - 4x + 4) + 5 - 4$

$x = 2$ (must be an equation) ***A1 N2***

[2 marks]

4b.

[4 marks]

Markscheme

(i)

$h = 2$ ***A1*** ***N1***

(ii) **METHOD 1**

valid attempt to find k ***(M1)***

eg $f(2)$

correct substitution into **their** function ***(A1)***

eg

$(2)^2 - 4(2) + 5$

$k = 1$ ***A1*** ***N2***

METHOD 2

valid attempt to complete the square ***(M1)***

eg

$x^2 - 4x + 4$

correct working ***(A1)***

eg

$(x^2 - 4x + 4) - 4 + 5,\ (x - 2)^2 + 1$

$k = 1$ ***A1*** ***N2***

[4 marks]

5a.

[2 marks]

Markscheme

recognition that the x-coordinate of the vertex is -1.5 (seen anywhere) ***(M1)***

eg axis of symmetry is -1.5, sketch, $f'(-1.5) = 0$

correct working to find the zeroes ***A1***

eg -1.5 ± 4.5

$x = -6$ and $x = 3$ ***AG*** ***N0***

[2 marks]

5b. *[4 marks]*

Markscheme

METHOD 1 (using factors)

attempt to write factors ***(M1)***

eg $(x-6)(x+3)$

correct factors ***A1***

eg $(x-3)(x+6)$

$q=3,\ r=-18$ ***A1A1*** ***N3***

METHOD 2 (using derivative or vertex)

valid approach to find q ***(M1)***

eg $f'(-1.5)=0,\ -\frac{q}{2a}=-1.5$

$q=3$ ***A1***

correct substitution ***A1***

eg $3^2+3(3)+r=0,\ (-6)^2+3(-6)+r=0$

$r=-18$ ***A1***

$q=3,\ r=-18$ ***N3***

METHOD 3 (solving simultaneously)

valid approach setting up system of two equations ***(M1)***

eg $9+3q+r=0,\ 36-6q+r=0$

one correct value

eg $q=3,\ r=-18$ ***A1***

correct substitution ***A1***

eg $3^2+3(3)+r=0,\ (-6)^2+3(-6)+r=0,\ 3^2+3q-18=0,\ 36-6q-18=0$

second correct value ***A1***

eg $q=3,\ r=-18$

$q=3,\ r=-18$ ***N3***

[4 marks]

6a. *[2 marks]*

Markscheme

$h=3,\ k=-1$ ***A1A1*** ***N2***

[2 marks]

6b. *[2 marks]*

Markscheme

$a=2,\ b=4\ (\text{or } a=4,\ b=2)$ ***A1A1*** ***N2***

[2 marks]

6c. *[2 marks]*

Markscheme

attempt to substitute $x = 0$ into their f ***(M1)***

eg $(0-3)^2 - 1,\ (0-2)(0-4)$

$y = 8$ ***A1 N2***

[2 marks]

7a. *[2 marks]*

Markscheme

valid approach ***(M1)***

eg horizontal translation 3 units to the right

$x = 3$ (must be an equation) ***A1 N2***

[2 marks]

7b. *[2 marks]*

Markscheme

valid approach ***(M1)***

eg $f(x) = 0,\ e^0 = x - 3$

$4,\ x = 4,\ (4,\ 0)$ ***A1 N2***

[2 marks]

7c. *[3 marks]*

Markscheme

attempt to substitute either **their correct** limits or the function into formula involving f^2 ***(M1)***

eg $\int_4^{10} f^2,\ \pi \int (2\ln(x-3))^2 \mathrm{d}x$

141.537

volume = 142 ***A2 N3***

[3 marks]

Total [7 marks]

8a. *[1 mark]*

Markscheme

y-intercept is $-6,\ (0,\ -6),\ y = -6$ ***A1***

[1 mark]

8b.

[3 marks]

Markscheme

valid attempt to solve ***(M1)***

eg $(x-2)(x+3)=0,\ x=\frac{-1\pm\sqrt{1+24}}{2}$, one correct answer

$x=2,\ x=-3$ ***A1A1*** ***N3***

[3 marks]

9a.

[2 marks]

Markscheme

$h=2,\ k=3$ ***A1A1*** ***N2***

[2 marks]

9b.

[3 marks]

Markscheme

attempt to substitute
$(1,7)$ in any order into **their**
$f(x)$ ***(M1)***

eg
$7=a(1-2)^2+3, 7=a(1-3)^2+2, 1=a(7-2)^2+3$

correct equation ***(A1)***

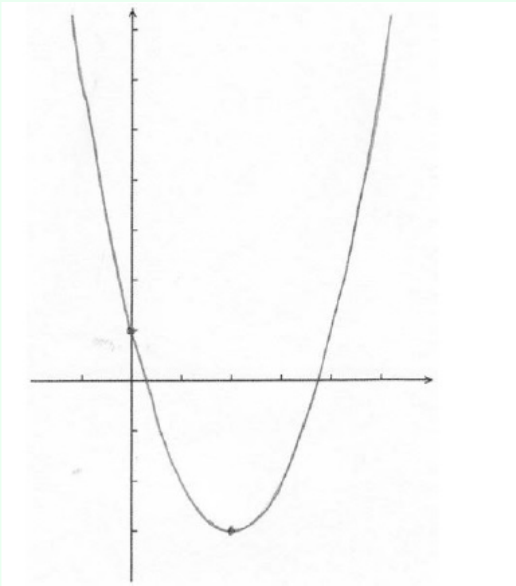
eg
$7=a+3$

a = 4 ***A1*** ***N2***

[3 marks]

10a. *[4 marks]*

Markscheme



A1A1A1A1 N4

Note: The shape **must** be an approximately correct upwards parabola.

Only if the shape is approximately correct, award the following:

A1 for vertex
$x \approx 2$, ***A1*** for *x*-intercepts between 0 and 1, and 3 and 4, ***A1*** for correct *y*-intercept
$(0, 1)$, ***A1*** for correct domain
$[-1, 5]$.

Scale not required on the axes, but approximate positions need to be clear.

[4 marks]

10b. *[1 mark]*

Markscheme

$p = 2$ ***A1 N1***

[1 mark]

10c. *[4 marks]*

Markscheme

correct vertical reflection, correct vertical translation ***(A1)(A1)***

e.g.
$-f(x)$,
$-((x-2)^2-3)$,
$-y$,
$-f(x)+6$,
$y+6$

transformations in correct order ***(A1)***

e.g.
$-(x^2-4x+1)+6$,
$-((x-2)^2-3)+6$

simplification which clearly leads to given answer ***A1***

e.g.
$-x^2+4x-1+6$,
$-(x^2-4x+4-3)+6$

$g(x)=-x^2+4x+5$ ***AG*** ***N0***

Note: If working shown, award ***A1A1A0A0*** if transformations correct, but done in reverse order, e.g. $-(x^2-4x+1+6)$.

[4 marks]

10d. *[3 marks]*

Markscheme

valid approach ***(M1)***

e.g. sketch,
$f=g$

$-0.449489\ldots$,
$4.449489\ldots$

$(2\pm\sqrt{6})$ (exact),
-0.449 $[-0.450, -0.449]$;
4.45 $[4.44, 4.45]$ ***A1A1*** ***N3***

[3 marks]

10e. *[3 marks]*

Markscheme

attempt to substitute limits or functions into area formula (accept absence of $\mathrm{d}x$) ***(M1)***

e.g.
$\int_a^b((-x^2+4x+5)-(x^2-4x+1))\mathrm{d}x$,
$\int_{4.45}^{-0.449}(f-g)$,
$\int(-2x^2+8x+4)\mathrm{d}x$

approach involving subtraction of integrals/areas (accept absence of $\mathrm{d}x$) ***(M1)***

e.g.
$\int_a^b(-x^2+4x+5)-\int_a^b(x^2-4x+1)$,
$\int(f-g)\mathrm{d}x$

$\text{area}=39.19183\ldots$

$\text{area}=39.2$
$[39.1, 39.2]$ ***A1*** ***N3***

[3 marks]

11a. *[3 marks]*

Markscheme

interchanging *x* and *y* (seen anywhere) ***(M1)***

e.g.
$x = 2y - 1$

correct manipulation ***(A1)***

e.g.
$x + 1 = 2y$

$f^{-1}(x) = \frac{x+1}{2}$ ***A1 N2***

[3 marks]

11b. *[3 marks]*

Markscheme

METHOD 1

attempt to find or
$g(1)$ or
$f(1)$ ***(M1)***

$g(1) = 5$ ***(A1)***

$f(5) = 9$ ***A1 N2***

[3 marks]

METHOD 2

attempt to form composite (in any order) ***(M1)***

e.g.
$2(3x^2 + 2) - 1$,
$3(2x - 1)^2 + 2$

$(f \circ g)(1) = 2(3 \times 1^2 + 2) - 1$
$(= 6 \times 1^2 + 3)$ ***(A1)***

$(f \circ g)(1) = 9$ ***A1 N2***

[3 marks]

12a. *[4 marks]*

Markscheme

METHOD 1

evidence of discriminant ***(M1)***

e.g.
$b^2 - 4ac$, discriminant = 0

correct substitution into discriminant ***A1***

e.g.
$k^2 - 4 \times \frac{1}{2} \times 8$,
$k^2 - 16 = 0$

$k = \pm 4$ ***A1A1*** ***N3***

METHOD 2

recognizing that equal roots means perfect square ***(R1)***

e.g. attempt to complete the square,
$\frac{1}{2}(x^2 + 2kx + 16)$

correct working

e.g.
$\frac{1}{2}(x + k)^2$,
$\frac{1}{2}k^2 = 8$ ***A1***

$k = \pm 4$ ***A1A1*** ***N3***

[4 marks]

12b. *[4 marks]*

Markscheme

evidence of appropriate approach ***(M1)***

e.g.
$b^2 - 4ac < 0$

correct working for *k* ***A1***

e.g.
$-4 < k < 4$,
$k^2 < 16$, list all correct values of *k*

$p = \frac{7}{11}$ ***A2*** ***N3***

[4 marks]

13a. *[3 marks]*

Markscheme

interchanging *x* and *y* (may be seen at any time) ***(M1)***

evidence of correct manipulation ***(A1)***

e.g.
$x = 2y + 4$

$f^{-1}(x) = \frac{x-4}{2}$ (accept
$y = \frac{x-4}{2}, \frac{x-4}{2}$) ***A1*** ***N2***

[3 marks]

13b. *[2 marks]*

Markscheme

attempt to form composite (in any order) ***(M1)***

e.g.
$f(7x^2), 2(7x^2)+4, 7(2x+4)^2$

$(f \circ g)(x) = 14x^2 + 4$ ***A1*** ***N2***

13c. *[2 marks]*

Markscheme

correct substitution ***(A1)***

e.g.
7×3.5^2 ,
$14(3.5)^2 + 4$

$(f \circ g)(3.5) = 175.5$ (accept 176) ***A1*** ***N2***

[2 marks]

14a. *[4 marks]*

Markscheme

attempt to apply rules of logarithms ***(M1)***

e.g.
$\ln a^b = b \ln a$,
$\ln ab = \ln a + \ln b$

correct application of
$\ln a^b = b \ln a$ (seen anywhere) ***A1***

e.g.
$3 \ln x = \ln x^3$

correct application of
$\ln ab = \ln a + \ln b$ (seen anywhere) ***A1***

e.g.
$\ln 5x^3 = \ln 5 + \ln x^3$

so
$\ln 5x^3 = \ln 5 + 3 \ln x$

$g(x) = f(x) + \ln 5$ (accept
$g(x) = 3 \ln x + \ln 5$) ***A1*** ***N1***

[4 marks]

14b. *[3 marks]*

Markscheme

transformation with correct name, direction, and value ***A3***

e.g. translation by
$\begin{pmatrix} 0 \\ \ln 5 \end{pmatrix}$, shift up by
$\ln 5$, vertical translation of
$\ln 5$

[3 marks]

15a. *[2 marks]*

Markscheme

substituting (0, 13) into function ***M1***

e.g.
$13 = Ae^0 + 3$

$13 = A + 3$ ***A1***

$A = 10$ ***AG*** ***N0***

[2 marks]

15b. *[3 marks]*

Markscheme

substituting into
$f(15) = 3.49$ ***A1***

e.g.
$3.49 = 10e^{15k} + 3$,
$0.049 = e^{15k}$

evidence of solving equation ***(M1)***

e.g. sketch, using
$\ln$

$k = -0.201$ (accept
$\frac{\ln 0.049}{15}$) ***A1*** ***N2***

[3 marks]

15c. *[5 marks]*

Markscheme

(i)
$f(x) = 10e^{-0.201x} + 3$

$f(x) = 10e^{-0.201x} \times -0.201$
$(= -2.01e^{-0.201x})$ ***A1A1A1*** ***N3***

Note: Award ***A1*** for
$10e^{-0.201x}$, ***A1*** for
$\times -0.201$, ***A1*** for the derivative of 3 is zero.

(ii) valid reason with reference to derivative ***R1*** ***N1***

e.g.
$f'(x) < 0$, derivative always negative

(iii)
$y = 3$ ***A1*** ***N1***

[5 marks]

15d. [6 marks]

Markscheme

finding limits
$3.8953\ldots$,
$8.6940\ldots$ (seen anywhere) ***A1A1***

evidence of integrating and subtracting functions ***(M1)***

correct expression ***A1***

e.g.
$\int_{3.90}^{8.69} g(x) - f(x)\mathrm{d}x$,
$\int_{3.90}^{8.69} \left[\left(-x^2 + 12x - 24\right) - \left(10\mathrm{e}^{-0.201x} + 3\right)\right]\mathrm{d}x$

area
$= 19.5$ ***A2 N4***

[6 marks]

16a. [2 marks]

Markscheme

interchanging
x and
y ***(M1)***

eg
$x = 3y - 2$

$f^{-1}(x) = \frac{x+2}{3}\ \left(\text{accept } y = \frac{x+2}{3}, \frac{x+2}{3}\right)$ ***A1 N2***

[2 marks]

16b. [2 marks]

Markscheme

attempt to form composite (in any order) ***(M1)***

eg
$g\left(\frac{x+2}{3}\right)$, $\frac{\frac{5}{3x}+2}{3}$

correct substitution ***A1***

eg
$\frac{5}{3\left(\frac{x+2}{3}\right)}$

$\left(g \circ f^{-1}\right)(x) = \frac{5}{x+2}$ ***AG N0***

[2 marks]

16c. [2 marks]

Markscheme

valid approach ***(M1)***

eg
$h(0)$, $\frac{5}{0+2}$

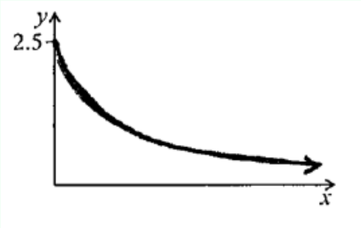
$y = \frac{5}{2}$ (accept $(0, 2.5)$) ***A1 N2***

[2 marks]

16d.

[3 marks]

Markscheme



A1A2 ***N3***

Notes: Award ***A1*** for approximately correct shape (reciprocal, decreasing, concave up).

Only if this ***A1*** is awarded, award ***A2*** for all the following approximately correct features: *y*-intercept at $(0, 2.5)$, asymptotic to *x*-axis, correct domain $x \geqslant 0$.

If only two of these features are correct, award ***A1***.

[3 marks]

16e.

[1 mark]

Markscheme

$x = \frac{5}{2}$ (accept $(2.5, 0)$) ***A1*** ***N1***

[1 mark]

16f.

[1 mark]

Markscheme

$x = 0$ (must be an equation) ***A1*** ***N1***

[1 mark]

16g. *[3 marks]*

Markscheme

METHOD 1

attempt to substitute
3 into
h (seen anywhere) ***(M1)***

eg
$h(3),\ \frac{5}{3+2}$

correct equation ***(A1)***

eg
$a = \frac{5}{3+2},\ h(3) = a$

$a = 1$ ***A1*** ***N2***

[3 marks]

METHOD 2

attempt to find inverse (may be seen in (d)) ***(M1)***

eg
$x = \frac{5}{y+2},\ h^{-1} = \frac{5}{x} - 2,\ \frac{5}{x} + 2$

correct equation,
$\frac{5}{x} - 2 = 3$ ***(A1)***

$a = 1$ ***A1*** ***N2***

[3 marks]