# Trig double angle identities *[203 marks]*

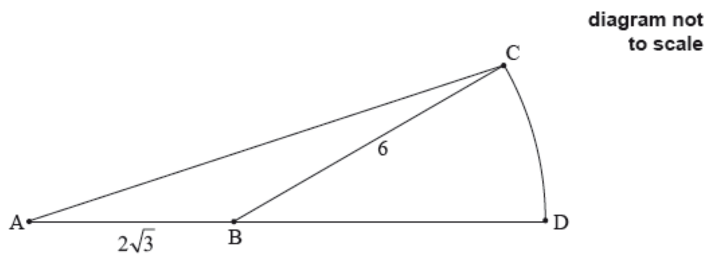
1. Solve $\log_2(2\sin x) + \log_2(\cos x) = -1$, for $2\pi < x < \frac{5\pi}{2}$. *[7 marks]*

Let $\sin\theta = \frac{\sqrt{5}}{3}$, where $\theta$ is acute.

2a. Find $\cos\theta$. *[3 marks]*

2b. Find $\cos 2\theta$. *[2 marks]*

The following diagram shows a triangle ABC and a sector BDC of a circle with centre B and radius 6 cm. The points A , B and D are on the same line.




$\text{AB} = 2\sqrt{3}\text{ cm}, \text{BC} = 6\text{ cm}, \text{area of triangle ABC} = 3\sqrt{3}\text{ cm}^2, \text{A}\hat{\text{B}}\text{C}$ is obtuse.

3a. Find $\text{A}\hat{\text{B}}\text{C}$. *[5 marks]*

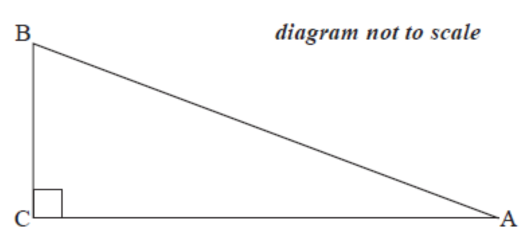
3b. Find the exact area of the sector BDC. *[3 marks]*

Given that
$\sin x = \frac{3}{4}$, where $x$ is an obtuse angle,

4a. find the value of $\cos x$; *[4 marks]*

4b. find the value of $\cos 2x$. *[3 marks]*

The following diagram shows a right-angled triangle,
$\text{ABC}$, where
$\sin \text{A} = \frac{5}{13}$.




5a. Show that
$\cos A = \frac{12}{13}$. *[2 marks]*

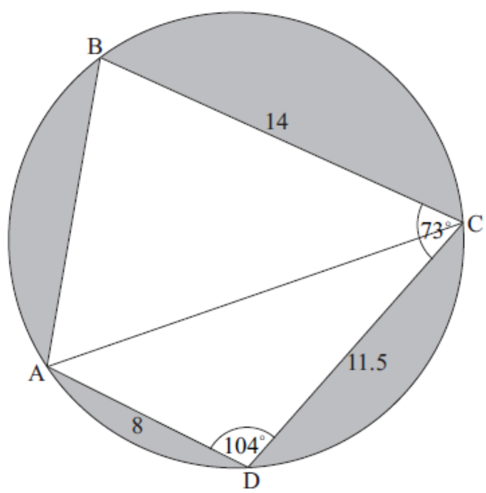
5b. Find $\cos 2A$. *[3 marks]*

In triangle $\mathrm{ABC}$, $\mathrm{AB} = 6\,\mathrm{cm}$ and $\mathrm{AC} = 8\,\mathrm{cm}$. The area of the triangle is $16\,\mathrm{cm}^2$.

6a. Find the two possible values for $\hat{A}$. *[4 marks]*

6b. Given that $\hat{A}$ is obtuse, find $\mathrm{BC}$. *[3 marks]*

The diagram shows a circle of radius 8 metres. The points ABCD lie on thecircumference of the circle.




BC = 14 m, CD = 11.5 m, AD = 8 m, $A\hat{D}C = 104^\circ$ , and $B\hat{C}D = 73^\circ$ .

7a. Find AC. *[3 marks]*

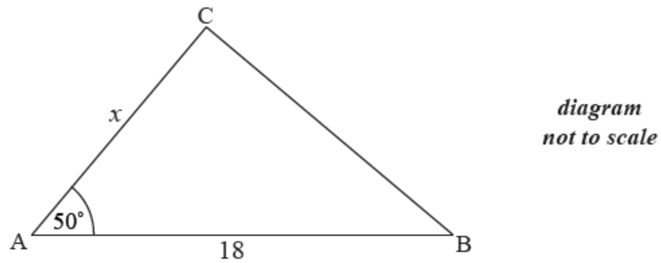
7b. (i) Find $A\hat{C}D$ . *[5 marks]*

(ii) Hence, find $A\hat{C}B$ .

7c. Find the area of triangle ADC. *[2 marks]*

7d. Hence or otherwise, find the total area of the shaded regions. *[4 marks]*

The following diagram shows a triangle ABC.




The area of triangle ABC is $80$ cm$^2$ , AB $= 18$ cm , AC $= x$ cm and $\mathrm{B\hat{A}C} = 50^\circ$ .

8a. Find $x$ . *[3 marks]*

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8b. Find BC. *[3 marks]*

9a. Let $\sin 100^\circ = m$. Find an expression for $\cos 100^\circ$ in terms of *m*. *[3 marks]*

9b. Let $\sin 100^\circ = m$ . Find an expression for $\tan 100^\circ$ in terms of *m*. *[1 mark]*

9c. Let $\sin 100^\circ = m$. Find an expression for $\sin 200^\circ$ in terms of *m*. *[2 marks]*

Let $f(x) = (\sin x + \cos x)^2$ .

10a. Show that $f(x)$ can be expressed as $1 + \sin 2x$ . *[2 marks]*

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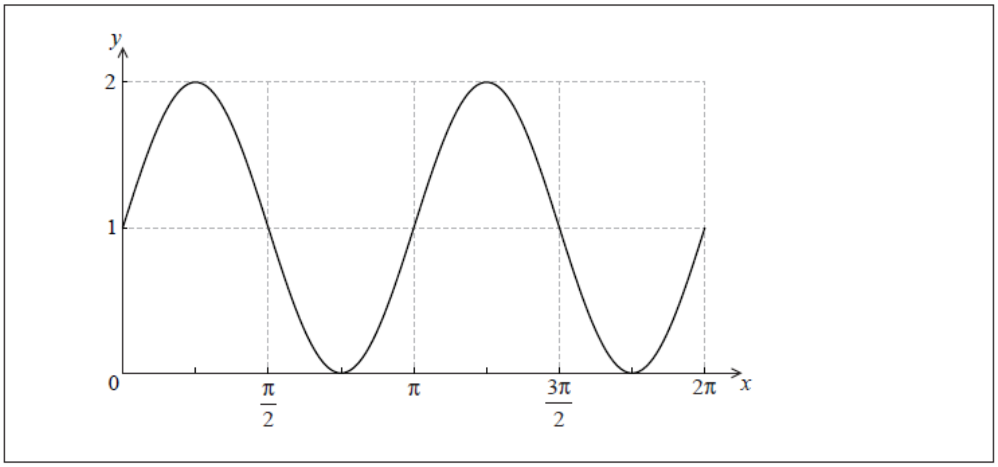
10b. The graph of *f* is shown below for $0 \le x \le 2\pi$ . *[2 marks]*




Let $g(x) = 1 + \cos x$ . On the same set of axes, sketch the graph of *g* for $0 \le x \le 2\pi$ .

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10c. The graph of *g* can be obtained from the graph of *f* under a horizontal stretch of scale factor *p* followed by a translation by the vector $\begin{pmatrix} k \\ 0 \end{pmatrix}$ . *[2 marks]*

Write down the value of *p* and a possible value of *k* .

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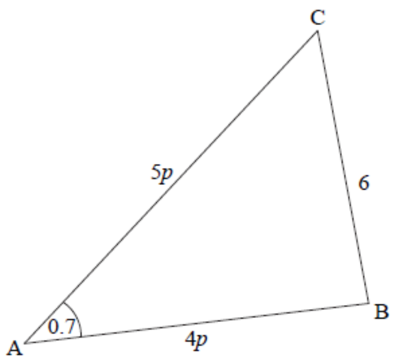
The following diagram shows a triangle ABC.

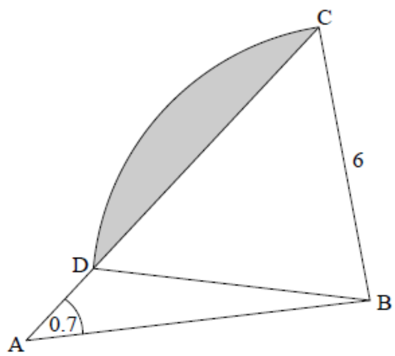



$\mathrm{BC} = 6$ ,
$\mathrm{C\widehat{A}B} = 0.7$ radians ,
$\mathrm{AB} = 4p$ ,
$\mathrm{AC} = 5p$ , where
$p > 0$ .

11a. (i) Show that $p^2(41 - 40\cos 0.7) = 36$ . *[4 marks]*

(ii) Find *p* .

Consider the circle with centre B that passes through the point C. The circle cuts the line CA at D, and $\mathrm{A\hat{D}B}$ is obtuse. Part of the circle is shown in the following diagram.



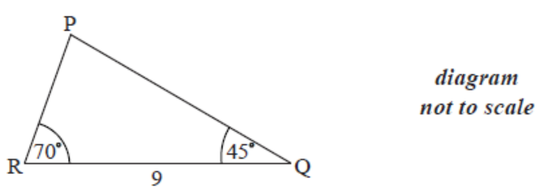

11b. Write down the length of BD. *[1 mark]*

11c. Find $\mathrm{A\hat{D}B}$ . *[4 marks]*

11d. (i) Show that $\mathrm{C\hat{B}D} = 1.29$ radians, correct to 2 decimal places. *[6 marks]*

(ii) Hence, find the area of the shaded region.

The following diagram shows $\Delta\mathrm{PQR}$ , where RQ = 9 cm, $\mathrm{P\hat{R}Q} = 70^{\circ}$ and $\mathrm{P\hat{Q}R} = 45^{\circ}$ .




12a. Find $\mathrm{R\hat{P}Q}$ . *[1 mark]*

12b. Find PR . *[3 marks]*

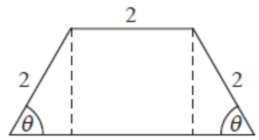
12c. Find the area of $\Delta\mathrm{PQR}$ . *[2 marks]*

Let $\sin\theta = \frac{2}{\sqrt{13}}$ , where $\frac{\pi}{2} < \theta < \pi$ .

13a. Find $\cos\theta$ . *[3 marks]*

13b. Find $\tan 2\theta$ . *[5 marks]*

The diagram below shows a plan for a window in the shape of a trapezium.




Three sides of the window are $2\text{ m}$ long. The angle between the sloping sides of thewindow and the base is $\theta$ , where $0 < \theta < \frac{\pi}{2}$ .

14a. Show that the area of the window is given by $y = 4\sin\theta + 2\sin 2\theta$ . *[5 marks]*

14b. Zoe wants a window to have an area of $5\text{ m}^2$. Find the two possible values of $\theta$ . *[4 marks]*

14c. John wants two windows which have the same area *A* but different values of $\theta$ . *[7 marks]*

Find all possible values for *A* .

15a. Show that $4 - \cos 2\theta + 5\sin\theta = 2\sin^2\theta + 5\sin\theta + 3$ . *[2 marks]*

15b. **Hence**, solve the equation $4 - \cos 2\theta + 5\sin\theta = 0$ for $0 \le \theta \le 2\pi$ . *[5 marks]*

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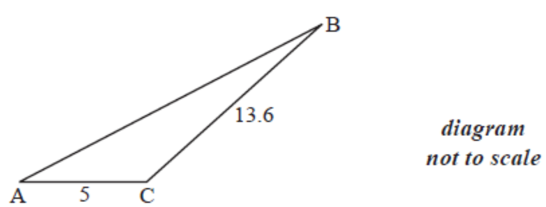
The following diagram shows the triangle ABC.




The angle at C is obtuse, $\text{AC} = 5\text{ cm}$, $\text{BC} = 13.6\text{ cm}$ and the area is $20\text{ cm}^2$ .

16a. Find $\text{A}\hat{\text{C}}\text{B}$ . *[4 marks]*

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16b. Find AB. *[3 marks]*

The straight line with equation $y = \frac{3}{4}x$ makes an acute angle $\theta$ with the *x*-axis.

17a. Write down the value of $\tan\theta$ . *[1 mark]*

17b. Find the value of [6 marks]

(i) $\sin 2\theta$ ;

(ii) $\cos 2\theta$ .

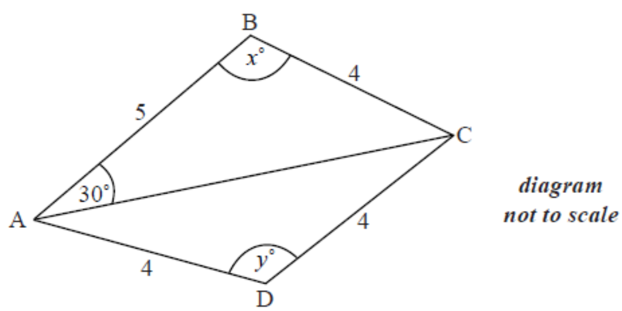
Let $f(x) = \cos 2x$ and $g(x) = 2x^2 - 1$ .

18a. Find $f\left(\frac{\pi}{2}\right)$ . [2 marks]

18b. Find $(g \circ f)\left(\frac{\pi}{2}\right)$ . [2 marks]

18c. Given that $(g \circ f)(x)$ can be written as $\cos(kx)$ , find the value of *k*, $k \in \mathbb{Z}$ . [3 marks]

The diagram below shows a quadrilateral ABCD with obtuse angles $\mathrm{A\hat{B}C}$ and $\mathrm{A\hat{D}C}$.




AB = 5 cm, BC = 4 cm, CD = 4 cm, AD = 4 cm ,
$\mathrm{B\hat{A}C} = 30^\circ$ ,
$\mathrm{A\hat{B}C} = x^\circ$ ,
$\mathrm{A\hat{D}C} = y^\circ$ .

19a. Use the cosine rule to show that $\mathrm{AC} = \sqrt{41 - 40\cos x}$ . [1 mark]

19b. Use the sine rule in triangle ABC to find another expression for AC. [2 marks]

19c. (i) Hence, find *x*, giving your answer to two decimal places. [6 marks]

(ii) Find AC .

19d. (i) Find *y*. [5 marks]

(ii) Hence, or otherwise, find the area of triangle ACD.

20. Solve $\cos 2x - 3\cos x - 3 - \cos^2 x = \sin^2 x$ , for $0 \le x \le 2\pi$ . [7 marks]

Let
$f(x) = \sin^3 x + \cos^3 x \tan x, \frac{\pi}{2} < x < \pi$ .

21a. Show that
$f(x) = \sin x$ . *[2 marks]*

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21b. Let
$\sin x = \frac{2}{3}$ . Show that
$f(2x) = -\frac{4\sqrt{5}}{9}$ . *[5 marks]*

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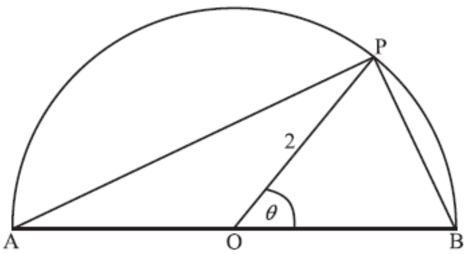
The following diagram shows a semicircle centre O, diameter [AB], with radius 2.

Let P be a point on the circumference, with
$\mathrm{P\widehat{O}B} = \theta$ radians.




22a. Find the area of the triangle OPB, in terms of
$\theta$ . *[2 marks]*

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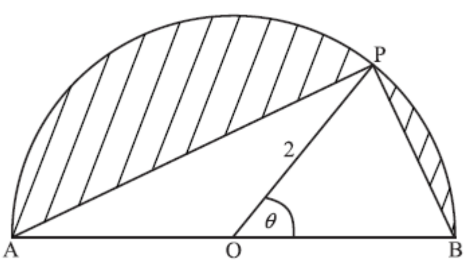
22b. Explain why the area of triangle OPA is the same as the area triangle OPB. *[3 marks]*

Let *S* be the total area of the two segments shaded in the diagram below.




22c. Show that
$S = 2(\pi - 2\sin\theta)$ . *[3 marks]*

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22d. Find the value of
$\theta$ when *S* is a local minimum, justifying that it is a minimum. *[8 marks]*

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22e. Find a value of
$\theta$ for which *S* has its greatest value. *[2 marks]*

23a. Given that
$\cos A = \frac{1}{3}$ and
$0 \le A \le \frac{\pi}{2}$ , find
$\cos 2A$ . *[3 marks]*

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23b. Given that $\sin B = \frac{2}{3}$ and $\frac{\pi}{2} \le B \le \pi$, find $\cos B$. *[3 marks]*

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The expression $6 \sin x \cos x$ can be expressed in the form $a \sin bx$.

24a. Find the value of *a* and of *b*. *[3 marks]*

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24b. Hence or otherwise, solve the equation $6 \sin x \cos x = \frac{3}{2}$, for $\frac{\pi}{4} \le x \le \frac{\pi}{2}$. *[4 marks]*

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