Cool review of ch 1, 2, 3 *[226 marks]*

Let $b = \log_2 a$, where $a > 0$. Write down each of the following expressions in terms of b.

1a. $\log_2 a^3$ *[2 marks]*

Markscheme

correct approach ***(A1)***

eg $3\log_2 a$

$\log_2 a^3 = 3b$ ***A1 N2***

[2 marks]

1b. $\log_2 8a$ *[2 marks]*

Markscheme

correct working ***(A1)***

eg $\log_2 8 + \log_2 a$, $\log_2 8 = 3$

$\log_2 8a = 3 + b$ ***A1 N2***

[2 marks]

1c. $\log_8 a$ *[2 marks]*

Markscheme

correct working ***(A1)***

eg $\frac{\log_2 a}{\log_2 8}$, $\frac{1}{3}\log_2 a$, $b\log_8 2$

$\log_8 a = \frac{b}{3}$ ***A1 N2***

[2 marks]

2. Solve $\log_2(2\sin x) + \log_2(\cos x) = -1$, for $2\pi < x < \frac{5\pi}{2}$. *[7 marks]*

Markscheme

correct application of $\log a + \log b = \log ab$ ***(A1)***

eg $\log_2(2\sin x\cos x),\ \log 2 + \log(\sin x) + \log(\cos x)$

correct equation without logs ***A1***

eg $2\sin x\cos x = 2^{-1},\ \sin x\cos x = \frac{1}{4},\ \sin 2x = \frac{1}{2}$

recognizing double-angle identity (seen anywhere) ***A1***

eg $\log(\sin 2x),\ 2\sin x\cos x = \sin 2x,\ \sin 2x = \frac{1}{2}$

evaluating $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}\ (30°)$ ***(A1)***

correct working ***A1***

eg $x = \frac{\pi}{12} + 2\pi,\ 2x = \frac{25\pi}{6},\ \frac{29\pi}{6},\ 750°,\ 870°,\ x = \frac{\pi}{12}$ **and** $x = \frac{5\pi}{12}$, one correct final answer

$x = \frac{25\pi}{12},\ \frac{29\pi}{12}$ (do not accept additional values) ***A2*** ***N0***

[7 marks]

Let $f'(x) = \frac{6-2x}{6x-x^2}$, for $0 < x < 6$.

The graph of f has a maximum point at P.

3a. Find the x-coordinate of P. *[3 marks]*

Markscheme

recognizing $f'(x) = 0$ ***(M1)***

correct working ***(A1)***

eg $6 - 2x = 0$

$x = 3$ ***A1*** ***N2***

[3 marks]

The y-coordinate of P is $\ln 27$.

3b. Find $f(x)$, expressing your answer as a single logarithm. *[8 marks]*

Markscheme

evidence of integration ***(M1)***

eg $\int f',\ \int \frac{6-2x}{6x-x^2}\mathrm{d}x$

using substitution ***(A1)***

eg $\int \frac{1}{u}\mathrm{d}u$ where $u = 6x - x^2$

correct integral ***A1***

eg $\ln(u) + c,\ \ln(6x - x^2)$

substituting $(3,\ \ln 27)$ into **their** integrated expression (must have c) ***(M1)***

eg $\ln(6 \times 3 - 3^2) + c = \ln 27,\ \ln(18 - 9) + \ln k = \ln 27$

correct working ***(A1)***

eg $c = \ln 27 - \ln 9$

EITHER

$c = \ln 3$ ***(A1)***

attempt to substitute **their** value of c into $f(x)$ ***(M1)***

eg $f(x) = \ln(6x - x^2) + \ln 3$ ***A1*** ***N4***

OR

attempt to substitute **their** value of c into $f(x)$ ***(M1)***

eg $f(x) = \ln(6x - x^2) + \ln 27 - \ln 9$

correct use of a log law ***(A1)***

eg $f(x) = \ln(6x - x^2) + \ln\left(\frac{27}{9}\right),\ f(x) = \ln\left(27(6x - x^2)\right) - \ln 9$

$f(x) = \ln\left(3(6x - x^2)\right)$ ***A1*** ***N4***

[8 marks]

3c. The graph of f is transformed by a vertical stretch with scale factor $\frac{1}{\ln 3}$. The image of P under this transformation has coordinates $(a,\ b)$.

Find the value of a and of b, where $a,\ b \in \mathbb{N}$.

Markscheme

$a = 3$ ***A1*** ***N1***

correct working ***A1***

eg $\frac{\ln 27}{\ln 3}$

correct use of log law ***(A1)***

eg $\frac{3\ln 3}{\ln 3}$, $\log_3 27$

$b = 3$ ***A1*** ***N2***

[4 marks]

Let $x = \ln 3$ and $y = \ln 5$. Write the following expressions in terms of x and y.

4a. $\ln\left(\frac{5}{3}\right)$. *[2 marks]*

Markscheme

correct approach ***(A1)***

eg $\ln 5 - \ln 3$

$\ln\left(\frac{5}{3}\right) = y - x$ ***A1*** ***N2***

[2 marks]

4b. $\ln 45$. *[4 marks]*

Markscheme

recognizing factors of 45 (may be seen in log expansion) ***(M1)***

eg $\ln(9 \times 5)$, $3 \times 3 \times 5$, $\log 3^2 \times \log 5$

correct application of $\log(ab) = \log a + \log b$ ***(A1)***

eg $\ln 9 + \ln 5$, $\ln 3 + \ln 3 + \ln 5$, $\ln 3^2 + \ln 5$

correct working ***(A1)***

eg $2\ln 3 + \ln 5$, $x + x + y$

$\ln 45 = 2x + y$ ***A1*** ***N3***

[4 marks]

Let
$\log_3 p = 6$ and
$\log_3 q = 7$.

5a. Find $\log_3 p^2$. *[2 marks]*

Markscheme

METHOD 1

evidence of correct formula ***(M1)***

eg $\log u^n = n \log u$, $2\log_3 p$

$\log_3 (p^2) = 12$ ***A1*** ***N2***

METHOD 2

valid method using $p = 3^6$ ***(M1)***

eg $\log_3 (3^6)^2$, $\log 3^{12}$, $12\log_3 3$

$\log_3 (p^2) = 12$ ***A1*** ***N2***

[2 marks]

5b. Find $\log_3 \left(\frac{p}{q}\right)$. *[2 marks]*

Markscheme

METHOD 1

evidence of correct formula ***(M1)***

eg $\log\left(\frac{p}{q}\right) = \log p - \log q$, $6 - 7$

$\log_3 \left(\frac{p}{q}\right) = -1$ ***A1*** ***N2***

METHOD 2

valid method using $p = 3^6$ and $q = 3^7$ ***(M1)***

eg $\log_3 \left(\frac{3^6}{3^7}\right)$, $\log 3^{-1}$, $-\log_3 3$

$\log_3 \left(\frac{p}{q}\right) = -1$ ***A1*** ***N2***

[2 marks]

5c. Find $\log_3 (9p)$. *[3 marks]*

Markscheme

METHOD 1

evidence of correct formula ***(M1)***

eg $\log_3 uv = \log_3 u + \log_3 v$, $\log 9 + \log p$

$\log_3 9 = 2$ (may be seen in expression) ***A1***

eg $2 + \log p$

$\log_3(9p) = 8$ ***A1*** ***N2***

METHOD 2

valid method using $p = 3^6$ ***(M1)***

eg $\log_3(9 \times 3^6)$, $\log_3(3^2 \times 3^6)$

correct working ***A1***

eg $\log_3 9 + \log_3 3^6$, $\log_3 3^8$

$\log_3(9p) = 8$ ***A1*** ***N2***

[3 marks]

Total [7 marks]

Let $f(x) = 1 + \mathrm{e}^{-x}$ and $g(x) = 2x + b$, for $x \in \mathbb{R}$, where b is a constant.

6a. Find $(g \circ f)(x)$. *[2 marks]*

Markscheme

attempt to form composite ***(M1)***

eg $g(1 + \mathrm{e}^{-x})$

correct function ***A1*** ***N2***

eg $(g \circ f)(x) = 2 + b + 2\mathrm{e}^{-x},\ 2(1 + \mathrm{e}^{-x}) + b$

[2 marks]

6b. Given that $\lim_{x \to +\infty}(g \circ f)(x) = -3$, find the value of b. *[4 marks]*

Markscheme

evidence of $\lim_{x\to\infty}(2+b+2e^{-x}) = 2+b+\lim_{x\to\infty}(2e^{-x})$ ***(M1)***

eg $2+b+2e^{-\infty}$, graph with horizontal asymptote when $x\to\infty$

Note: Award ***M0*** if candidate clearly has incorrect limit, such as $x\to 0,\ e^{\infty},\ 2e^{0}$.

evidence that $e^{-x}\to 0$ (seen anywhere) ***(A1)***

eg $\lim_{x\to\infty}(e^{-x})=0,\ 1+e^{-x}\to 1,\ 2(1)+b=-3,\ e^{\text{large negative number}}\to 0$, graph of $y=e^{-x}$ or

$y=2e^{-x}$ with asymptote $y=0$, graph of composite function with asymptote $y=-3$

correct working ***(A1)***

eg $2+b=-3$

$b=-5$ ***A1 N2***

[4 marks]

Let $f(x)=8x+3$ and $g(x)=4x$, for $x\in\mathbb{R}$.

7a. Write down $g(2)$. *[1 mark]*

Markscheme

$g(2)=8$ ***A1 N1***

[1 mark]

7b. Find $(f\circ g)(x)$. *[2 marks]*

Markscheme

attempt to form composite (in any order) ***(M1)***

eg $f(4x),\ 4\times(8x+3)$

$(f\circ g)(x)=32x+3$ ***A1 N2***

[2 marks]

7c. Find $f^{-1}(x)$. *[2 marks]*

Markscheme

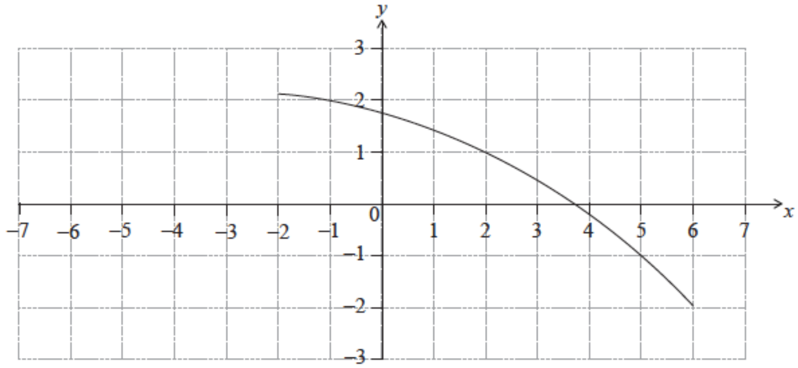
interchanging x and y (may be seen at any time) ***(M1)***

eg $x = 8y + 3$

$f^{-1}(x) = \frac{x-3}{8}$ $\left(\text{accept } \frac{x-3}{8},\ y = \frac{x-3}{8}\right)$ ***A1*** ***N2***

[2 marks]

The following diagram shows the graph of a function f.



8a. Find $f^{-1}(-1)$. *[2 marks]*

Markscheme

valid approach ***(M1)***

eg horizontal line on graph at $-1,\ f(a) = -1,\ (-1, 5)$

$f^{-1}(-1) = 5$ ***A1*** ***N2***

[2 marks]

8b. Find $(f \circ f)(-1)$. *[3 marks]*

Markscheme

attempt to find $f(-1)$ ***(M1)***

eg line on graph

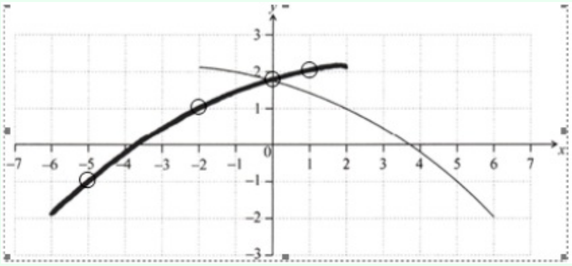
$f(-1) = 2$ ***(A1)***

$(f \circ f)(-1) = 1$ ***A1*** ***N3***

[3 marks]

8c. On the same diagram, sketch the graph of $y = f(-x)$. *[2 marks]*

Markscheme



A1A1 *N2*

Note: The shape **must** be an approximately correct shape (concave down and increasing). **Only** if the shape is approximately correct, award the following for points in circles:

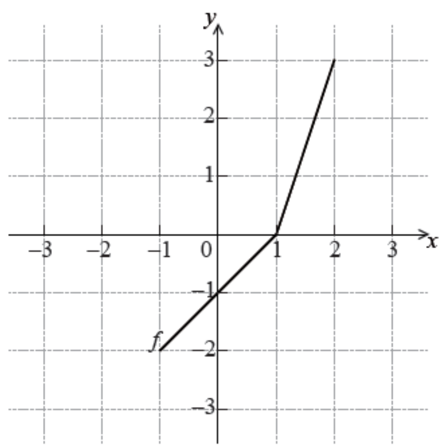
A1 for the y-intercept,

A1 for any **two** of these points $(-5,\ -1),\ (-2,\ 1),\ (1,\ 2)$.

[2 marks]

Total [7 marks]

The diagram below shows the graph of a function f, for $-1 \le x \le 2$.



9a. Write down the value of $f(2)$. *[1 mark]*

Markscheme

$f(2) = 3$ ***A1*** ***N1***

[1 mark]

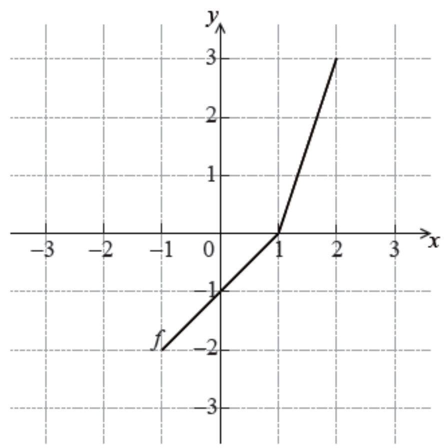
9b. Write down the value of $f^{-1}(-1)$. *[2 marks]*

Markscheme

$f^{-1}(-1) = 0$ ***A2*** ***N2***

[2 marks]

9c. Sketch the graph of f^{-1} on the grid below. *[3 marks]*



Markscheme

EITHER

attempt to draw $y = x$ on grid ***(M1)***

OR

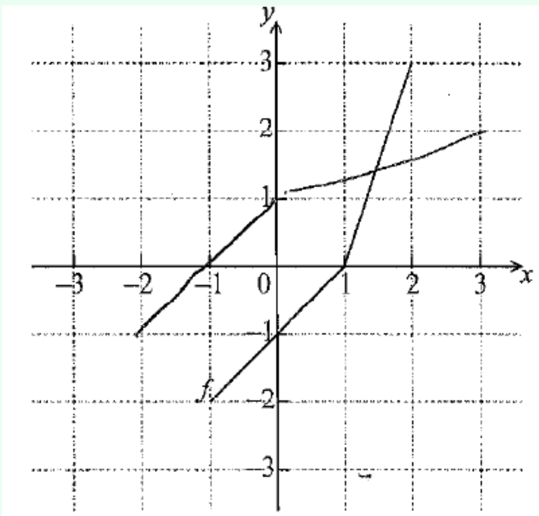
attempt to reverse ***x*** and ***y*** coordinates ***(M1)***

eg writing or plotting **at least two** of the points

$(-2, -1)$, $(-1, 0)$, $(0, 1)$, $(3, 2)$

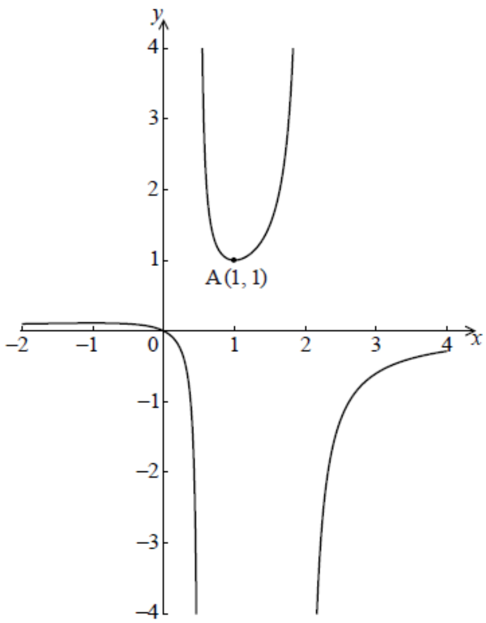
THEN

correct graph ***A2*** ***N3***



[3 marks]

Let
$f(x) = \frac{x}{-2x^2+5x-2}$ for
$-2 \leq x \leq 4$,
$x \neq \frac{1}{2}$,
$x \neq 2$. The graph of
f is given below.



The graph of
f has a local minimum at A(
1,
1) and a local maximum at B.

10a. Use the quotient rule to show that $f'(x) = \frac{2x^2-2}{(-2x^2+5x-2)^2}$. *[6 marks]*

Markscheme

correct derivatives **applied** in quotient rule ***(A1)A1A1***

$1,\ -4x+5$

Note: Award ***(A1)*** for 1, ***A1*** for $-4x$ and ***A1*** for 5, **only** if it is clear candidates are using the quotient rule.

correct substitution into quotient rule ***A1***

e.g. $\frac{1\times(-2x^2+5x-2)-x(-4x+5)}{(-2x^2+5x-2)^2}$, $\frac{-2x^2+5x-2-x(-4x+5)}{(-2x^2+5x-2)^2}$

correct working ***(A1)***

e.g. $\frac{-2x^2+5x-2-(-4x^2+5x)}{(-2x^2+5x-2)^2}$

expression clearly leading to the answer ***A1***

e.g. $\frac{-2x^2+5x-2+4x^2-5x}{(-2x^2+5x-2)^2}$

$f'(x)=\frac{2x^2-2}{(-2x^2+5x-2)^2}$ ***AG*** ***N0***

[6 marks]

10b. Hence find the coordinates of B. *[7 marks]*

Markscheme

evidence of attempting to solve $f'(x)=0$ ***(M1)***

e.g. $2x^2-2=0$

evidence of correct working ***A1***

e.g. $x^2=1, \frac{\pm\sqrt{16}}{4}, 2(x-1)(x+1)$

correct solution to quadratic ***(A1)***

e.g. $x=\pm1$

correct *x*-coordinate $x=-1$ (may be seen in coordinate form $\left(-1,\frac{1}{9}\right)$) ***A1*** ***N2***

attempt to substitute -1 into *f* (do not accept any other value) ***(M1)***

e.g. $f(-1)=\frac{-1}{-2\times(-1)^2+5\times(-1)-2}$

correct working

e.g. $\frac{-1}{-2-5-2}$ ***A1***

correct *y*-coordinate $y=\frac{1}{9}$ (may be seen in coordinate form $\left(-1,\frac{1}{9}\right)$) ***A1*** ***N2***

[7 marks]

10c. Given that the line $y=k$ does not meet the graph of *f* , find the possible values of *k* . *[3 marks]*

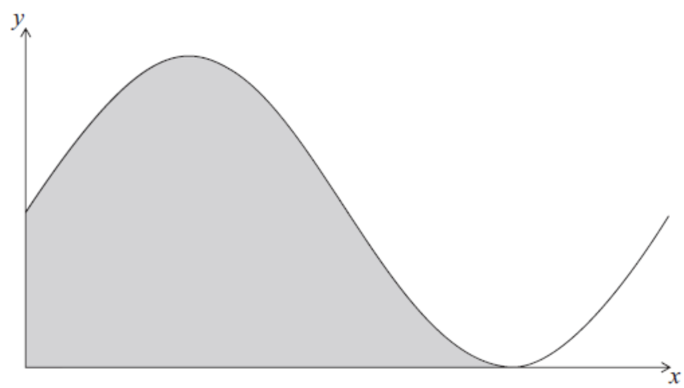
Markscheme

recognizing values between max and min ***(R1)***

$\frac{1}{9} < k < 1$ ***A2 N3***

[3 marks]

Let
$f(x) = 6 + 6\sin x$. Part of the graph of *f* is shown below.



The shaded region is enclosed by the curve of *f* , the *x*-axis, and the *y*-axis.

11a. Solve for $0 \le x < 2\pi$ *[5 marks]*

(i) $6 + 6\sin x = 6$;

(ii) $6 + 6\sin x = 0$.

Markscheme

(i) $\sin x = 0$ ***A1***

$x = 0$, $x = \pi$ ***A1A1 N2***

(ii) $\sin x = -1$ ***A1***

$x = \frac{3\pi}{2}$ ***A1 N1***

[5 marks]

11b. Write down the exact value of the *x*-intercept of *f* , for $0 \le x < 2\pi$. *[1 mark]*

Markscheme

$\frac{3\pi}{2}$ ***A1 N1***

[1 mark]

11c. The area of the shaded region is k. Find the value of k, giving your answer in terms of π. *[6 marks]*

Markscheme

evidence of using anti-differentiation ***(M1)***

e.g. $\int_0^{\frac{3\pi}{2}}(6+6\sin x)\mathrm{d}x$

correct integral $6x-6\cos x$ (seen anywhere) ***A1A1***

correct substitution ***(A1)***

e.g. $6\left(\frac{3\pi}{2}\right)-6\cos\left(\frac{3\pi}{2}\right)-(-6\cos 0)$, $9\pi-0+6$

$k=9\pi+6$ ***A1A1*** ***N3***

[6 marks]

11d. Let $g(x)=6+6\sin\left(x-\frac{\pi}{2}\right)$. The graph of *f* is transformed to the graph of *g*. *[2 marks]*

Give a full geometric description of this transformation.

Markscheme

translation of $\begin{pmatrix}\frac{\pi}{2}\\0\end{pmatrix}$ ***A1A1*** ***N2***

[2 marks]

11e. Let $g(x)=6+6\sin\left(x-\frac{\pi}{2}\right)$. The graph of *f* is transformed to the graph of *g*. *[3 marks]*

Given that $\int_p^{p+\frac{3\pi}{2}}g(x)\mathrm{d}x=k$ and $0\le p<2\pi$, write down the two values of *p*.

Markscheme

recognizing that the area under *g* is the same as the shaded region in *f* ***(M1)***

$p=\frac{\pi}{2}$, $p=0$ ***A1A1*** ***N3***

[3 marks]

Let $f(x)=3x^2-6x+p$. The equation $f(x)=0$ has two equal roots.

12a. Write down the **value** of the discriminant. *[2 marks]*

Markscheme

correct value 0, or $36-12p$ ***A2*** ***N2***

[2 marks]

12b. Hence, show that $p=3$. *[1 mark]*

Markscheme

correct equation which clearly leads to $p=3$ ***A1***

eg $36-12p=0,\ 36=12p$

$p=3$ ***AG*** ***N0***

[1 mark]

12c. The graph of f has its vertex on the x-axis. *[4 marks]*

Find the coordinates of the vertex of the graph of f.

Markscheme

METHOD 1

valid approach ***(M1)***

eg $x = -\frac{b}{2a}$

correct working ***A1***

eg $-\frac{(-6)}{2(3)},\ x = \frac{6}{6}$

correct answers ***A1A1*** ***N2***

eg $x = 1,\ y = 0;\ (1,\ 0)$

METHOD 2

valid approach ***(M1)***

eg $f(x) = 0$, factorisation, completing the square

correct working ***A1***

eg $x^2 - 2x + 1 = 0,\ (3x - 3)(x - 1),\ f(x) = 3(x - 1)^2$

correct answers ***A1A1*** ***N2***

eg $x = 1,\ y = 0;\ (1,\ 0)$

METHOD 3

valid approach using derivative ***(M1)***

eg $f'(x) = 0,\ 6x - 6$

correct equation ***A1***

eg $6x - 6 = 0$

correct answers ***A1A1*** ***N2***

eg $x = 1,\ y = 0;\ (1,\ 0)$

[4 marks]

12d. The graph of f has its vertex on the x-axis. *[1 mark]*

Write down the solution of $f(x) = 0$.

Markscheme

$x = 1$ ***A1*** ***N1***

[1 mark]

12e. The graph of f has its vertex on the x-axis. *[1 mark]*

The function can be written in the form $f(x) = a(x - h)^2 + k$. Write down the value of a.

Markscheme

$a = 3$ ***A1*** ***N1***

[1 mark]

12f. The graph of f has its vertex on the x-axis. *[1 mark]*

The function can be written in the form $f(x) = a(x - h)^2 + k$. Write down the value of h.

Markscheme

$h = 1$ ***A1*** ***N1***

[1 mark]

12g. The graph of f has its vertex on the x-axis. *[1 mark]*

The function can be written in the form $f(x) = a(x - h)^2 + k$. Write down the value of k.

Markscheme

$k = 0$ ***A1*** ***N1***

[1 mark]

12h. The graph of f has its vertex on the x-axis. *[4 marks]*

The graph of a function g is obtained from the graph of f by a reflection of f in the x-axis, followed by a translation by the vector $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$. Find g, giving your answer in the form $g(x) = Ax^2 + Bx + C$.

Markscheme

attempt to apply vertical reflection ***(M1)***

eg $-f(x),\ -3(x-1)^2$, sketch

attempt to apply vertical shift 6 units up ***(M1)***

eg $-f(x)+6$, vertex $(1,6)$

transformations performed correctly (in correct order) ***(A1)***

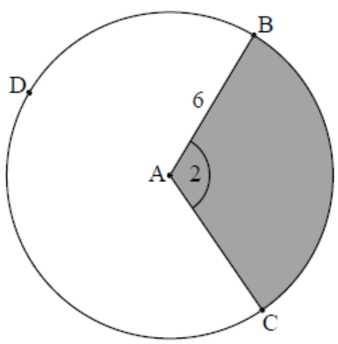
eg $-3(x-1)^2+6,\ -3x^2+6x-3+6$

$g(x)=-3x^2+6x+3$ ***A1 N3***

[4 marks]

The following diagram shows a circle with centre A and radius 6 cm.

diagram not to scale



The points B, C, and D lie on the circle, and $B\hat{A}C$ = 2 radians.

13a. Find the area of the shaded sector. *[2 marks]*

Markscheme

correct substitution ***(A1)***

eg $\frac{1}{2}(2)\left(6^2\right)$

area = 36 (cm^2) ***A1 N2***

[2 marks]

13b. Find the perimeter of the non-shaded sector ABDC. *[4 marks]*

Markscheme

valid approach to find major arc length ***(M1)***

eg angle = $2\pi - 2$, circumference – arc BC

correct working for major arc length ***(A1)***

eg $6(2\pi - 2)$, $(2 \times 6 \times \pi) - (6 \times 2)$, $12\pi - 12$

valid approach to find perimeter of a sector (seen anywhere) ***(M1)***

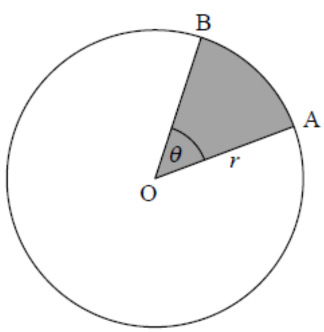
eg arc + 2(radius), $12\pi - 12 + 2(6)$

perimeter = 12π ***A1 N1***

[4 marks]

14. The following diagram shows a circle with centre O and radius r cm. *[7 marks]*

diagram not to scale



The points A and B lie on the circumference of the circle, and $A\hat{O}B = \theta$. The area of the shaded sector AOB is 12 cm^2 and the length of arc AB is 6 cm.

Find the value of r.

Markscheme

evidence of correctly substituting into circle formula (may be seen later) ***A1A1***

eg $\frac{1}{2}\theta r^2 = 12,\ r\theta = 6$

attempt to eliminate one variable ***(M1)***

eg $r = \frac{6}{\theta},\ \theta = \frac{1}{r},\ \frac{\frac{1}{2}\theta r^2}{r\theta} = \frac{12}{6}$

correct elimination ***(A1)***

eg $\frac{1}{2} \times \frac{6}{r} \times r^2 = 12,\ \frac{1}{2}\theta \times \left(\frac{6}{\theta}\right)^2 = 12,\ A = \frac{1}{2} \times r^2 \times \frac{l}{r},\ \frac{r^2}{2r} = 2$

correct equation ***(A1)***

eg $\frac{1}{2} \times 6r = 12,\ \frac{1}{2} \times \frac{36}{\theta} = 12,\ 12 = \frac{1}{2} \times r^2 \times \frac{6}{r}$

correct working ***(A1)***

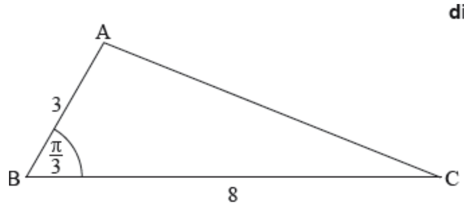
eg $3r = 12,\ \frac{18}{\theta} = 12,\ \frac{r}{2} = 2,\ 24 = 6r$

r = 4 (cm) ***A1 N2***

[7 marks]

The following diagram shows triangle ABC, with $\mathrm{AB} = 3\ \mathrm{cm},\ \mathrm{BC} = 8\ \mathrm{cm}$, and $\mathrm{A\hat{B}C} = \frac{\pi}{3}$.

diagram not to scale



15a. Show that $\mathrm{AC} = 7\ \mathrm{cm}$. *[4 marks]*

Markscheme

evidence of choosing the cosine rule ***(M1)***

eg $c^2 = a^2 + b^2 - ab\cos C$

correct substitution into RHS of cosine rule ***(A1)***

eg $3^2 + 8^2 - 2 \times 3 \times 8 \times \cos\frac{\pi}{3}$

evidence of correct value for $\cos\frac{\pi}{3}$ (may be seen anywhere, including in cosine rule) ***A1***

eg $\cos\frac{\pi}{3} = \frac{1}{2}$, $\mathrm{AC}^2 = 9 + 64 - \left(48 \times \frac{1}{2}\right)$, $9 + 64 - 24$

correct working clearly leading to answer ***A1***

eg $\mathrm{AC}^2 = 49,\ b = \sqrt{49}$

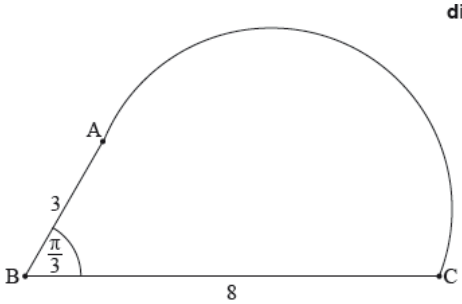
$\mathrm{AC} = 7\ (\mathrm{cm})$ ***AG*** ***N0***

Note: Award no marks if the only working seen is $\mathrm{AC}^2 = 49$ or $\mathrm{AC} = \sqrt{49}$ (or similar).

[4 marks]

15b. The shape in the following diagram is formed by adding a semicircle with diameter [AC] to the triangle. *[3 marks]*

diagram not to scale



Find the exact perimeter of this shape.

Markscheme

correct substitution for semicircle ***(A1)***

eg $\text{semicircle} = \frac{1}{2}(2\pi \times 3.5),\ \frac{1}{2} \times \pi \times 7,\ 3.5\pi$

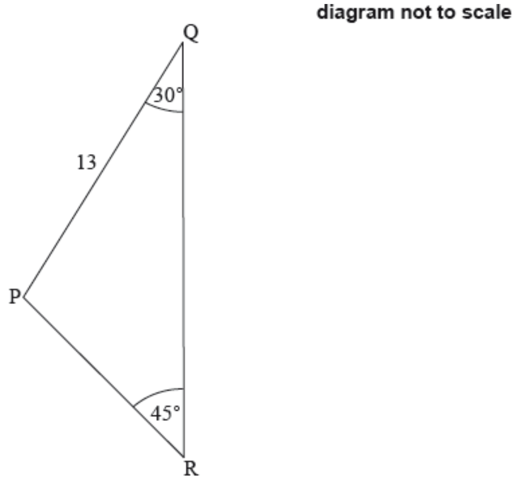
valid approach (seen anywhere) ***(M1)***

eg $\text{perimeter} = \mathrm{AB} + \mathrm{BC} + \text{semicircle},\ 3 + 8 + \left(\frac{1}{2} \times 2 \times \pi \times \frac{7}{2}\right),\ 8 + 3 + 3.5\pi$

$11 + \frac{7}{2}\pi\ (= 3.5\pi + 11)\ (\mathrm{cm})$ ***A1*** ***N2***

[3 marks]

16. The following diagram shows triangle PQR. *[6 marks]*



$P\hat{Q}R = 30°$, $Q\hat{R}P = 45°$ and $PQ = 13\text{ cm}$.

Find PR.

Markscheme

METHOD 1

evidence of choosing the sine rule ***(M1)***

eg $\frac{a}{\sin A} = \frac{b}{\sin B}$

correct substitution ***A1***

eg $\frac{x}{\sin 30} = \frac{13}{\sin 45}, \frac{13 \sin 30}{\sin 45}$

$\sin 30 = \frac{1}{2},\ \sin 45 = \frac{1}{\sqrt{2}}$ ***(A1)(A1)***

correct working ***A1***

eg $\frac{1}{2} \times \frac{13}{\frac{1}{\sqrt{2}}}, \frac{1}{2} \times 13 \times \frac{2}{\sqrt{2}}, 13 \times \frac{1}{2} \times \sqrt{2}$

correct answer ***A1*** ***N3***

eg $\mathrm{PR} = \frac{13\sqrt{2}}{2}, \frac{13}{\sqrt{2}}\ (\mathrm{cm})$

METHOD 2 (using height of Δ PQR)

valid approach to find height of ΔPQR ***(M1)***

eg $\sin 30 = \frac{x}{13},\ \cos 60 = \frac{x}{13}$

$\sin 30 = \frac{1}{2}$ or $\cos 60 = \frac{1}{2}$ ***(A1)***

$\text{height} = 6.5$ ***A1***

correct working ***A1***

eg $\sin 45 = \frac{6.5}{\mathrm{PR}}, \sqrt{6.5^2 + 6.5^2}$

correct working ***(A1)***

eg $\sin 45 = \frac{1}{\sqrt{2}},\ \cos 45 = \frac{1}{\sqrt{2}}, \sqrt{\frac{169 \times 2}{4}}$

correct answer ***A1*** ***N3***

eg $\mathrm{PR} = \frac{13\sqrt{2}}{2}, \frac{13}{\sqrt{2}}\ (\mathrm{cm})$

[6 marks]

Note: In this question, distance is in metres and time is in seconds.

Two particles P_1 and P_2 start moving from a point A at the same time, along different straight lines.

After t seconds, the position of P_1 is given by ***r*** = $\begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} + t \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$.

17a. Find the coordinates of A. *[2 marks]*

Markscheme

recognizing $t = 0$ at A ***(M1)***

A is $(4,\ -1,\ 3)$ ***A1*** ***N2***

[2 marks]

Two seconds after leaving A, P_1 is at point B.

17b. Find $\overrightarrow{\mathrm{AB}}$; *[3 marks]*

Markscheme

METHOD 1

valid approach ***(M1)***

eg $\begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} + 2\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$, $(6,\ 3,\ -1)$

correct approach to find $\overrightarrow{\mathrm{AB}}$ ***(A1)***

eg $\mathrm{AO} + \mathrm{OB},\ \mathrm{B} - \mathrm{A},\ \begin{pmatrix} 6 \\ 3 \\ -1 \end{pmatrix} - \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}$

$\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 2 \\ 4 \\ -4 \end{pmatrix}$ ***A1*** ***N2***

METHOD 2

recognizing $\overrightarrow{\mathrm{AB}}$ is two times the direction vector ***(M1)***

correct working ***(A1)***

eg $\overrightarrow{\mathrm{AB}} = 2\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$

$\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 2 \\ 4 \\ -4 \end{pmatrix}$ ***A1*** ***N2***

[3 marks]

17c. Find $\left|\overrightarrow{\mathrm{AB}}\right|$. *[2 marks]*

Markscheme

correct substitution *(A1)*

eg $\left|\overrightarrow{\mathrm{AB}}\right| = \sqrt{2^2+4^2+4^2},\ \sqrt{4+16+16},\ \sqrt{36}$

$\left|\overrightarrow{\mathrm{AB}}\right| = 6$ ***A1*** ***N2***

[2 marks]

Two seconds after leaving A, P_2 is at point C, where $\overrightarrow{\mathrm{AC}} = \begin{pmatrix}3\\0\\4\end{pmatrix}$.

17d. Find $\cos \mathrm{B\hat{A}C}$. *[5 marks]*

Markscheme

METHOD 1 (vector approach)

valid approach involving $\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\mathrm{AC}}$ ***(M1)***

eg $\overrightarrow{\mathrm{AB}} \bullet \overrightarrow{\mathrm{AC}}$, $\frac{\overrightarrow{\mathrm{BA}} \bullet \overrightarrow{\mathrm{AC}}}{\mathrm{AB} \times \mathrm{AC}}$

finding scalar product and $\left|\overrightarrow{\mathrm{AC}}\right|$ ***(A1)(A1)***

scalar product $2(3) + 4(0) - 4(4)\ (= -10)$

$\left|\overrightarrow{\mathrm{AC}}\right| = \sqrt{3^2 + 0^2 + 4^2}\ (= 5)$

substitution of **their** scalar product and magnitudes into cosine formula ***(M1)***

eg $\cos \mathrm{B\hat{A}C} = \frac{6+0-16}{6\sqrt{3^2+4^2}}$

$\cos B\hat{A}C = -\frac{10}{30} \left(= -\frac{1}{3}\right)$ ***A1 N2***

METHOD 2 (triangle approach)

valid approach involving cosine rule ***(M1)***

eg $\cos \mathrm{B\hat{A}C} = \frac{\mathrm{AB}^2 + \mathrm{AC}^2 - \mathrm{BC}^2}{2 \times \mathrm{AB} \times \mathrm{AC}}$

finding lengths AC and BC ***(A1)(A1)***

$\mathrm{AC} = 5,\ \mathrm{BC} = 9$

substitution of **their** lengths into cosine formula ***(M1)***

eg $\cos \mathrm{B\hat{A}C} = \frac{5^2+6^2-9^2}{2 \times 5 \times 6}$

$\cos \mathrm{B\hat{A}C} = -\frac{20}{60}\ \left(= -\frac{1}{3}\right)$ ***A1 N2***

[5 marks]

17e. Hence or otherwise, find the distance between P_1 and P_2 two seconds after they leave A. *[4 marks]*

Markscheme

Note: Award relevant marks for working seen to find BC in part (c) (if cosine rule used in part (c)).

METHOD 1 (using cosine rule)

recognizing need to find BC ***(M1)***

choosing cosine rule ***(M1)***

eg $c^2 = a^2 + b^2 - 2ab\cos \mathrm{C}$

correct substitution into RHS ***A1***

eg $\mathrm{BC}^2 = (6)^2 + (5)^2 - 2(6)(5)\left(-\frac{1}{3}\right)$, $36 + 25 + 20$

distance is 9 ***A1*** ***N2***

METHOD 2 (finding magnitude of $\overrightarrow{BC}$)

recognizing need to find BC ***(M1)***

valid approach ***(M1)***

eg attempt to find $\overrightarrow{\mathrm{OB}}$ or $\overrightarrow{\mathrm{OC}}$, $\overrightarrow{\mathrm{OB}} = \begin{pmatrix} 6 \\ 3 \\ -1 \end{pmatrix}$ or $\overrightarrow{\mathrm{OC}} = \begin{pmatrix} 7 \\ -1 \\ 7 \end{pmatrix}$, $\overrightarrow{\mathrm{BA}} + \overrightarrow{\mathrm{AC}}$

correct working ***A1***

eg $\overrightarrow{\mathrm{BC}} = \begin{pmatrix} 1 \\ -4 \\ 8 \end{pmatrix}$, $\overrightarrow{\mathrm{CB}} = \begin{pmatrix} -1 \\ 4 \\ -8 \end{pmatrix}$, $\sqrt{1^2 + 4^2 + 8^2} = \sqrt{81}$

distance is 9 ***A1*** ***N2***

METHOD 3 (finding coordinates and using distance formula)

recognizing need to find BC ***(M1)***

valid approach ***(M1)***

eg attempt to find coordinates of B or C, $\mathrm{B}(6,\ 3,\ -1)$ or $\mathrm{C}(7,\ -1,\ 7)$

correct substitution into distance formula ***A1***

eg $\mathrm{BC} = \sqrt{(6-7)^2 + (3-(-1))^2 + (-1-7)^2}$, $\sqrt{1^2 + 4^2 + 8^2} = \sqrt{81}$

distance is 9 ***A1*** ***N2***

[4 marks]

Let $\overrightarrow{OA} = \begin{pmatrix} -1 \\ 0 \\ 4 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix}$.

18a. (i) Find $\overrightarrow{AB}$. *[4 marks]*

(ii) Find $\left|\overrightarrow{AB}\right|$.

Markscheme

(i) valid approach to find $\overrightarrow{AB}$

eg $\overrightarrow{OB} - \overrightarrow{OA}$, $\begin{pmatrix} 4-(-1) \\ 1-0 \\ 3-4 \end{pmatrix}$

$\overrightarrow{AB} = \begin{pmatrix} 5 \\ 1 \\ -1 \end{pmatrix}$ ***A1 N2***

(ii) valid approach to find $\left|\overrightarrow{AB}\right|$ ***(M1)***

eg $\sqrt{(5)^2 + (1)^2 + (-1)^2}$

$\left|\overrightarrow{AB}\right| = \sqrt{27}$ ***A1 N2***

[4 marks]

The point C is such that $\overrightarrow{AC} = \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$.

18b. Show that the coordinates of C are $(-2,\ 1,\ 3)$. *[1 mark]*

Markscheme

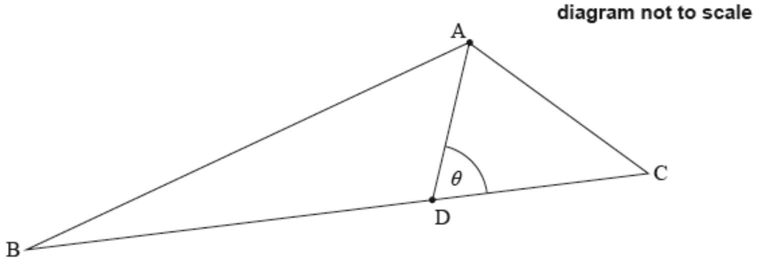
correct approach ***A1***

eg $\overrightarrow{OC} = \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \\ 4 \end{pmatrix}$

C has coordinates $(-2,\ 1,\ 3)$ ***AG N0***

[1 mark]

The following diagram shows triangle ABC. Let D be a point on [BC], with acute angle $\mathrm{ADC} = \theta$.



18c. Write down an expression in terms of θ for *[2 marks]*

(i) angle ADB;

(ii) area of triangle ABD.

Markscheme

(i)

$\mathrm{A\hat{D}B} = \pi - \theta, \hat{\mathrm{D}} = 180 - \theta$ ***A1 N1***

(ii) any correct expression for the area involving θ ***A1 N1***

eg $\text{area} = \frac{1}{2} \times \mathrm{AD} \times \mathrm{BD} \times \sin(180 - \theta),\ \frac{1}{2}ab\sin\theta,\ \frac{1}{2}\left|\overrightarrow{\mathrm{DA}}\right|\left|\overrightarrow{\mathrm{DB}}\right|\sin(\pi - \theta)$

[2 marks]

18d. Given that $\frac{\text{area } \Delta\mathrm{ABD}}{\text{area } \Delta\mathrm{ACD}} = 3$, show that $\frac{\mathrm{BD}}{\mathrm{BC}} = \frac{3}{4}$. *[5 marks]*

Markscheme

METHOD 1 (using sine formula for area)

correct expression for the area of triangle ACD (seen anywhere) ***(A1)***

eg $\frac{1}{2}\mathrm{AD} \times \mathrm{DC} \times \sin\theta$

correct equation involving areas ***A1***

eg $\frac{\frac{1}{2}\mathrm{AD}\times\mathrm{BD}\times\sin(\pi-\theta)}{\frac{1}{2}\mathrm{AD}\times\mathrm{DC}\times\sin\theta} = 3$

recognizing that $\sin(\pi - \theta) = \sin\theta$ (seen anywhere) ***(A1)***

$\frac{\mathrm{BD}}{\mathrm{DC}} = 3$ (seen anywhere) ***(A1)***

correct approach using ratio ***A1***

eg $3\overrightarrow{\mathrm{DC}} + \overrightarrow{\mathrm{DC}} = \overrightarrow{\mathrm{BC}},\ \overrightarrow{\mathrm{BC}} = 4\overrightarrow{\mathrm{DC}}$

correct ratio $\frac{\mathrm{BD}}{\mathrm{BC}} = \frac{3}{4}$ ***AG*** ***N0***

METHOD 2 (Geometric approach)

recognising $\Delta\mathrm{ABD}$ and $\Delta\mathrm{ACD}$ have same height ***(A1)***

eg use of h for both triangles, $\frac{\frac{1}{2}\mathrm{BD}\times h}{\frac{1}{2}\mathrm{CD}\times h} = 3$

correct approach ***A2***

eg $\mathrm{BD} = 3x$ and $\mathrm{DC} = x,\ \frac{\mathrm{BD}}{\mathrm{DC}} = 3$

correct working ***A2***

eg $\mathrm{BC} = 4x,\ \mathrm{BD} + \mathrm{DC} = 4\mathrm{DC},\ \frac{\mathrm{BD}}{\mathrm{BC}} = \frac{3x}{4x},\ \frac{\mathrm{BD}}{\mathrm{BC}} = \frac{3\mathrm{DC}}{4\mathrm{DC}}$

$\frac{\mathrm{BD}}{\mathrm{BC}} = \frac{3}{4}$ ***AG*** ***N0***

[5 marks]

18e. Hence or otherwise, find the coordinates of point D. *[4 marks]*

Markscheme

correct working (seen anywhere) ***(A1)***

eg $\overrightarrow{\mathrm{BD}} = \frac{3}{4}\overrightarrow{\mathrm{BC}}$, $\overrightarrow{\mathrm{OD}} = \overrightarrow{\mathrm{OB}} + \frac{3}{4}\begin{pmatrix}-6\\0\\0\end{pmatrix}$, $\overrightarrow{\mathrm{CD}} = \frac{1}{4}\overrightarrow{\mathrm{CB}}$

valid approach (seen anywhere) ***(M1)***

eg $\overrightarrow{\mathrm{OD}} = \overrightarrow{\mathrm{OB}} + \overrightarrow{\mathrm{BD}}$, $\overrightarrow{\mathrm{BC}} = \begin{pmatrix}-6\\0\\0\end{pmatrix}$

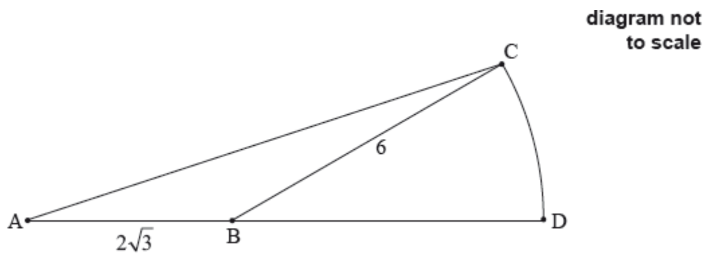
correct working to find x-coordinate ***(A1)***

eg $\begin{pmatrix}4\\1\\3\end{pmatrix} + \frac{3}{4}\begin{pmatrix}-6\\0\\0\end{pmatrix}$, $x = 4 + \frac{3}{4}(-6)$, $-2 + \frac{1}{4}(6)$

D is $\left(-\frac{1}{2}, 1, 3\right)$ ***A1*** ***N3***

[4 marks]

The following diagram shows a triangle ABC and a sector BDC of a circle with centre B and radius 6 cm. The points A , B and D are on the same line.



$\mathrm{AB} = 2\sqrt{3}$ cm, $\mathrm{BC} = 6$ cm, area of triangle ABC $= 3\sqrt{3}$ cm^2, $\mathrm{A\hat{B}C}$ is obtuse.

19a. Find $\mathrm{A\hat{B}C}$. *[5 marks]*

Markscheme

METHOD 1

correct substitution into formula for area of triangle ***(A1)***

eg $\frac{1}{2}(6)\left(2\sqrt{3}\right)\sin B,\ 6\sqrt{3}\sin B,\ \frac{1}{2}(6)\left(2\sqrt{3}\right)\sin B = 3\sqrt{3}$

correct working ***(A1)***

eg $6\sqrt{3}\sin B = 3\sqrt{3},\ \sin B = \frac{3\sqrt{3}}{\frac{1}{2}(6)2\sqrt{3}}$

$\sin B = \frac{1}{2}$ ***(A1)***

$\frac{\pi}{6}(30°)$ ***(A1)***

$\mathrm{A\hat{B}C} = \frac{5\pi}{6}(150°)$ ***A1 N3***

METHOD 2

(using height of triangle ABC by drawing perpendicular segment from C to AD)

correct substitution into formula for area of triangle ***(A1)***

eg $\frac{1}{2}\left(2\sqrt{3}\right)(h) = 3\sqrt{3},\ h\sqrt{3}$

correct working ***(A1)***

eg $h\sqrt{3} = 3\sqrt{3}$

height of triangle is 3 ***A1***

$\mathrm{C\hat{B}D} = \frac{\pi}{6}(30°)$ ***(A1)***

$\mathrm{A\hat{B}C} = \frac{5\pi}{6}(150°)$ ***A1 N3***

[5 marks]

19b. Find the exact area of the sector BDC. *[3 marks]*

Markscheme

recognizing supplementary angle ***(M1)***

eg $\mathrm{C\hat{B}D} = \frac{\pi}{6}$, sector $= \frac{1}{2}(180 - \mathrm{A\hat{B}C})(6^2)$

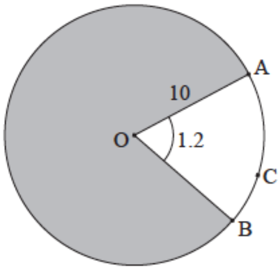
correct substitution into formula for area of sector ***(A1)***

eg $\frac{1}{2} \times \frac{\pi}{6} \times 6^2,\ \pi(6^2)\left(\frac{30}{360}\right)$

area $= 3\pi\ (\mathrm{cm}^2)$ ***A1 N2***

[3 marks]

The following diagram shows a circle with centre O and a radius of 10 cm. Points A, B and C lie on the circle.



Angle AOB is 1.2 radians.

20a. Find the length of arc ACB. *[2 marks]*

Markscheme

correct substitution *(A1)*

eg $10(1.2)$

ACB is $12\ (\text{cm})$ ***A1*** ***N2***

[2 marks]

20b. Find the perimeter of the shaded region. *[3 marks]*

Markscheme

valid approach to find major arc ***(M1)***

eg circumference $-\text{AB}$, major angle $\text{AOB} \times \text{radius}$

correct working for arc length ***(A1)***

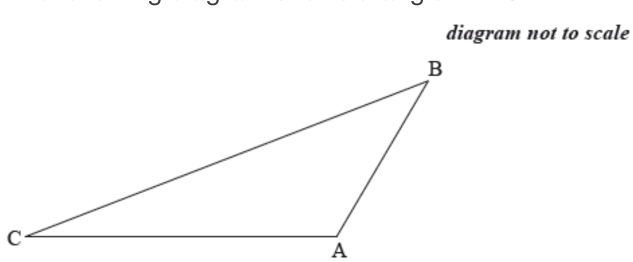
eg $2\pi(10) - 12,\ 10(2 \times 3.142 - 1.2),\ 2\pi(10) - 12 + 20$

perimeter is $20\pi + 8\quad (= 70.8)\ (\text{cm})$ ***A1*** ***N2***

[3 marks]

Total [5 marks]

21. The following diagram shows triangle ABC. *[6 marks]*



Let $\overrightarrow{\text{AB}} \bullet \overrightarrow{\text{AC}} = -5\sqrt{3}$ and $\left|\overrightarrow{\text{AB}}\right|\left|\overrightarrow{\text{AC}}\right| = 10$. Find the area of triangle ABC.

Markscheme

attempt to find $\cos \mathrm{C\hat{A}B}$ (seen anywhere) ***(M1)***

eg $\cos\theta = \frac{\overrightarrow{\mathrm{AB}} \bullet \overrightarrow{\mathrm{AC}}}{\left|\overrightarrow{\mathrm{AB}}\right|\left|\overrightarrow{\mathrm{AC}}\right|}$

$\cos \mathrm{C\hat{A}B} = \frac{-5\sqrt{3}}{10}$ $\left(= -\frac{\sqrt{3}}{2}\right)$ ***A1***

valid attempt to find $\sin \mathrm{C\hat{A}B}$ ***(M1)***

eg triangle, Pythagorean identity, $\mathrm{C\hat{A}B} = \frac{5\pi}{6},\ 150^\circ$

$\sin \mathrm{C\hat{A}B} = \frac{1}{2}$ ***(A1)***

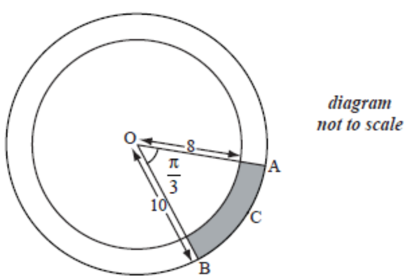
correct substitution into formula for area ***(A1)***

eg $\frac{1}{2} \times 10 \times \frac{1}{2},\ \frac{1}{2} \times 10 \times \sin\frac{\pi}{6}$

$\text{area} = \frac{10}{4}$ $\left(= \frac{5}{2}\right)$ ***A1*** ***N3***

[6 marks]

The diagram shows two concentric circles with centre O.



The radius of the smaller circle is 8 cm and the radius of the larger circle is 10 cm.

Points A, B and C are on the circumference of the larger circle such that
$\mathrm{A\hat{O}B}$ is
$\frac{\pi}{3}$ radians.

22a. Find the length of the arc ACB . *[2 marks]*

Markscheme

correct substitution in $l = r\theta$ ***(A1)***

e.g. $10 \times \frac{\pi}{3}$, $\frac{1}{6} \times 2\pi \times 10$

arc length
$= \frac{20\pi}{6}$
$\left(= \frac{10\pi}{3}\right)$ ***A1*** ***N2***

[2 marks]

22b. Find the area of the shaded region. *[4 marks]*

Markscheme

area of large sector $= \frac{1}{2} \times 10^2 \times \frac{\pi}{3}\ \left(= \frac{100\pi}{6}\right)$ ***(A1)***

area of small sector $= \frac{1}{2} \times 8^2 \times \frac{\pi}{3}\ \left(= \frac{64\pi}{6}\right)$ ***(A1)***

evidence of valid approach (seen anywhere) ***M1***

e.g. subtracting areas of two sectors, $\frac{1}{2} \times \frac{\pi}{3}(10^2 - 8^2)$

area shaded
$= 6\pi$ (accept
$\frac{36\pi}{6}$, etc.) ***A1*** ***N3***

[4 marks]

The vertices of the triangle PQR are defined by the position vectors

$\overrightarrow{\mathrm{OP}} = \begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix}$,

$\overrightarrow{\mathrm{OQ}} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$ and

$\overrightarrow{\mathrm{OR}} = \begin{pmatrix} 6 \\ -1 \\ 5 \end{pmatrix}$.

23a. Find *[3 marks]*

(i) $\overrightarrow{\mathrm{PQ}}$;

(ii) $\overrightarrow{\mathrm{PR}}$.

Markscheme

(i) evidence of approach ***(M1)***

e.g. $\overrightarrow{\mathrm{PQ}} = \overrightarrow{\mathrm{PO}} + \overrightarrow{\mathrm{OQ}}$, $\mathrm{Q} - \mathrm{P}$

$\overrightarrow{\mathrm{PQ}} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$ ***A1*** ***N2***

(ii) $\overrightarrow{\mathrm{PR}} = \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix}$ ***A1*** ***N1***

[3 marks]

23b. Show that $\cos \mathrm{R\hat{P}Q} = \frac{1}{2}$. *[7 marks]*

Markscheme

METHOD 1

choosing correct vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ ***(A1)(A1)***

finding $\overrightarrow{PQ} \bullet \overrightarrow{PR}$, $\left|\overrightarrow{PQ}\right|$, $\left|\overrightarrow{PR}\right|$ ***(A1) (A1)(A1)***

$\overrightarrow{PQ} \bullet \overrightarrow{PR} = -2 + 4 + 4 (= 6)$

$\left|\overrightarrow{PQ}\right| = \sqrt{(-1)^2 + 2^2 + 1^2} \left(= \sqrt{6}\right)$, $\left|\overrightarrow{PR}\right| = \sqrt{2^2 + 2^2 + 4^2} \left(= \sqrt{24}\right)$

substituting into formula for angle between two vectors ***M1***

e.g. $\cos R\hat{P}Q = \frac{6}{\sqrt{6} \times \sqrt{24}}$

simplifying to expression clearly leading to $\frac{1}{2}$ ***A1***

e.g. $\frac{6}{\sqrt{6} \times 2\sqrt{6}}$, $\frac{6}{\sqrt{144}}$, $\frac{6}{12}$

$\cos R\hat{P}Q = \frac{1}{2}$ ***AG N0***

METHOD 2

evidence of choosing cosine rule (seen anywhere) ***(M1)***

$\overrightarrow{QR} = \begin{pmatrix} 3 \\ 0 \\ 3 \end{pmatrix}$ ***A1***

$\left|\overrightarrow{QR}\right| = \sqrt{18}$, $\left|\overrightarrow{PQ}\right| = \sqrt{6}$ and $\left|\overrightarrow{PR}\right| = \sqrt{24}$ ***(A1)(A1)(A1)***

$\cos R\hat{P}Q = \frac{\left(\sqrt{6}\right)^2 + \left(\sqrt{24}\right)^2 - \left(\sqrt{18}\right)^2}{2\sqrt{6} \times \sqrt{24}}$ ***A1***

$\cos R\hat{P}Q = \frac{6+24-18}{24} \left(= \frac{12}{24}\right)$ ***A1***

$\cos R\hat{P}Q = \frac{1}{2}$ ***AG N0***

[7 marks]

23c. (i) Find $\sin R\hat{P}Q$. *[6 marks]*

(ii) Hence, find the area of triangle PQR, giving your answer in the form $a\sqrt{3}$.

Markscheme

(i) **METHOD 1**

evidence of appropriate approach ***(M1)***

e.g. using $\sin^2 R\hat{P}Q + \cos^2 R\hat{P}Q = 1$, diagram

substituting correctly ***(A1)***

e.g. $\sin R\hat{P}Q = \sqrt{1 - \left(\frac{1}{2}\right)^2}$

$\sin R\hat{P}Q = \sqrt{\frac{3}{4}} \left(= \frac{\sqrt{3}}{2}\right)$ ***A1 N3***

METHOD 2

since $\cos \hat{P} = \frac{1}{2}$, $\hat{P} = 60°$ ***(A1)***

evidence of approach

e.g. drawing a right triangle, finding the missing side ***(A1)***

$\sin \hat{P} = \frac{\sqrt{3}}{2}$ ***A1 N3***

(ii) evidence of appropriate approach ***(M1)***

e.g. attempt to substitute into $\frac{1}{2}ab\sin C$

correct substitution

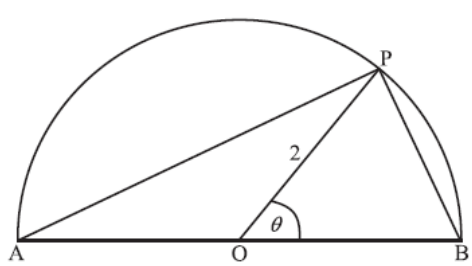
e.g. area $= \frac{1}{2}\sqrt{6} \times \sqrt{24} \times \frac{\sqrt{3}}{2}$ ***A1***

area $= 3\sqrt{3}$ ***A1 N2***

[6 marks]

The following diagram shows a semicircle centre O, diameter [AB], with radius 2.

Let P be a point on the circumference, with
$P\hat{O}B = \theta$ radians.



24a. Find the area of the triangle OPB, in terms of θ . *[2 marks]*

Markscheme

evidence of using area of a triangle *(M1)*

e.g. $A = \frac{1}{2} \times 2 \times 2 \times \sin\theta$

$A = 2\sin\theta$ ***A1*** ***N2***

[2 marks]

24b. Explain why the area of triangle OPA is the same as the area triangle OPB. *[3 marks]*

Markscheme

METHOD 1

$\mathrm{P\hat{O}A} = \pi - \theta$ ***(A1)***

area $\Delta\mathrm{OPA} = \frac{1}{2} 2 \times 2 \times \sin(\pi - \theta)\ (= 2\sin(\pi - \theta))$ ***A1***

since $\sin(\pi - \theta) = \sin\theta$ ***R1***

then both triangles have the same area ***AG*** ***N0***

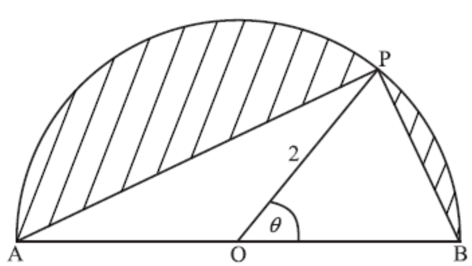
METHOD 2

triangle OPA has the same height and the same base as triangle OPB ***R3***

then both triangles have the same area ***AG*** ***N0***

[3 marks]

Let *S* be the total area of the two segments shaded in the diagram below.



24c. Show that $S = 2(\pi - 2\sin\theta)$. *[3 marks]*

Markscheme

area semicircle $= \frac{1}{2} \times \pi(2)^2 \ (= 2\pi)$ ***A1***

$\text{area } \Delta\text{APB} = 2\sin\theta + 2\sin\theta \ (= 4\sin\theta)$ ***A1***

$S = \text{area of semicircle} - \text{area } \Delta\text{APB} \ (= 2\pi - 4\sin\theta)$ ***M1***

$S = 2(\pi - 2\sin\theta)$ ***AG N0***

[3 marks]

24d. Find the value of θ when S is a local minimum, justifying that it is a minimum. *[8 marks]*

Markscheme

METHOD 1

attempt to differentiate ***(M1)***

e.g. $\frac{\mathrm{d}S}{\mathrm{d}\theta} = -4\cos\theta$

setting derivative equal to 0 ***(M1)***

correct equation ***A1***

e.g. $-4\cos\theta = 0$, $\cos\theta = 0$, $4\cos\theta = 0$

$\theta = \frac{\pi}{2}$ ***A1 N3***

EITHER

evidence of using second derivative ***(M1)***

$S''(\theta) = 4\sin\theta$ ***A1***

$S''\left(\frac{\pi}{2}\right) = 4$ ***A1***

it is a minimum because $S''\left(\frac{\pi}{2}\right) > 0$ ***R1 N0***

OR

evidence of using first derivative ***(M1)***

for $\theta < \frac{\pi}{2}, S'(\theta) < 0$ (may use diagram) ***A1***

for $\theta > \frac{\pi}{2}, S'(\theta) > 0$ (may use diagram) ***A1***

it is a minimum since the derivative goes from negative to positive ***R1 N0***

METHOD 2

$2\pi - 4\sin\theta$ is minimum when $4\sin\theta$ is a maximum ***R3***

$4\sin\theta$ is a maximum when $\sin\theta = 1$ ***(A2)***

$\theta = \frac{\pi}{2}$ ***A3 N3***

[8 marks]

24e. Find a value of θ for which S has its greatest value. *[2 marks]*

Markscheme

S is greatest when $4\sin\theta$ is smallest (or equivalent)   ***(R1)***

$\theta = 0$ (or π)   ***A1***   ***N2***

[2 marks]