Trig double angle identities *[203 marks]*

1. Solve $\log_2(2\sin x) + \log_2(\cos x) = -1$, for $2\pi < x < \frac{5\pi}{2}$. *[7 marks]*

Markscheme

correct application of $\log a + \log b = \log ab$ ***(A1)***

eg $\log_2(2\sin x\cos x),\ \log 2 + \log(\sin x) + \log(\cos x)$

correct equation without logs ***A1***

eg $2\sin x\cos x = 2^{-1},\ \sin x\cos x = \frac{1}{4},\ \sin 2x = \frac{1}{2}$

recognizing double-angle identity (seen anywhere) ***A1***

eg $\log(\sin 2x),\ 2\sin x\cos x = \sin 2x,\ \sin 2x = \frac{1}{2}$

evaluating $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}\ (30^\circ)$ ***(A1)***

correct working ***A1***

eg $x = \frac{\pi}{12} + 2\pi,\ 2x = \frac{25\pi}{6},\ \frac{29\pi}{6},\ 750^\circ,\ 870^\circ,\ x = \frac{\pi}{12}$ **and** $x = \frac{5\pi}{12}$, one correct final answer

$x = \frac{25\pi}{12},\ \frac{29\pi}{12}$ (do not accept additional values) ***A2 N0***

[7 marks]

Let $\sin\theta = \frac{\sqrt{5}}{3}$, where θ is acute.

2a. Find $\cos\theta$. *[3 marks]*

Markscheme

evidence of valid approach ***(M1)***

eg
right triangle, $\cos^2\theta = 1 - \sin^2\theta$

correct working ***(A1)***

eg
missing side is 2,
$\sqrt{1 - \left(\frac{\sqrt{5}}{3}\right)^2}$

$\cos\theta = \frac{2}{3}$ ***A1 N2***

[3 marks]

2b. Find $\cos 2\theta$. *[2 marks]*

Markscheme

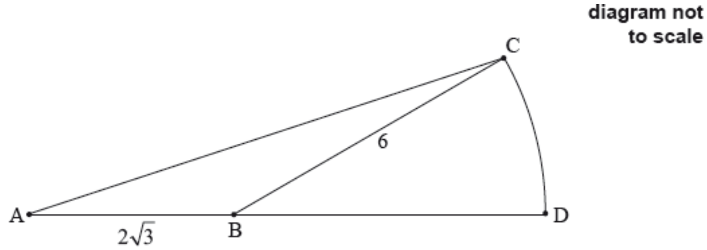
correct substitution into formula for $\cos 2\theta$ ***(A1)***

eg $2 \times \left(\frac{2}{3}\right)^2 - 1,\ 1 - 2\left(\frac{\sqrt{5}}{3}\right)^2,\ \left(\frac{2}{3}\right)^2 - \left(\frac{\sqrt{5}}{3}\right)^2$

$\cos 2\theta = -\frac{1}{9}$ ***A1 N2***

[2 marks]

The following diagram shows a triangle ABC and a sector BDC of a circle with centre B and radius 6 cm. The points A , B and D are on the same line.



$\text{AB} = 2\sqrt{3}\text{ cm}, \text{BC} = 6\text{ cm}, \text{area of triangle ABC} = 3\sqrt{3}\text{ cm}^2, \text{A}\hat{\text{B}}\text{C}$ is obtuse.

3a. Find $\text{A}\hat{\text{B}}\text{C}$. *[5 marks]*

Markscheme

METHOD 1

correct substitution into formula for area of triangle ***(A1)***

eg $\frac{1}{2}(6)\left(2\sqrt{3}\right)\sin B,\ 6\sqrt{3}\sin B,\ \frac{1}{2}(6)\left(2\sqrt{3}\right)\sin B = 3\sqrt{3}$

correct working ***(A1)***

eg $6\sqrt{3}\sin B = 3\sqrt{3},\ \sin B = \frac{3\sqrt{3}}{\frac{1}{2}(6)2\sqrt{3}}$

$\sin B = \frac{1}{2}$ ***(A1)***

$\frac{\pi}{6}(30^\circ)$ ***(A1)***

$\text{A}\hat{\text{B}}\text{C} = \frac{5\pi}{6}(150^\circ)$ ***A1 N3***

METHOD 2

(using height of triangle ABC by drawing perpendicular segment from C to AD)

correct substitution into formula for area of triangle ***(A1)***

eg $\frac{1}{2}\left(2\sqrt{3}\right)(h) = 3\sqrt{3},\ h\sqrt{3}$

correct working ***(A1)***

eg $h\sqrt{3} = 3\sqrt{3}$

height of triangle is 3 ***A1***

$\text{C}\hat{\text{B}}\text{D} = \frac{\pi}{6}(30^\circ)$ ***(A1)***

$\text{A}\hat{\text{B}}\text{C} = \frac{5\pi}{6}(150^\circ)$ ***A1 N3***

[5 marks]

3b. Find the exact area of the sector BDC. *[3 marks]*

Markscheme

recognizing supplementary angle ***(M1)***

eg $\text{C}\hat{\text{B}}\text{D} = \frac{\pi}{6},\ \text{sector} = \frac{1}{2}(180 - \text{A}\hat{\text{B}}\text{C})(6^2)$

correct substitution into formula for area of sector ***(A1)***

eg $\frac{1}{2} \times \frac{\pi}{6} \times 6^2,\ \pi(6^2)\left(\frac{30}{360}\right)$

$\text{area} = 3\pi\ (\text{cm}^2)$ ***A1 N2***

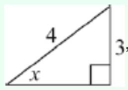
[3 marks]

Given that
$\sin x = \frac{3}{4}$, where x is an obtuse angle,

4a. find the value of $\cos x$; *[4 marks]*

Markscheme

valid approach ***(M1)***

eg 

$3,\ \sin^2 x + \cos^2 x = 1$

correct working ***(A1)***

eg $4^2 - 3^2,\ \cos^2 x = 1 - \left(\frac{3}{4}\right)^2$

correct calculation ***(A1)***

eg $\frac{\sqrt{7}}{4},\ \cos^2 x = \frac{7}{16}$

$\cos x = -\frac{\sqrt{7}}{4}$ ***A1 N3***

[4 marks]

4b. find the value of $\cos 2x$. *[3 marks]*

Markscheme

correct substitution (accept missing minus with cos) ***(A1)***

eg $1 - 2\left(\frac{3}{4}\right)^2,\ 2\left(-\frac{\sqrt{7}}{4}\right)^2 - 1,\ \left(\frac{\sqrt{7}}{4}\right)^2 - \left(\frac{3}{4}\right)^2$

correct working ***A1***

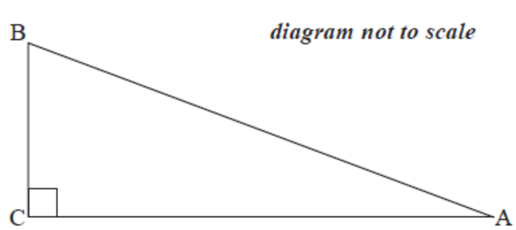
eg $2\left(\frac{7}{16}\right) - 1,\ 1 - \frac{18}{16},\ \frac{7}{16} - \frac{9}{16}$

$\cos 2x = -\frac{2}{16}\ \left(= -\frac{1}{8}\right)$ ***A1 N2***

[3 marks]

Total [7 marks]

The following diagram shows a right-angled triangle,
ABC, where
$\sin \mathrm{A} = \frac{5}{13}$.



5a. Show that
$\cos A = \frac{12}{13}$. *[2 marks]*

Markscheme

METHOD 1

approach involving Pythagoras' theorem ***(M1)***

eg
$5^2 + x^2 = 13^2$, labelling correct sides on triangle

finding third side is 12 (may be seen on diagram) ***A1***

$\cos A = \frac{12}{13}$ ***AG N0***

METHOD 2

approach involving
$\sin^2\theta + \cos^2\theta = 1$ ***(M1)***

eg
$\left(\frac{5}{13}\right)^2 + \cos^2\theta = 1,\ x^2 + \frac{25}{169} = 1$

correct working ***A1***

eg
$\cos^2\theta = \frac{144}{169}$

$\cos A = \frac{12}{13}$ ***AG N0***

[2 marks]

5b. Find
$\cos 2A$. *[3 marks]*

Markscheme

correct substitution into
$\cos 2\theta$ ***(A1)***

eg
$1 - 2\left(\frac{5}{13}\right)^2,\ 2\left(\frac{12}{13}\right)^2 - 1,\ \left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2$

correct working ***(A1)***

eg
$1 - \frac{50}{169},\ \frac{288}{169} - 1,\ \frac{144}{169} - \frac{25}{169}$

$\cos 2A = \frac{119}{169}$ ***A1 N2***

[3 marks]

In triangle
ABC,
$\mathrm{AB} = 6\,\mathrm{cm}$ and
$\mathrm{AC} = 8\,\mathrm{cm}$. The area of the triangle is
$16\,\mathrm{cm}^2$.

6a. Find the two possible values for
$\hat{A}$. *[4 marks]*

Markscheme

correct substitution into area formula *(A1)*

eg
$\frac{1}{2}(6)(8)\sin A = 16,\ \sin A = \frac{16}{24}$

correct working *(A1)*

eg
$A = \arcsin\left(\frac{2}{3}\right)$

$A = 0.729727656\ldots, 2.41186499\ldots;$
$(41.8103149^{\circ}, 138.1896851^{\circ})$

$A = 0.730;$
2.41 ***A1A1*** ***N3***

(accept degrees *ie*
$41.8^{\circ};$
138°)

[4 marks]

6b. Given that
$\hat{A}$ is obtuse, find
BC. *[3 marks]*

Markscheme

evidence of choosing cosine rule *(M1)*

eg
$\mathrm{BC}^2 = \mathrm{AB}^2 + \mathrm{AC}^2 - 2(\mathrm{AB})(\mathrm{AC})\cos A,\ a^2 + b^2 - 2ab\cos C$

correct substitution into RHS (angle must be obtuse) *(A1)*

eg
$\mathrm{BC}^2 = 6^2 + 8^2 - 2(6)(8)\cos 2.41,\ 6^2 + 8^2 - 2(6)(8)\cos 138^{\circ},$

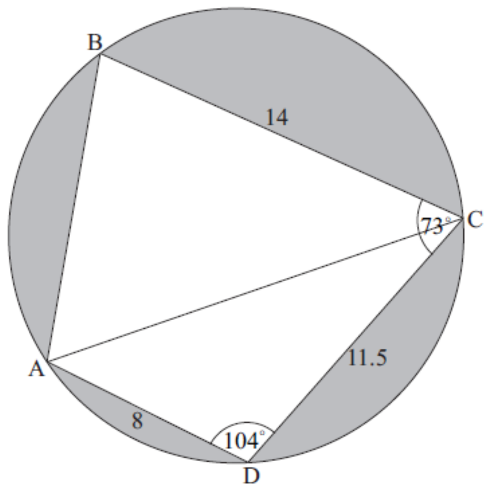
$\mathrm{BC} = \sqrt{171.55}$

$\mathrm{BC} = 13.09786$

$\mathrm{BC} = 13.1\ \mathrm{cm}$ ***A1*** ***N2***

[3 marks]

The diagram shows a circle of radius
8 metres. The points ABCD lie on thecircumference of the circle.



BC =
14 m, CD =
11.5 m, AD =
8 m,
$A\hat{D}C = 104^\circ$, and
$B\hat{C}D = 73^\circ$.

7a. Find AC. *[3 marks]*

Markscheme

evidence of choosing cosine rule ***(M1)***

eg
$c^2 = a^2 + b^2 - 2ab\cos C$,
$\mathrm{CD}^2 + \mathrm{AD}^2 - 2 \times \mathrm{CD} \times \mathrm{AD}\cos D$

correct substitution ***A1***

eg
$11.5^2 + 8^2 - 2 \times 11.5 \times 8\cos 104$,
$196.25 - 184\cos 104$

AC
$= 15.5$ (m) ***A1*** ***N2***

[3 marks]

7b. (i) Find
$A\hat{C}D$. *[5 marks]*

(ii) Hence, find
$A\hat{C}B$.

Markscheme

(i) **METHOD 1**

evidence of choosing sine rule ***(M1)***

eg
$\frac{\sin A}{a} = \frac{\sin B}{b}$,
$\frac{\sin \text{A}\hat{\text{C}}\text{D}}{\text{AD}} = \frac{\sin D}{\text{AC}}$

correct substitution ***A1***

eg
$\frac{\sin \text{A}\hat{\text{C}}\text{D}}{8} = \frac{\sin 104}{15.516\ldots}$

$\text{A}\hat{\text{C}}\text{D} = 30.0°$ ***A1 N2***

METHOD 2

evidence of choosing cosine rule ***(M1)***

eg
$c^2 = a^2 + b^2 - 2ab\cos C$

correct substitution ***A1***

e.g.
$8^2 = 11.5^2 + 15.516\ldots^2 - 2(11.5)(15.516\ldots)\cos C$

$\text{A}\hat{\text{C}}\text{D} = 30.0°$ ***A1 N2***

(ii) subtracting **their**
$\text{A}\hat{\text{C}}\text{D}$ from
73 ***(M1)***

eg
$73 - \text{A}\hat{\text{C}}\text{D}$,
$70 - 30.017\ldots$

$\text{A}\hat{\text{C}}\text{B} = 43.0°$ ***A1 N2***

[5 marks]

7c. Find the area of triangle ADC. *[2 marks]*

Markscheme

correct substitution ***(A1)***

eg area
$\Delta\text{ADC} = \frac{1}{2}(8)(11.5)\sin 104$

area
$= 44.6$ (m^2) ***A1 N2***

[2 marks]

7d. Hence or otherwise, find the total area of the shaded regions. *[4 marks]*

Markscheme

attempt to subtract ***(M1)***

eg
circle $-$ ABCD ,
$\pi r^2 - \Delta\text{ADC} - \Delta\text{ACB}$

area
$\Delta\text{ACB} = \frac{1}{2}(15.516\ldots)(14)\sin 42.98$ ***(A1)***

correct working ***A1***

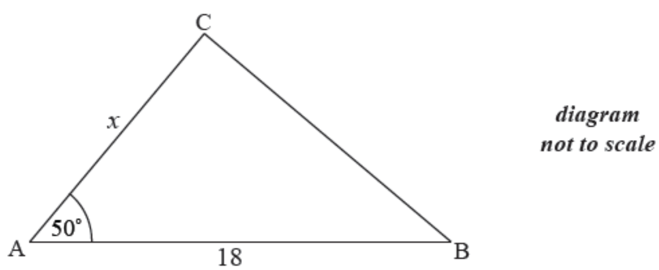
eg
$\pi(8)^2 - 44.6336\ldots - \frac{1}{2}(15.516\ldots)(14)\sin 42.98$,
$64\pi - 44.6 - 74.1$

shaded area is
82.4 (m^2) ***A1*** ***N3***

[4 marks]

Total [6 marks]

The following diagram shows a triangle ABC.



The area of triangle ABC is
80 cm^2 , AB
$= 18$ cm , AC
$= x$ cm and
$\text{B}\hat{\text{A}}\text{C} = 50°$.

8a. Find
x . *[3 marks]*

Markscheme

correct substitution into area formula ***(A1)***

eg
$\frac{1}{2}(18x)\sin 50$

setting **their** area expression equal to
80 ***(M1)***

eg
$9x\sin 50 = 80$

$x = 11.6$ ***A1*** ***N2***

[3 marks]

8b. Find BC. *[3 marks]*

Markscheme

evidence of choosing cosine rule ***(M1)***

eg
$c^2 = a^2 + b^2 + 2ab \sin C$

correct substitution into right hand side (may be in terms of x) ***(A1)***

eg
$11.6^2 + 18^2 - 2(11.6)(18) \cos 50$

BC
$= 13.8$ ***A1 N2***

[3 marks]

9a. Let $\sin 100^\circ = m$. Find an expression for $\cos 100^\circ$ in terms of *m*. *[3 marks]*

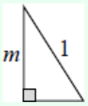
Markscheme

Note: All answers must be given in terms of *m*. If a candidate makes an error that means there is no *m* in their answer, do not award the final ***A1FT*** mark.

METHOD 1

valid approach involving Pythagoras ***(M1)***

e.g.

$\sin^2 x + \cos^2 x = 1$, labelled diagram 

correct working (may be on diagram) ***(A1)***

e.g.
$m^2 + (\cos 100)^2 = 1$,
$\sqrt{1 - m^2}$

$\cos 100 = -\sqrt{1 - m^2}$ ***A1 N2***

[3 marks]

METHOD 2

valid approach involving tan identity ***(M1)***

e.g.
$\tan = \frac{\sin}{\cos}$

correct working ***(A1)***

e.g.
$\cos 100 = \frac{\sin 100}{\tan 100}$

$\cos 100 = \frac{m}{\tan 100}$ ***A1 N2***

[3 marks]

9b. Let $\sin 100^\circ = m$. Find an expression for $\tan 100^\circ$ in terms of *m*. *[1 mark]*

Markscheme

METHOD 1

$\tan 100 = -\frac{m}{\sqrt{1-m^2}}$ (accept $\frac{m}{-\sqrt{1-m^2}}$) ***A1 N1***

[1 mark]

METHOD 2

$\tan 100 = \frac{m}{\cos 100}$ ***A1 N1***

[1 mark]

9c. Let $\sin 100° = m$. Find an expression for $\sin 200°$ in terms of *m*. *[2 marks]*

Markscheme

METHOD 1

valid approach involving double angle formula ***(M1)***

e.g. $\sin 2\theta = 2\sin\theta cos\theta$

$\sin 200 = -2m\sqrt{1-m^2}$ (accept $2m\left(-\sqrt{1-m^2}\right)$) ***A1 N2***

Note: If candidates find $\cos 100 = \sqrt{1-m^2}$, award full ***FT*** in parts (b) and (c), even though the values may not have appropriate signs for the angles.

[2 marks]

METHOD 2

valid approach involving double angle formula ***(M1)***

e.g. $\sin 2\theta = 2\sin\theta\cos\theta$, $2m \times \frac{m}{\tan 100}$

$\sin 200 = \frac{2m^2}{\tan 100} (= 2m\cos 100)$ ***A1 N2***

[2 marks]

Let $f(x) = (\sin x + \cos x)^2$.

10a. Show that $f(x)$ can be expressed as $1 + \sin 2x$. *[2 marks]*

Markscheme

attempt to expand ***(M1)***

e.g.
$(\sin x + \cos x)(\sin x + \cos x)$; at least 3 terms

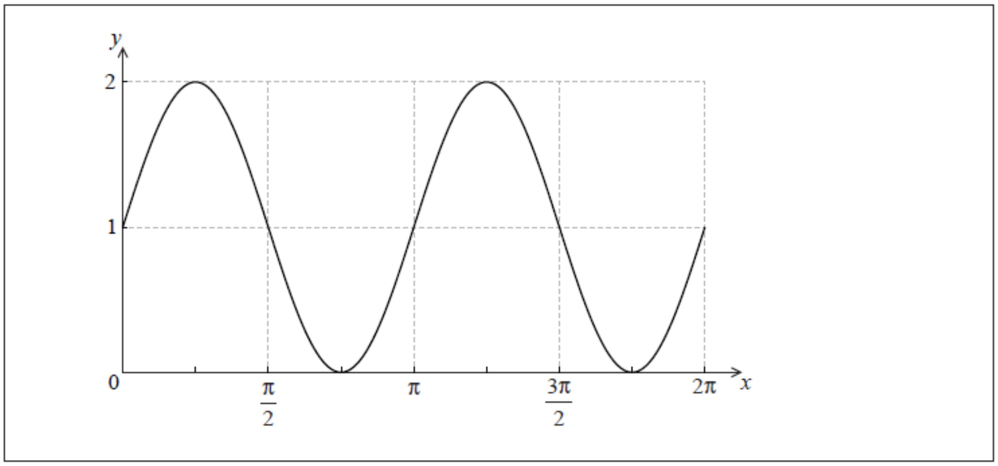
correct expansion ***A1***

e.g.
$\sin^2 x + 2\sin x\cos x + \cos^2 x$

$f(x) = 1 + \sin 2x$ ***AG*** ***N0***

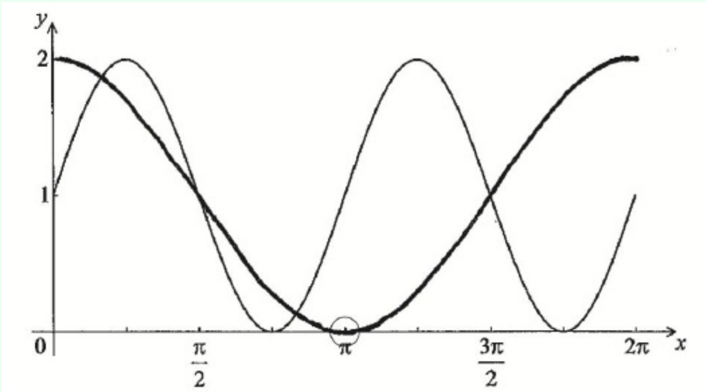
[2 marks]

10b. The graph of *f* is shown below for $0 \le x \le 2\pi$. *[2 marks]*



Let
$g(x) = 1 + \cos x$. On the same set of axes, sketch the graph of *g* for
$0 \le x \le 2\pi$.

Markscheme



A1A1 ***N2***

Note: Award ***A1*** for correct sinusoidal shape with period
2π and range
$[0, 2]$, ***A1*** for minimum in circle.

10c. The graph of g can be obtained from the graph of f under a horizontal stretch of scale factor p followed by a translation by the vector $\begin{pmatrix} k \\ 0 \end{pmatrix}$. *[2 marks]*

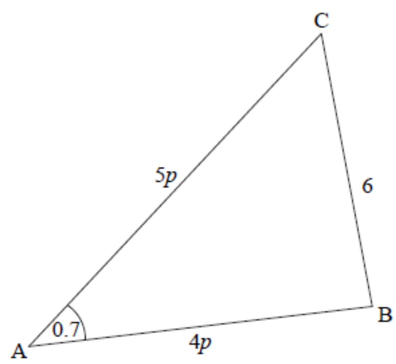
Write down the value of p and a possible value of k.

Markscheme

$p = 2$,
$k = -\frac{\pi}{2}$ ***A1A1 N2***

[2 marks]

The following diagram shows a triangle ABC.



$\text{BC} = 6$,
$\text{C}\widehat{\text{A}}\text{B} = 0.7$ radians,
$\text{AB} = 4p$,
$\text{AC} = 5p$, where
$p > 0$.

11a. (i) Show that $p^2(41 - 40\cos 0.7) = 36$. *[4 marks]*

(ii) Find p.

Markscheme

(i) evidence of valid approach ***(M1)***

e.g. choosing cosine rule

correct substitution ***(A1)***

e.g. $6^2 = (5p)^2 + (4p)^2 - 2 \times (4p) \times (5p)\cos 0.7$

simplification ***A1***

e.g. $36 = 25p^2 + 16p^2 - 40p^2 \cos 0.7$

$p^2(41 - 40\cos 0.7) = 36$ ***AG N0***

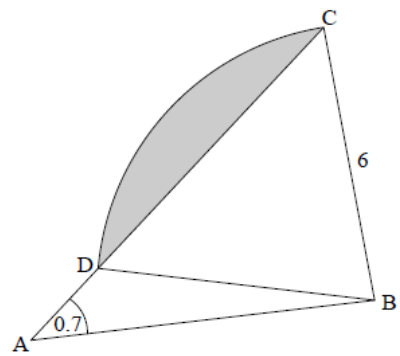
(ii) $1.85995\ldots$

$p = 1.86$ ***A1 N1***

Note: Award ***A0*** for $p = \pm 1.86$, i.e. not rejecting the negative value.

[4 marks]

Consider the circle with centre B that passes through the point C. The circle cuts thdine CA at D, and $\mathrm{A\hat{D}B}$ is obtuse. Part of the circle is shown in the following diagram.



11b. Write down the length of BD. *[1 mark]*

Markscheme

$\mathrm{BD} = 6$ ***A1 N1***

[1 mark]

11c. Find $\mathrm{A\hat{D}B}$. *[4 marks]*

Markscheme

evidence of valid approach ***(M1)***

e.g. choosing sine rule

correct substitution ***A1***

e.g. $\frac{\sin \mathrm{A\hat{D}B}}{4p} = \frac{\sin 0.7}{6}$

acute $\mathrm{A\hat{D}B} = 0.9253166\ldots$ ***(A1)***

$\pi - 0.9253166\ldots = 2.216275\ldots$

$\mathrm{A\hat{D}B} = 2.22$ ***A1 N3***

[4 marks]

11d. (i) Show that $\mathrm{C\hat{B}D} = 1.29$ radians, correct to 2 decimal places. *[6 marks]*

(ii) Hence, find the area of the shaded region.

Markscheme

(i) evidence of valid approach ***(M1)***

e.g. recognize isosceles triangle, base angles equal

$\pi - 2(0.9253\ldots)$ ***A1***

$\mathrm{C\hat{B}D} = 1.29$ ***AG*** ***N0***

(ii) area of sector BCD ***(A1)***

e.g.
$0.5 \times (1.29) \times (6)^2$

area of triangle BCD ***(A1)***

e.g.
$0.5 \times (6)^2 \sin 1.29$

evidence of subtraction ***M1***

$5.92496\ldots$

$5.937459\ldots$

$\text{area} = 5.94$ ***A1*** ***N3***

[6 marks]

The following diagram shows
$\Delta\mathrm{PQR}$, where RQ = 9 cm,
$\mathrm{P\hat{R}Q} = 70^\circ$ and
$\mathrm{P\hat{Q}R} = 45^\circ$.

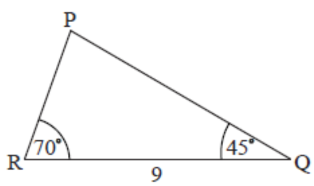


diagram not to scale

12a. Find
$\mathrm{R\hat{P}Q}$. *[1 mark]*

Markscheme

$\mathrm{R\hat{P}Q} = 65^\circ$ ***A1*** ***N1***

[1 mark]

12b. Find PR . *[3 marks]*

Markscheme

evidence of choosing sine rule ***(M1)***

correct substitution ***A1***

e.g.
$\frac{\text{PR}}{\sin 45^\circ} = \frac{9}{\sin 65^\circ}$

7.021854078

$\text{PR} = 7.02$ ***A1*** ***N2***

[3 marks]

12c. Find the area of ΔPQR . *[2 marks]*

Markscheme

correct substitution ***(A1)***

e.g.
$\text{area} = \frac{1}{2} \times 9 \times 7.02 \ldots \times \sin 70^\circ$

29.69273008

$\text{area} = 29.7$ ***A1*** ***N2***

[2 marks]

Let $\sin\theta = \frac{2}{\sqrt{13}}$, where $\frac{\pi}{2} < \theta < \pi$.

13a. Find $\cos\theta$. *[3 marks]*

Markscheme

METHOD 1

evidence of choosing
$\sin^2\theta + \cos^2\theta = 1$ ***(M1)***

correct working ***(A1)***

e.g.
$\cos^2\theta = \frac{9}{13}$,
$\cos\theta = \pm\frac{3}{\sqrt{13}}$,
$\cos\theta = \sqrt{\frac{9}{13}}$

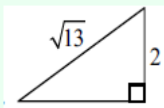
$\cos\theta = -\frac{3}{\sqrt{13}}$ ***A1 N2***

Note: If no working shown, award ***N1*** for
$\frac{3}{\sqrt{13}}$.

METHOD 2

approach involving Pythagoras' theorem ***(M1)***

e.g.
$2^2 + x^2 = 13$,



finding third side equals 3 ***(A1)***

$\cos\theta = -\frac{3}{\sqrt{13}}$ ***A1 N2***

Note: If no working shown, award ***N1*** for
$\frac{3}{\sqrt{13}}$.

[3 marks]

13b. Find
$\tan 2\theta$. *[5 marks]*

Markscheme

correct substitution into
$\sin 2\theta$ (seen anywhere) ***(A1)***

e.g.
$2\left(\frac{2}{\sqrt{13}}\right)\left(-\frac{3}{\sqrt{13}}\right)$

correct substitution into
$\cos 2\theta$ (seen anywhere) ***(A1)***

e.g.
$\left(-\frac{3}{\sqrt{13}}\right)^2-\left(\frac{2}{\sqrt{13}}\right)^2$,
$2\left(-\frac{3}{\sqrt{13}}\right)^2-1$,
$1-2\left(\frac{2}{\sqrt{13}}\right)^2$

valid attempt to find
$\tan 2\theta$ ***(M1)***

e.g.
$\frac{2\left(\frac{2}{\sqrt{13}}\right)\left(-\frac{3}{\sqrt{13}}\right)}{\left(-\frac{3}{\sqrt{13}}\right)^2-\left(\frac{2}{\sqrt{13}}\right)^2}$,
$\frac{2\left(-\frac{2}{3}\right)}{1-\left(-\frac{2}{3}\right)^2}$

correct working ***A1***

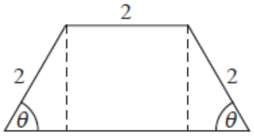
e.g.
$\frac{\frac{(2)(2)(-3)}{13}}{\frac{9}{13}-\frac{4}{13}}$,
$\frac{-\frac{12}{\left(\sqrt{13}\right)^2}}{\frac{18}{13}-1}$,
$\frac{-\frac{12}{13}}{\frac{5}{13}}$

$\tan 2\theta=-\frac{12}{5}$ ***A1 N4***

Note: If students find answers for
$\cos\theta$ which are not in the range
$[-1, 1]$, award full ***FT*** in (b) for correct ***FT*** working shown.

[5 marks]

The diagram below shows a plan for a window in the shape of a trapezium.



Three sides of the window are
2 m long. The angle between the sloping sides of thewindow and the base is
θ , where
$0<\theta<\frac{\pi}{2}$.

14a. Show that the area of the window is given by
$y=4\sin\theta+2\sin 2\theta$. *[5 marks]*

Markscheme

evidence of finding height, *h* ***(A1)***

e.g.
$\sin \theta = \frac{h}{2}$,
$2 \sin \theta$

evidence of finding base of triangle, *b* ***(A1)***

e.g.
$\cos \theta = \frac{b}{2}$,
$2 \cos \theta$

attempt to substitute valid values into a formula for the area of the window ***(M1)***

e.g. two triangles plus rectangle, trapezium area formula

correct expression (must be in terms of
θ) ***A1***

e.g.
$2 \left(\frac{1}{2} \times 2 \cos \theta \times 2 \sin \theta \right) + 2 \times 2 \sin \theta$,
$\frac{1}{2} (2 \sin \theta)(2 + 2 + 4 \cos \theta)$

attempt to replace
$2 \sin \theta \cos \theta$ by
$\sin 2\theta$ ***M1***

e.g.
$4 \sin \theta + 2(2 \sin \theta \cos \theta)$

$y = 4 \sin \theta + 2 \sin 2\theta$ ***AG*** ***N0***

[5 marks]

14b. Zoe wants a window to have an area of
5 m^2. Find the two possible values of
θ . *[4 marks]*

Markscheme

correct equation ***A1***

e.g.
$y = 5$,
$4 \sin \theta + 2 \sin 2\theta = 5$

evidence of attempt to solve ***(M1)***

e.g. a sketch,
$4 \sin \theta + 2 \sin \theta - 5 = 0$

$\theta = 0.856$
(49.0°) ,
$\theta = 1.25$
(71.4°) ***A1A1*** ***N3***

[4 marks]

14c. John wants two windows which have the same area *A* but different values of
θ . *[7 marks]*

Find all possible values for *A* .

Markscheme

recognition that lower area value occurs at
$\theta = \frac{\pi}{2}$ ***(M1)***

finding value of area at
$\theta = \frac{\pi}{2}$ ***(M1)***

e.g.
$4\sin\left(\frac{\pi}{2}\right) + 2\sin\left(2 \times \frac{\pi}{2}\right)$, draw square

$A = 4$ ***(A1)***

recognition that maximum value of *y* is needed ***(M1)***

$A = 5.19615\ldots$ ***(A1)***

$4 < A < 5.20$ (accept
$4 < A < 5.19$) ***A2 N5***

[7 marks]

15a. Show that
$4 - \cos 2\theta + 5\sin\theta = 2\sin^2\theta + 5\sin\theta + 3$. *[2 marks]*

Markscheme

attempt to substitute
$1 - 2\sin^2\theta$ for
$\cos 2\theta$ ***(M1)***

correct substitution ***A1***

e.g.
$4 - (1 - 2\sin^2\theta) + 5\sin\theta$

$4 - \cos 2\theta + 5\sin\theta = 2\sin^2\theta + 5\sin\theta + 3$ ***AG N0***

[2 marks]

15b. **Hence**, solve the equation
$4 - \cos 2\theta + 5\sin\theta = 0$ for
$0 \le \theta \le 2\pi$. *[5 marks]*

Markscheme

evidence of appropriate approach to solve ***(M1)***

e.g. factorizing, quadratic formula

correct working ***A1***

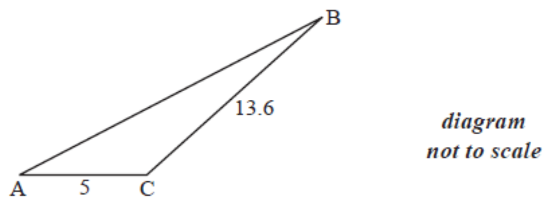
e.g.
$(2\sin\theta + 3)(\sin\theta + 1)$,
$(2x + 3)(x + 1) = 0$,
$\sin x = \frac{-5 \pm \sqrt{1}}{4}$

correct solution
$\sin\theta = -1$ (do not penalise for including
$\sin\theta = -\frac{3}{2}$ ***(A1)***

$\theta = \frac{3\pi}{2}$ ***A2 N3***

[5 marks]

The following diagram shows the triangle ABC.



The angle at C is obtuse,
$\text{AC} = 5\text{ cm}$,
$\text{BC} = 13.6\text{ cm}$ and the area is
20 cm^2 .

16a. Find
$\text{A}\hat{\text{C}}\text{B}$. *[4 marks]*

Markscheme

correct substitution into the formula for the area of a triangle ***A1***

e.g.
$\frac{1}{2} \times 5 \times 13.6 \times \sin C = 20$,
$\frac{1}{2} \times 5 \times h = 20$

attempt to solve ***(M1)***

e.g.
$\sin C = 0.5882\ldots$,
$\sin C = \frac{8}{13.6}$

$\hat{C} = 36.031\ldots^\circ$ (
$0.6288\ldots\text{ radians}$) ***(A1)***

$\text{A}\hat{\text{C}}\text{B} = 144^\circ$
(2.51 radians) ***A1 N3***

[4 marks]

16b. Find AB. *[3 marks]*

Markscheme

evidence of choosing the cosine rule ***(M1)***

correct substitution ***A1***

e.g.
$(\text{AB})^2 = 5^2 + 13.6^2 - 2(5)(13.6)\cos 143.968\ldots$

$\text{AB} = 17.9$ ***A1 N2***

[3 marks]

The straight line with equation
$y = \frac{3}{4}x$ makes an acute angle
θ with the *x*-axis.

17a. Write down the value of
$\tan\theta$. *[1 mark]*

Markscheme

$\tan\theta = \frac{3}{4}$ (do not accept
$\frac{3}{4}x$) ***A1*** ***N1***

[1 mark]

17b. Find the value of *[6 marks]*

(i)
$\sin 2\theta$;

(ii)
$\cos 2\theta$.

Markscheme

(i)
$\sin\theta = \frac{3}{5}$,
$\cos\theta = \frac{4}{5}$ ***(A1)(A1)***

correct substitution ***A1***

e.g.
$\sin 2\theta = 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)$

$\sin 2\theta = \frac{24}{25}$ ***A1*** ***N3***

(ii) correct substitution ***A1***

e.g.
$\cos 2\theta = 1 - 2\left(\frac{3}{5}\right)^2$,
$\left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

$\cos 2\theta = \frac{7}{25}$ ***A1*** ***N1***

[6 marks]

Let
$f(x) = \cos 2x$ and
$g(x) = 2x^2 - 1$.

18a. Find
$f\left(\frac{\pi}{2}\right)$. *[2 marks]*

Markscheme

$f\left(\frac{\pi}{2}\right) = \cos\pi$ ***(A1)***

$= -1$ ***A1*** ***N2***

[2 marks]

18b. Find
$(g \circ f)\left(\frac{\pi}{2}\right)$. *[2 marks]*

Markscheme

$(g \circ f)\left(\frac{\pi}{2}\right) = g(-1)$
$(= 2(-1)^2 - 1)$ ***(A1)***

$= 1$ ***A1*** ***N2***

[2 marks]

18c. Given that $(g \circ f)(x)$ can be written as $\cos(kx)$, find the value of *k*, $k \in \mathbb{Z}$. *[3 marks]*

Markscheme

$(g \circ f)(x) = 2(\cos(2x))^2 - 1$
$(= 2\cos^2(2x) - 1)$ ***A1***

evidence of
$2\cos^2\theta - 1 = \cos 2\theta$ (seen anywhere) ***(M1)***

$(g \circ f)(x) = \cos 4x$

$k = 4$ ***A1*** ***N2***

[3 marks]

The diagram below shows a quadrilateral ABCD with obtuse angles $\mathrm{A\hat{B}C}$ and $\mathrm{A\hat{D}C}$.

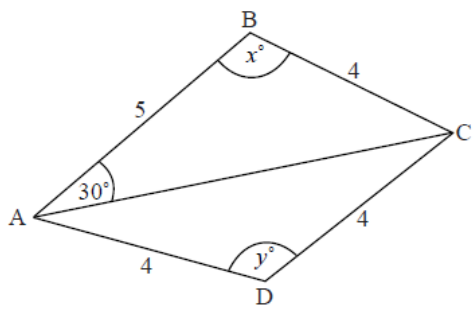


diagram not to scale

AB = 5 cm, BC = 4 cm, CD = 4 cm, AD = 4 cm ,
$\mathrm{B\hat{A}C} = 30^\circ$,
$\mathrm{A\hat{B}C} = x^\circ$,
$\mathrm{A\hat{D}C} = y^\circ$.

19a. Use the cosine rule to show that $\mathrm{AC} = \sqrt{41 - 40\cos x}$. *[1 mark]*

Markscheme

correct substitution ***A1***

e.g.
$25 + 16 - 40\cos x$,
$5^2 + 4^2 - 2 \times 4 \times 5\cos x$

$\mathrm{AC} = \sqrt{41 - 40\cos x}$ ***AG***

[1 mark]

19b. Use the sine rule in triangle ABC to find another expression for AC. *[2 marks]*

Markscheme

correct substitution ***A1***

e.g.
$\frac{\text{AC}}{\sin x} = \frac{4}{\sin 30}$,
$\frac{1}{2}\text{AC} = 4\sin x$

$\text{AC} = 8\sin x$ (accept $\frac{4\sin x}{\sin 30}$) ***A1 N1***

[2 marks]

19c. (i) Hence, find *x*, giving your answer to two decimal places. *[6 marks]*

(ii) Find AC .

Markscheme

(i) evidence of appropriate approach using AC ***M1***

e.g.
$8\sin x = \sqrt{41 - 40\cos x}$, sketch showing intersection

correct solution
$8.682\ldots$,
$111.317\ldots$ ***(A1)***

obtuse value
$111.317\ldots$ ***(A1)***

$x = 111.32$ to 2 dp (do **not** accept the radian answer 1.94) ***A1 N2***

(ii) substituting value of *x* into either expression for AC ***(M1)***

e.g.
$\text{AC} = 8\sin 111.32$

$\text{AC} = 7.45$ ***A1 N2***

[6 marks]

19d. (i) Find *y*. *[5 marks]*

(ii) Hence, or otherwise, find the area of triangle ACD.

Markscheme

(i) evidence of choosing cosine rule ***(M1)***

e.g.
$\cos B = \frac{a^2+c^2-b^2}{2ac}$

correct substitution ***A1***

e.g.
$\frac{4^2+4^2-7.45^2}{2\times4\times4}$,
$7.45^2 = 32 - 32\cos y$,
$\cos y = -0.734\ldots$

$y = 137$ ***A1 N2***

(ii) correct substitution into area formula ***(A1)***

e.g.
$\frac{1}{2} \times 4 \times 4 \times \sin 137$,
$8 \sin 137$

area
$= 5.42$ ***A1 N2***

[5 marks]

---

20. Solve
$\cos 2x - 3\cos x - 3 - \cos^2 x = \sin^2 x$, for
$0 \le x \le 2\pi$.

[7 marks]

Markscheme

evidence of substituting for
$\cos 2x$ ***(M1)***

evidence of substituting into
$\sin^2 x + \cos^2 x = 1$ ***(M1)***

correct equation in terms of
$\cos x$ (seen anywhere) ***A1***

e.g.
$2\cos^2 x - 1 - 3\cos x - 3 = 1$,
$2\cos^2 x - 3\cos x - 5 = 0$

evidence of appropriate approach to solve ***(M1)***

e.g. factorizing, quadratic formula

appropriate working ***A1***

e.g.
$(2\cos x - 5)(\cos x + 1) = 0$,
$(2x - 5)(x + 1)$,
$\cos x = \frac{3 \pm \sqrt{49}}{4}$

correct solutions to the equation

e.g.
$\cos x = \frac{5}{2}$,
$\cos x = -1$,
$x = \frac{5}{2}$,
$x = -1$ ***(A1)***

$x = \pi$ ***A1 N4***

[7 marks]

Let
$f(x) = \sin^3 x + \cos^3 x \tan x, \frac{\pi}{2} < x < \pi$.

21a. Show that
$f(x) = \sin x$. *[2 marks]*

Markscheme

changing
$\tan x$ into
$\frac{\sin x}{\cos x}$ ***A1***

e.g.
$\sin^3 x + \cos^3 x \frac{\sin x}{\cos x}$

simplifying ***A1***

e.g
$\sin x(\sin^2 x + \cos^2 x)$,
$\sin^3 x + \sin x - \sin^3 x$

$f(x) = \sin x$ ***AG*** ***N0***

[2 marks]

21b. Let
$\sin x = \frac{2}{3}$. Show that
$f(2x) = -\frac{4\sqrt{5}}{9}$. *[5 marks]*

Markscheme

recognizing
$f(2x) = \sin 2x$, seen anywhere ***(A1)***

evidence of using double angle identity
$\sin(2x) = 2 \sin x \cos x$, seen anywhere ***(M1)***

evidence of using Pythagoras with
$\sin x = \frac{2}{3}$ ***M1***

e.g. sketch of right triangle,
$\sin^2 x + \cos^2 x = 1$

$\cos x = -\frac{\sqrt{5}}{3}$ (accept
$\frac{\sqrt{5}}{3}$) ***(A1)***

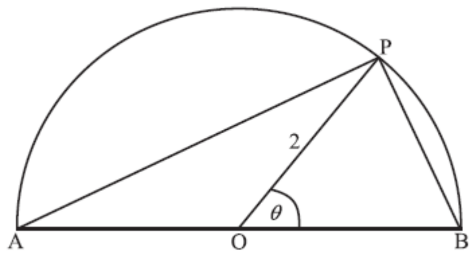
$f(2x) = 2\left(\frac{2}{3}\right)\left(-\frac{\sqrt{5}}{3}\right)$ ***A1***

$f(2x) = -\frac{4\sqrt{5}}{9}$ ***AG*** ***N0***

[5 marks]

The following diagram shows a semicircle centre O, diameter [AB], with radius 2.

Let P be a point on the circumference, with
$\text{P}\hat{\text{O}}\text{B} = \theta$ radians.



22a. Find the area of the triangle OPB, in terms of θ .

[2 marks]

Markscheme

evidence of using area of a triangle ***(M1)***

e.g.
$A = \frac{1}{2} \times 2 \times 2 \times \sin\theta$

$A = 2\sin\theta$ ***A1 N2***

[2 marks]

22b. Explain why the area of triangle OPA is the same as the area triangle OPB.

[3 marks]

Markscheme

METHOD 1

$\text{P}\hat{\text{O}}\text{A} = \pi - \theta$ ***(A1)***

area $\Delta\text{OPA} = \frac{1}{2}2 \times 2 \times \sin(\pi - \theta)$
$(= 2\sin(\pi - \theta))$ ***A1***

since
$\sin(\pi - \theta) = \sin\theta$ ***R1***

then both triangles have the same area ***AG N0***

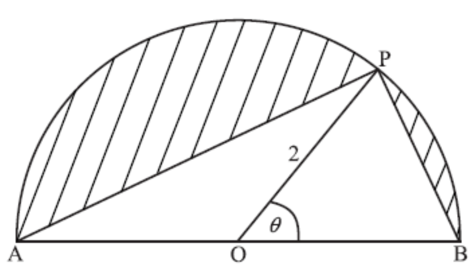
METHOD 2

triangle OPA has the same height and the same base as triangle OPB ***R3***

then both triangles have the same area ***AG N0***

[3 marks]

Let S be the total area of the two segments shaded in the diagram below.



22c. Show that
$S = 2(\pi - 2\sin\theta)$.

[3 marks]

Markscheme

area semicircle
$=\frac{1}{2}\times\pi(2)^2$
$(=2\pi)$ ***A1***

area $\Delta\mathrm{APB}=2\sin\theta+2\sin\theta$
$(=4\sin\theta)$ ***A1***

$S=\text{area of semicircle}-\text{area }\Delta\mathrm{APB}$
$(=2\pi-4\sin\theta)$ ***M1***

$S=2(\pi-2\sin\theta)$ ***AG N0***

[3 marks]

22d. Find the value of θ when S is a local minimum, justifying that it is a minimum. *[8 marks]*

Markscheme

METHOD 1

attempt to differentiate ***(M1)***

e.g.
$\frac{\mathrm{d}S}{\mathrm{d}\theta}=-4\cos\theta$

setting derivative equal to 0 ***(M1)***

correct equation ***A1***

e.g.
$-4\cos\theta=0$,
$\cos\theta=0$,
$4\cos\theta=0$

$\theta=\frac{\pi}{2}$ ***A1 N3***

EITHER

evidence of using second derivative ***(M1)***

$S''(\theta)=4\sin\theta$ ***A1***

$S''\left(\frac{\pi}{2}\right)=4$ ***A1***

it is a minimum because
$S''\left(\frac{\pi}{2}\right)>0$ ***R1 N0***

OR

evidence of using first derivative ***(M1)***

for
$\theta<\frac{\pi}{2}, S'(\theta)<0$ (may use diagram) ***A1***

for
$\theta>\frac{\pi}{2}, S'(\theta)>0$ (may use diagram) ***A1***

it is a minimum since the derivative goes from negative to positive ***R1 N0***

METHOD 2

$2\pi-4\sin\theta$ is minimum when
$4\sin\theta$ is a maximum ***R3***

$4\sin\theta$ is a maximum when
$\sin\theta=1$ ***(A2)***

$\theta=\frac{\pi}{2}$ ***A3 N3***

[8 marks]

22e. Find a value of θ for which S has its greatest value. *[2 marks]*

Markscheme

S is greatest when $4\sin\theta$ is smallest (or equivalent) ***(R1)***

$\theta = 0$ (or π) ***A1 N2***

[2 marks]

23a. Given that $\cos A = \frac{1}{3}$ and $0 \le A \le \frac{\pi}{2}$, find $\cos 2A$. *[3 marks]*

Markscheme

evidence of choosing the formula $\cos 2A = 2\cos^2 A - 1$ ***(M1)***

Note: If they choose another correct formula, do not award the ***M1*** unless there is evidence of finding $\sin^2 A = 1 - \frac{1}{9}$

correct substitution ***A1***

e.g. $\cos 2A = \left(\frac{1}{3}\right)^2 - \frac{8}{9}$, $\cos 2A = 2 \times \left(\frac{1}{3}\right)^2 - 1$

$\cos 2A = -\frac{7}{9}$ ***A1 N2***

[3 marks]

23b. Given that $\sin B = \frac{2}{3}$ and $\frac{\pi}{2} \le B \le \pi$, find $\cos B$. *[3 marks]*

Markscheme

METHOD 1

evidence of using
$\sin^2 B + \cos^2 B = 1$ ***(M1)***

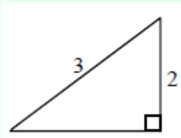
e.g.
$\left(\frac{2}{3}\right)^2 + \cos^2 B = 1$,
$\sqrt{\frac{5}{9}}$ (seen anywhere),

$\cos B = \pm\sqrt{\frac{5}{9}}$
$\left(= \pm\frac{\sqrt{5}}{3}\right)$ ***(A1)***

$\cos B = -\sqrt{\frac{5}{9}}$
$\left(= -\frac{\sqrt{5}}{3}\right)$ ***A1 N2***

METHOD 2

diagram ***M1***



for finding third side equals
$\sqrt{5}$ ***(A1)***

$\cos B = -\frac{\sqrt{5}}{3}$ ***A1 N2***

[3 marks]

The expression
$6\sin x \cos x$ can be expressed in the form
$a \sin bx$.

24a. Find the value of *a* and of *b* . *[3 marks]*

Markscheme

recognizing double angle ***M1***

e.g.
$3 \times 2\sin x \cos x$,
$3\sin 2x$

$a = 3$,
$b = 2$ ***A1A1 N3***

[3 marks]

24b. Hence or otherwise, solve the equation
$6\sin x \cos x = \frac{3}{2}$, for
$\frac{\pi}{4} \le x \le \frac{\pi}{2}$. *[4 marks]*

Markscheme

substitution
$3\sin 2x = \frac{3}{2}$ ***M1***

$\sin 2x = \frac{1}{2}$ ***A1***

finding the angle ***A1***

e.g.
$\frac{\pi}{6}$,
$2x = \frac{5\pi}{6}$

$x = \frac{5\pi}{12}$ ***A1*** ***N2***

Note: Award ***A0*** if other values are included.

[4 marks]