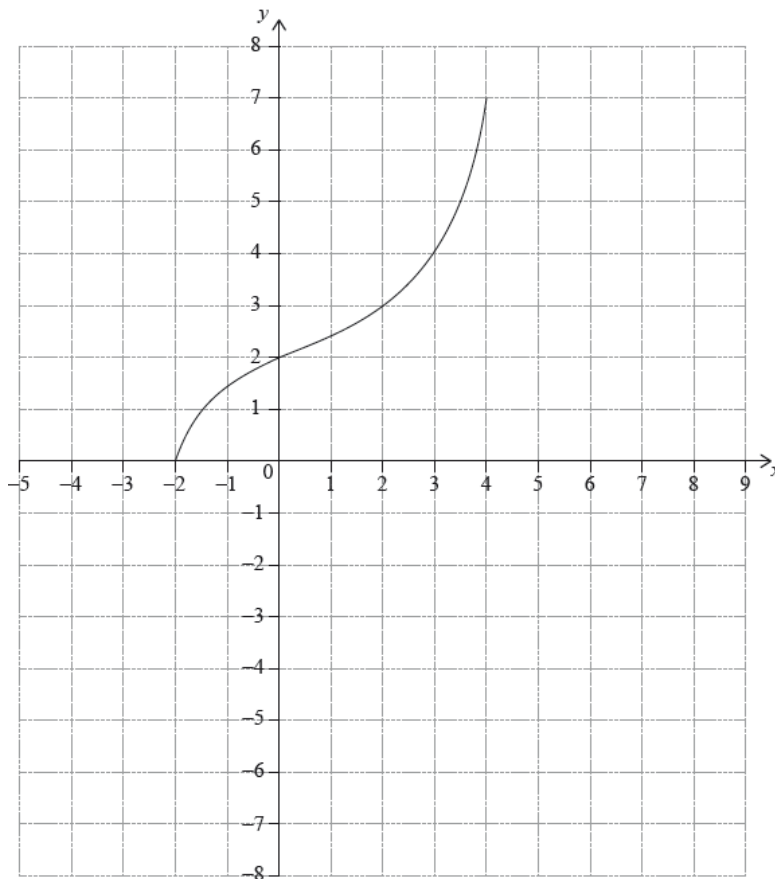


Functions Prep [134 marks]

Let $f(x) = ax^2 - 4x - c$. A horizontal line, L , intersects the graph of f at $x = -1$ and $x = 3$.

- 1a. The equation of the axis of symmetry is $x = p$. Find p . [2 marks]
- 1b. Hence, show that $a = 2$. [2 marks]
- 1c. The equation of L is $y = 5$. Find the value of c . [3 marks]

The following diagram shows the graph of a function f , with domain $-2 \leq x \leq 4$.



The points $(-2, 0)$ and $(4, 7)$ lie on the graph of f .

- 2a. Write down the range of f . [1 mark]
- 2b. Write down $f(2)$; [1 mark]
- 2c. Write down $f^{-1}(2)$. [1 mark]
- 2d. On the grid, sketch the graph of f^{-1} . [3 marks]

A quadratic function f can be written in the form $f(x) = a(x - p)(x - 3)$. The graph of f has axis of symmetry $x = 2.5$ and y -intercept at $(0, -6)$

3a. Find the value of p . [3 marks]

3b. Find the value of a . [3 marks]

3c. The line $y = kx - 5$ is a tangent to the curve of f . Find the values of k . [8 marks]

Let $f(x) = x^2 - 4x + 5$.

4a. Find the equation of the axis of symmetry of the graph of f . [2 marks]

The function can also be expressed in the form $f(x) = (x - h)^2 + k$.

4b. (i) Write down the value of h . [4 marks]

(ii) Find the value of k .

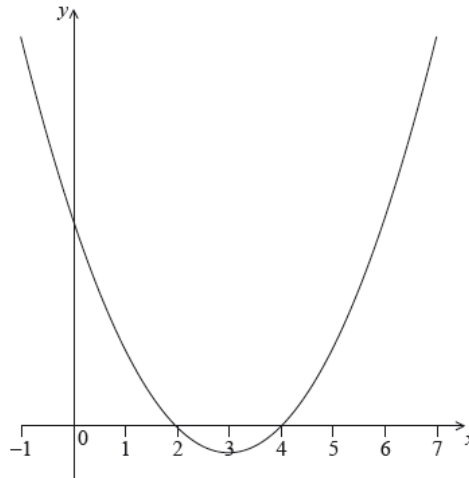
Consider $f(x) = x^2 + qx + r$. The graph of f has a minimum value when $x = -1.5$.

The distance between the two zeros of f is 9.

5a. Show that the two zeros are 3 and -6 . [2 marks]

5b. Find the value of q and of r . [4 marks]

The following diagram shows part of the graph of a quadratic function f .



The vertex is at $(3, -1)$ and the x -intercepts at 2 and 4.

The function f can be written in the form $f(x) = (x - h)^2 + k$.

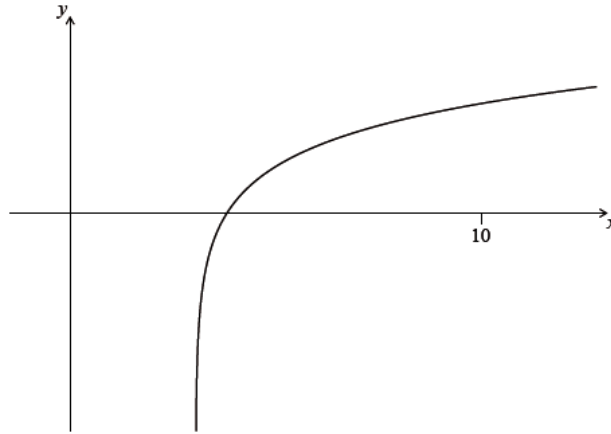
6a. Write down the value of h and of k . [2 marks]

The function can also be written in the form $f(x) = (x - a)(x - b)$.

6b. Write down the value of a and of b . [2 marks]

6c. Find the y -intercept. [2 marks]

Let $f(x) = 2\ln(x - 3)$, for $x > 3$. The following diagram shows part of the graph of f .



- 7a. Find the equation of the vertical asymptote to the graph of f . [2 marks]
- 7b. Find the x -intercept of the graph of f . [2 marks]
- 7c. The region enclosed by the graph of f , the x -axis and the line $x = 10$ is rotated 360° about the x -axis. Find the volume of the solid formed. [3 marks]

Let $f(x) = x^2 + x - 6$.

- 8a. Write down the y -intercept of the graph of f . [1 mark]
- 8b. Solve $f(x) = 0$. [3 marks]

Let $f(x) = a(x - h)^2 + k$. The vertex of the graph of f is at $(2, 3)$ and the graph passes through $(1, 7)$.

- 9a. Write down the value of h and of k . [2 marks]
- 9b. Find the value of a . [3 marks]

Consider the function $f(x) = x^2 - 4x + 1$.

- 10a. Sketch the graph of f , for $-1 \leq x \leq 5$. [4 marks]
- 10b. This function can also be written as $f(x) = (x - p)^2 - 3$. [1 mark]
Write down the value of p .

- 10c. The graph of g is obtained by reflecting the graph of f in the x -axis, followed by a translation of $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$. [4 marks]

Show that

$$g(x) = -x^2 + 4x + 5.$$

- 10d. The graph of g is obtained by reflecting the graph of f in the x -axis, followed by a translation of $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$. [3 marks]

The graphs of f and g intersect at two points.

Write down the x -coordinates of these two points.

- 10e. The graph of g is obtained by reflecting the graph of f in the x -axis, followed by a translation of $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$. [3 marks]

Let R be the region enclosed by the graphs of f and g .

Find the area of R .

Let

$$f(x) = 2x - 1 \text{ and}$$

$$g(x) = 3x^2 + 2.$$

- 11a. Find $f^{-1}(x)$. [3 marks]

- 11b. Find $(f \circ g)(1)$. [3 marks]

Let

$$f(x) = \frac{1}{2}x^2 + kx + 8, \text{ where}$$

$$k \in \mathbb{Z}.$$

- 12a. Find the values of k such that $f(x) = 0$ has two equal roots. [4 marks]

- 12b. Each value of k is equally likely for $-5 \leq k \leq 5$. Find the probability that $f(x) = 0$ has no roots. [4 marks]

Let

$$f(x) = 2x + 4 \text{ and}$$

$$g(x) = 7x^2.$$

- 13a. Find $f^{-1}(x)$. [3 marks]

- 13b. Find $(f \circ g)(x)$. [2 marks]

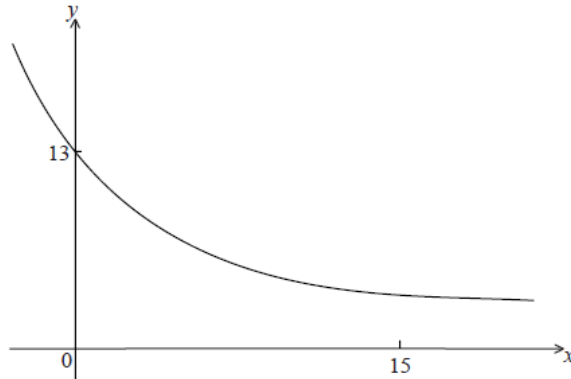
- 13c. Find $(f \circ g)(3.5)$. [2 marks]

Let
 $f(x) = 3 \ln x$ and
 $g(x) = \ln 5x^3$.

14a. Express $g(x)$ in the form $f(x) + \ln a$, where $a \in \mathbb{Z}^+$. [4 marks]

14b. The graph of g is a transformation of the graph of f . Give a full geometric description of this transformation. [3 marks]

Let
 $f(x) = Ae^{kx} + 3$. Part of the graph of f is shown below.



The y -intercept is at $(0, 13)$.

15a. Show that $A = 10$. [2 marks]

15b. Given that $f(15) = 3.49$ (correct to 3 significant figures), find the value of k . [3 marks]

15c. (i) Using your value of k , find $f'(x)$. [5 marks]
 (ii) Hence, explain why f is a decreasing function.
 (iii) Write down the equation of the horizontal asymptote of the graph f .

15d. Let $g(x) = -x^2 + 12x - 24$. [6 marks]
 Find the area enclosed by the graphs of f and g .

Let
 $f(x) = 3x - 2$ and
 $g(x) = \frac{5}{3x}$, for
 $x \neq 0$.

16a. Find $f^{-1}(x)$. [2 marks]

16b. Show that $(g \circ f^{-1})(x) = \frac{5}{x+2}$. [2 marks]

Let

$$h(x) = \frac{5}{x+2}, \text{ for}$$

$x \geq 0$. The graph of h has a horizontal asymptote at

$$y = 0.$$

- 16c. Find the y -intercept of the graph of h . [2 marks]
- 16d. Hence, sketch the graph of h . [3 marks]
- 16e. For the graph of h^{-1} , write down the x -intercept; [1 mark]
- 16f. For the graph of h^{-1} , write down the equation of the vertical asymptote. [1 mark]
- 16g. Given that $h^{-1}(a) = 3$, find the value of a . [3 marks]