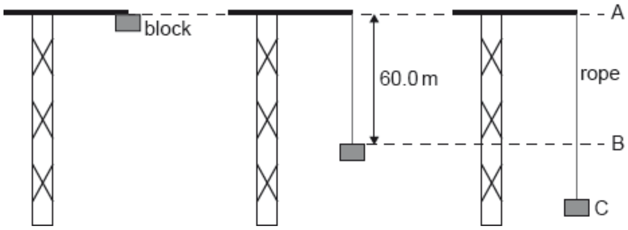
SHMP2 *[93 marks]*

An elastic climbing rope is tested by fixing one end of the rope to the top of a crane. The other end of the rope is connected to a block which is initially at position A. The block is released from rest. The mass of the rope is negligible.



The unextended length of the rope is 60.0 m. From position A to position B, the block falls freely.

1a. At position B the rope starts to extend. Calculate the speed of the block at position B. *[2 marks]*

Markscheme

use of conservation of energy

OR

$v^2 = u^2 + 2as$

v = «$\sqrt{2 \times 60.0 \times 9.81}$» = 34.3 «$ms^{-1}$»

[2 marks]

At position C the speed of the block reaches zero. The time taken for the block to fall between B and C is 0.759 s. The mass of the block is 80.0 kg.

1b. Determine the magnitude of the average resultant force acting on the block between B and C. *[2 marks]*

Markscheme

use of impulse $F_{ave} \times \Delta t = \Delta p$

OR

use of $F = ma$ with average acceleration

OR

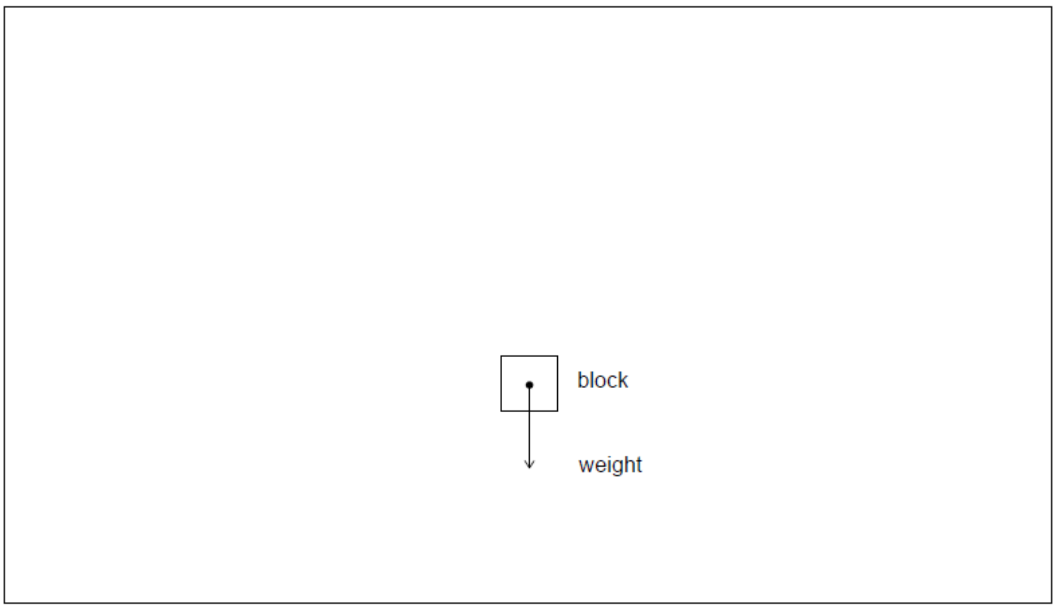
$F = \frac{80.0 \times 34.3}{0.759}$

3620«N»

Allow ECF from (a).

[2 marks]

1c. Sketch on the diagram the average resultant force acting on the block between B and C. The arrow on the diagram represents the weight of the block. *[2 marks]*



Markscheme

upwards

clearly longer than weight

For second marking point allow ECF from (b)(i) providing line is upwards.

[2 marks]

1d. Calculate the magnitude of the average force exerted by the rope on the block between B and C. *[2 marks]*

Markscheme

$3620 + 80.0 \times 9.81$

4400 «N»

Allow ECF from (b)(i).

[2 marks]

For the rope and block, describe the energy changes that take place

1e. between A and B. *[1 mark]*

Markscheme

(loss in) gravitational potential energy (of block) into kinetic energy (of block)

Must see names of energy (gravitational potential energy and kinetic energy) – Allow for reasonable variations of terminology (eg energy of motion for KE).

[1 mark]

1f. between B and C. *[1 mark]*

Markscheme

(loss in) gravitational potential and kinetic energy of block into elastic potential energy of rope

See note for 1(c)(i) for naming convention.

Must see either the block or the rope (or both) mentioned in connection with the appropriate energies.

[1 mark]

1g. The length reached by the rope at C is 77.4 m. Suggest how energy considerations could be used to determine the elastic constant of the rope. *[2 marks]*

Markscheme

k can be determined using EPE = $\frac{1}{2}kx^2$

correct statement or equation showing

GPE at A = EPE at C

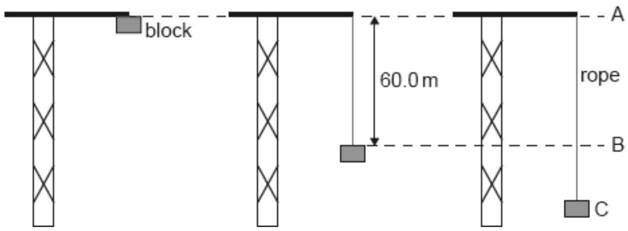
OR

(GPE + KE) at B = EPE at C

Candidate must clearly indicate the energy associated with either position A or B for MP2.

[2 marks]

An elastic climbing rope is tested by fixing one end of the rope to the top of a crane. The other end of the rope is connected to a block which is initially at position A. The block is released from rest. The mass of the rope is negligible.



The unextended length of the rope is 60.0 m. From position A to position B, the block falls freely.

In another test, the block hangs in equilibrium at the end of the same elastic rope. The elastic constant of the rope is 400 Nm^{-1}. The block is pulled 3.50 m vertically below the equilibrium position and is then released from rest.

1h. Calculate the time taken for the block to return to the equilibrium position for the first time. *[2 marks]*

Markscheme

$T = 2\pi\sqrt{\frac{80.0}{400}}$ = 2.81 «s»

time = $\frac{T}{4}$ = 0.702 «s»

*Award **[0]** for kinematic solutions that assume a constant acceleration.*

[2 marks]

1i. Calculate the speed of the block as it passes the equilibrium position. *[2 marks]*

Markscheme

ALTERNATIVE 1

$\omega = \frac{2\pi}{2.81} = 2.24$ «rad s^{-1}»

$v = 2.24 \times 3.50 = 7.84$ «ms^{-1}»

ALTERNATIVE 2

$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$ ***OR*** $\frac{1}{2}400 \times 3.5^2 = \frac{1}{2}80v^2$

$v = 7.84$ «ms^{-1}»

*Award **[0]** for kinematic solutions that assume a constant acceleration.*

Allow ECF for T from (e)(i).

[2 marks]

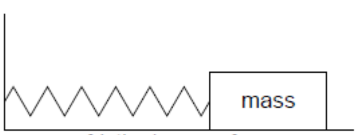
A student is investigating a method to measure the mass of a wooden block by timing the period of its oscillations on a spring.

2a. Describe the conditions required for an object to perform simple harmonic motion (SHM). *[2 marks]*

Markscheme

acceleration/restoring force is proportional to displacement
and in the opposite direction/directed towards equilibrium

A 0.52 kg mass performs simple harmonic motion with a period of 0.86 s when attached to the spring. A wooden block attached to the same spring oscillates with a period of 0.74 s.



2b. Calculate the mass of the wooden block. *[2 marks]*

Markscheme

ALTERNATIVE 1

$\frac{T_1^2}{T_2^2} = \frac{m_1}{m_2}$

mass = 0.38 / 0.39 «kg»

ALTERNATIVE 2

«use of *T* $= 2\pi\sqrt{\frac{m}{k}}$» *k* = 28 «Nm^{-1}»

«use of *T* $= 2\pi\sqrt{\frac{m}{k}}$» *m* = 0.38 / 0.39 «kg»

Allow ECF from MP1.

2c. In carrying out the experiment the student displaced the block horizontally by 4.8 cm from the equilibrium position. Determine the total energy in the oscillation of the wooden block. *[3 marks]*

Markscheme

ω = «$\frac{2\pi}{0.74}$» = 8.5 «rads^{-1}»

total energy = $\frac{1}{2} \times 0.39 \times 8.5^2 \times (4.8 \times 10^{-2})^2$

= 0.032 «J»

Allow ECF from (b) and incorrect ω.

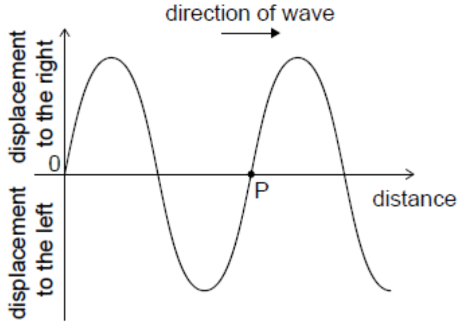
Allow answer using k from part (b).

2d. A second identical spring is placed in parallel and the experiment in (b) is repeated. Suggest how this change affects the fractional uncertainty in the mass of the block. *[3 marks]*

Markscheme

spring constant/k/stiffness would increase
T would be smaller
fractional uncertainty in *T* would be greater, so fractional uncertainty of mass of block would be greater

With the block stationary a longitudinal wave is made to travel through the original spring from left to right. The diagram shows the variation with distance *x* of the displacement *y* of the coils of the spring at an instant of time.



A point on the graph has been labelled that represents a point P on the spring.

2e. State the direction of motion of P on the spring. *[1 mark]*

Markscheme

left

2f. Explain whether P is at the centre of a compression or the centre of a rarefaction. *[2 marks]*

Markscheme

coils to the right of P move right and the coils to the left move left

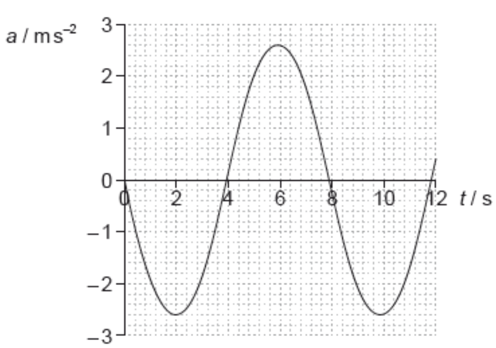
hence P at centre of rarefaction

Do not allow a bald statement of rarefaction or answers that don't include reference to the movement of coils.

Allow ECF from MP1 if the movement of the coils imply a compression.

This question is about simple harmonic motion (SHM).

The graph shows the variation with time t of the acceleration a of an object X undergoing simple harmonic motion (SHM).



3a. Define *simple harmonic motion (SHM)*. *[2 marks]*

Markscheme

force/acceleration proportional to the displacement/distance from a (fixed/equilibrium) point/mean position;

directed towards this (equilibrium) point / in opposite direction to displacement/ distance;

Allow algebra only if symbols are fully explained.

3b. X has a mass of 0.28 kg. Calculate the maximum force acting on X. *[1 mark]*

Markscheme

0.73 (N); *(allow answer in range of 0.71 to 0.75 (N))*

3c. Determine the maximum displacement of X. Give your answer to an appropriate number of significant figures. *[4 marks]*

Markscheme

use of $a_0 = -\omega^2 x_0$;

$T = 7.9\ (\mathrm{s})$ ***or*** $\omega = 0.795$ ***or*** $\frac{\pi}{4}\ (\mathrm{rad\,s^{-1}});\}$ *(allow answers in the range of T = 7.8 to 8.0 (s)* ***or*** *ω = 0.785 to 0.805 (rad* s^{-1}*))*

$x_0 = 4.1(1)\ (\mathrm{m});$ *(allow answers in the range of 4.0 to 4.25 (m))*

two significant figures in final answer whatever the value;

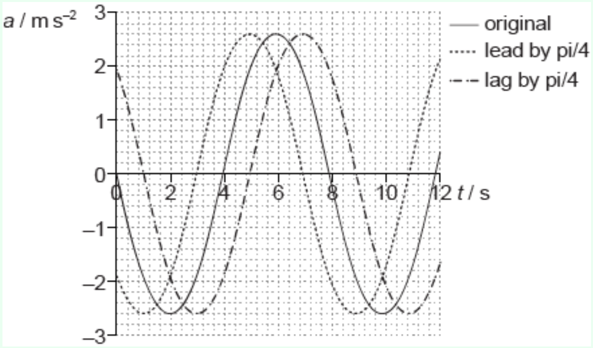
Award ***[4]*** *for a bald correct answer.*

3d. A second object Y oscillates with the same frequency as X but with a phase difference of $\frac{\pi}{4}$. Sketch, using the graph opposite, how the acceleration of object Y varies with t. *[2 marks]*

Markscheme

shape correct, constant amplitude for new curve, minimum of 10 s shown; } *(there must be some consistent lead or lag and no change in T)*

lead/lag of 1 s (to within half a square by eye);



This question is in **two** parts. **Part 1** is about wave motion. **Part 2** is about the melting of the Pobeda ice island.

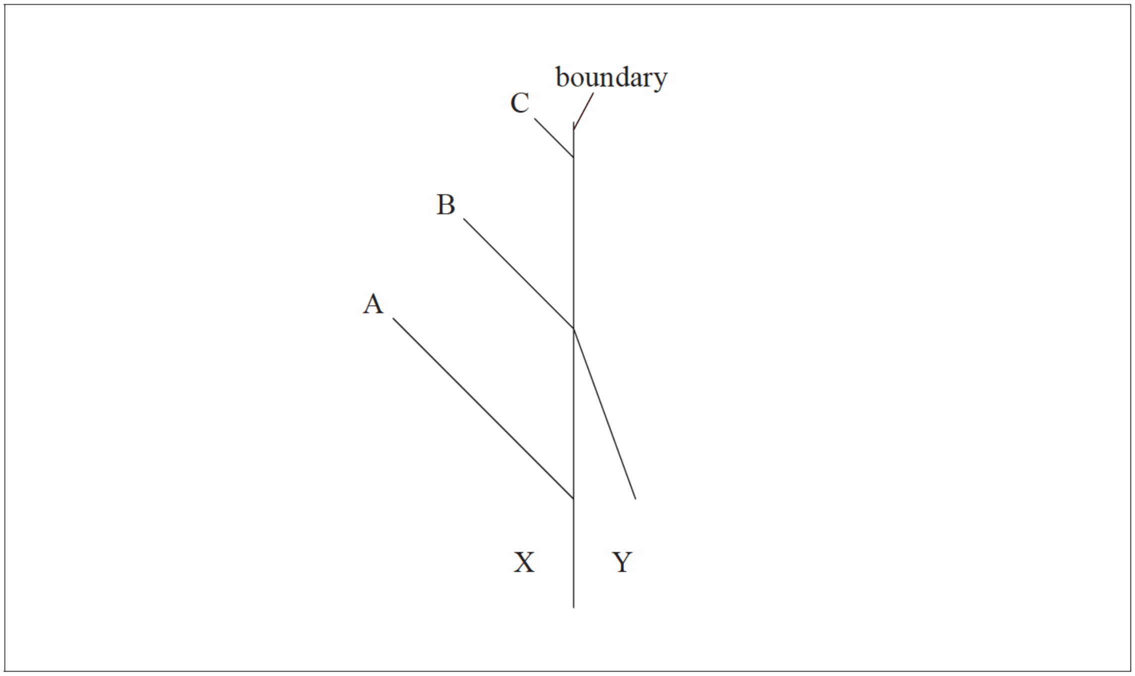
Part 1 Wave motion

4a. State what is meant by the terms ray and wavefront and state the relationship between them. *[3 marks]*

Markscheme

ray: direction of wave travel / energy propagation;
wavefront: line that joins points with same phase/of same crest/trough;
ray normal/at right angles/perpendicular to wavefront;

4b. The diagram shows three wavefronts, A, B and C, of a wave at a particular instant in time incident on a boundary between media X and Y. Wavefront B is also shown in medium Y. *[4 marks]*



(i) Draw a line to show wavefront C in medium Y.

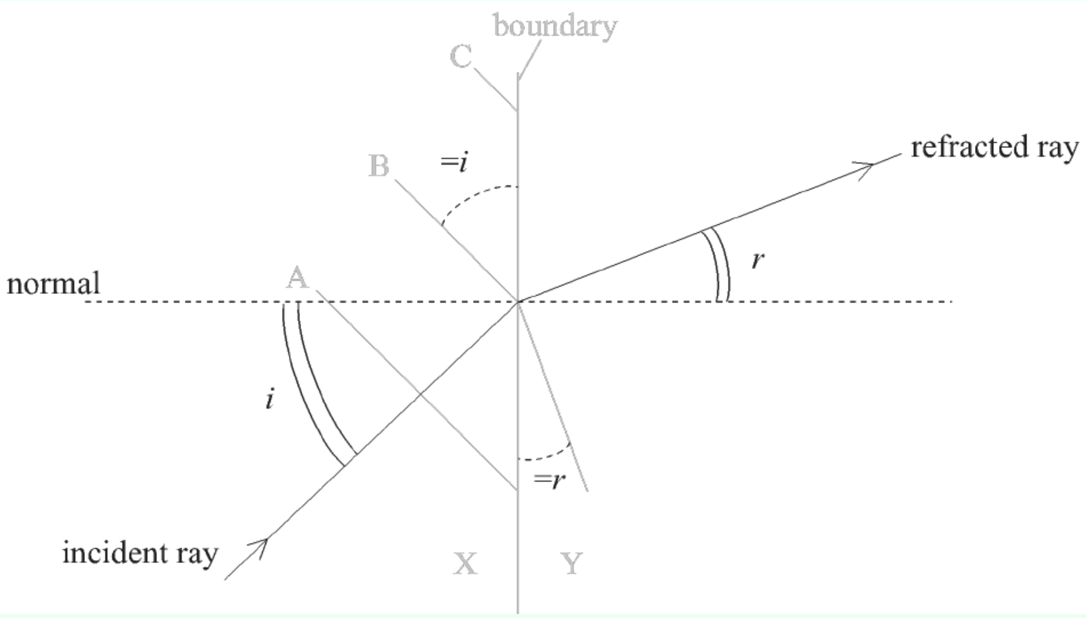
(ii) The refractive index of X is n_X and the refractive index of Y is n_Y. By making appropriate measurements, calculate $\frac{n_X}{n_Y}$.

Markscheme

(i) line parallel to existing line in Y and continuous at boundary; *(both needed)*

(ii) measures "wavelength" correctly in media X and Y; } *(by eye)*
(look for ratio of 0.5: 1 in responses)
$\frac{n_X}{n_Y} = \frac{\lambda_Y}{\lambda_X}$;
0.5:1; *(accept answers in the range of 0.47 to 0.53)*

or



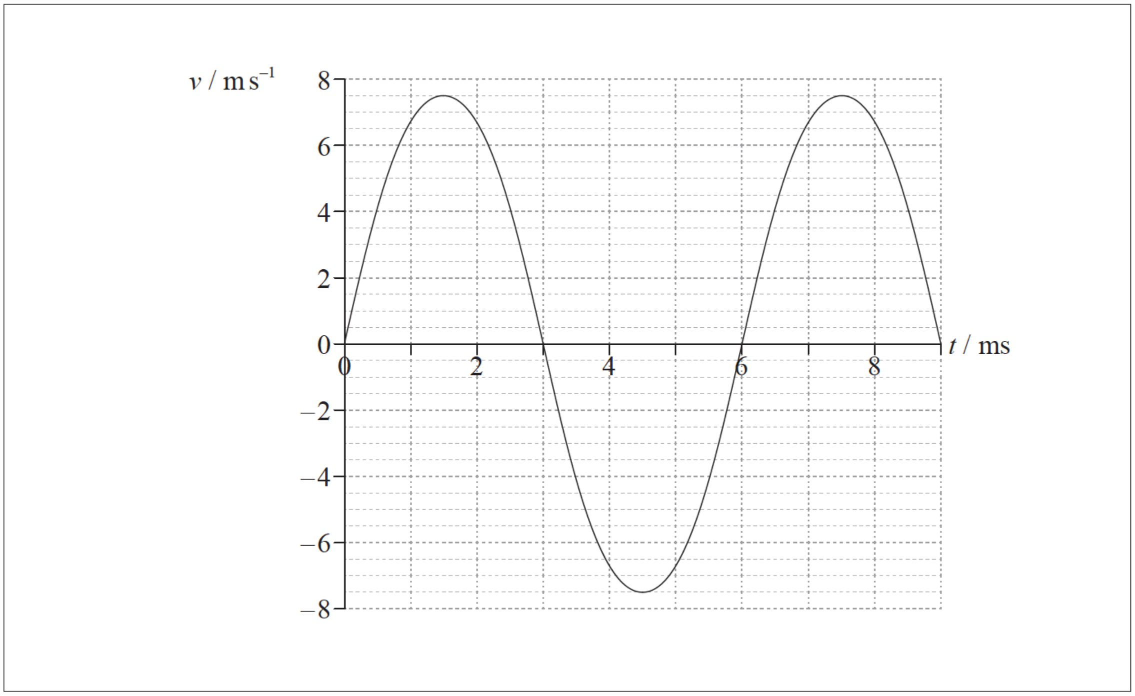
justification that angles needed for calculation are either pair of *i* and *r* as shown and angles measured correctly;
$\frac{n_X}{n_Y} = \frac{\sin r}{\sin i}$;
0.5:1;

4c. Describe the difference between transverse waves and longitudinal waves. *[2 marks]*

Markscheme

mention of perpendicular/right angle/90 ° angle for transverse and parallel for longitudinal;
clear comparison between direction of energy propagation and direction of vibration/oscillation of particles for both waves;

4d. The graph below shows the variation of the velocity *v* with time *t* for one oscillating particle of a medium. *[3 marks]*



(i) Calculate the frequency of oscillation of the particle.

(ii) Identify on the graph, with the letter M, a time at which the displacement of the particle is a maximum.

Markscheme

(i) time period=6.0ms;
167Hz;

(ii) M where line crosses *x*-axis;

(iii) counts rectangles (14±2) to first peak;
one rectangle equivalent to 0.5 mm;
7.2 mm;

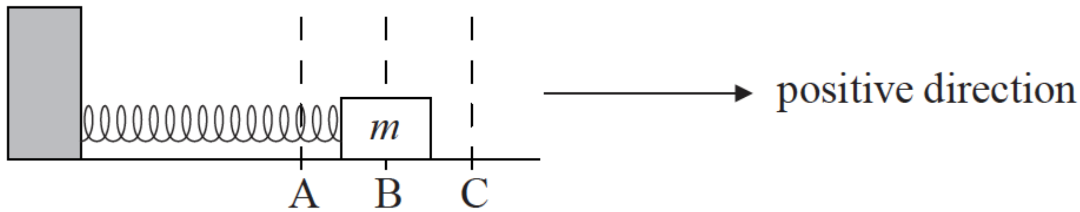
or

$\omega = (2\pi f =) 330\pi$;
$a = \left(\frac{v}{w} =\right) \frac{7.5}{330\pi}$;
7.2 mm;
Allow any valid algebraic method, eg $v = \omega\sqrt{(x_0{}^2 - x^2)}$.

This question is in **two** parts. **Part 1** is about simple harmonic motion and the superposition of waves. **Part 2** is about gravitational fields.

Part 1 Simple harmonic motion and the superposition of waves

An object of mass *m* is placed on a frictionless surface and attached to a light horizontal spring. The other end of the spring is fixed.



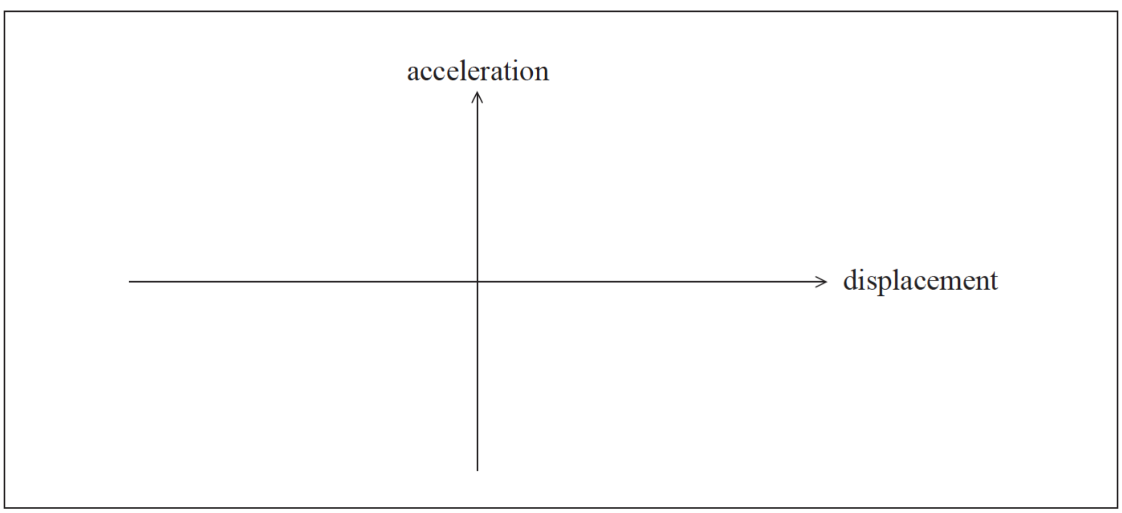
The equilibrium position is at B. The direction B to C is taken to be positive. The object is released from position A and executes simple harmonic motion between positions A and C.

5a. Define *simple harmonic motion*. *[2 marks]*

Markscheme

the force/acceleration is proportional to the displacement from the equilibrium position/centre;
the force/acceleration is directed towards the equilibrium position/centre / the force/acceleration is in the opposite direction to the displacement;

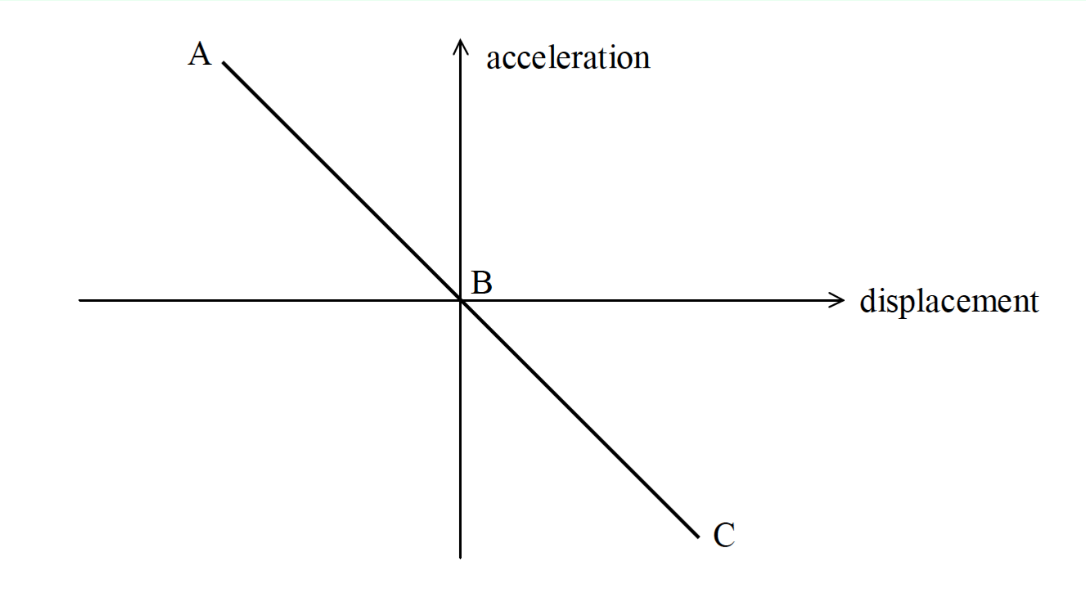
5b. (i) On the axes below, sketch a graph to show how the acceleration of the mass varies with displacement from the equilibrium position B. *[3 marks]*



(ii) On your graph, label the points that correspond to the positions A, B and C.

Markscheme

(i) straight line through the origin;
with negative gradient;



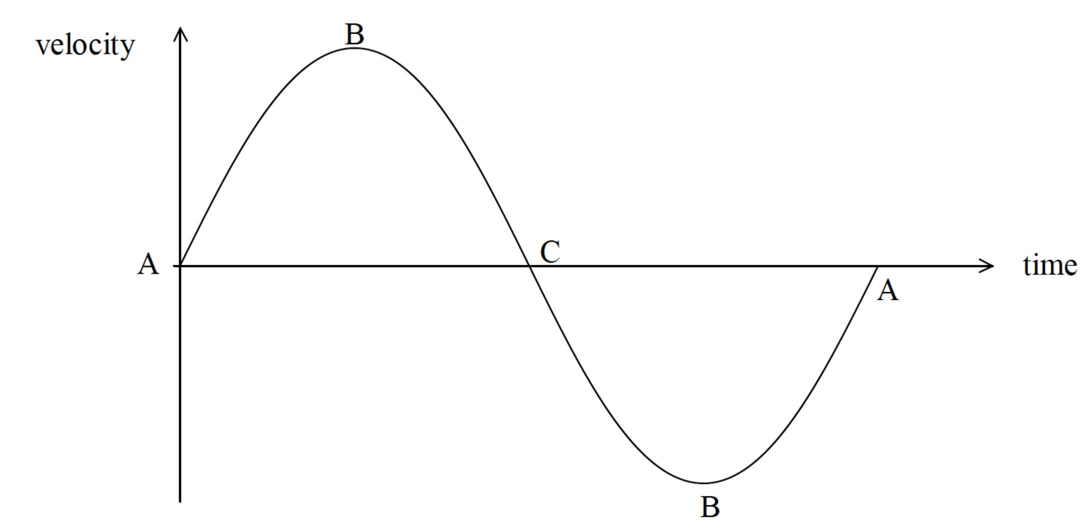
(ii) all three labels correct;

5c. (i) On the axes below, sketch a graph to show how the velocity of the mass varies with time from the moment of release from A until the mass returns to A for the first time. *[3 marks]*

(ii) On your graph, label the points that correspond to the positions A, B and C.

Markscheme

(i) positive sine graph;
drawn correctly for one period;



(ii) all three labels correct;
Accept either of the As and either of the Bs.
Accept either B if shown on the time axis in the correct position.

5d. The period of oscillation is 0.20s and the distance from A to B is 0.040m. Determine the maximum speed of the mass. *[3 marks]*

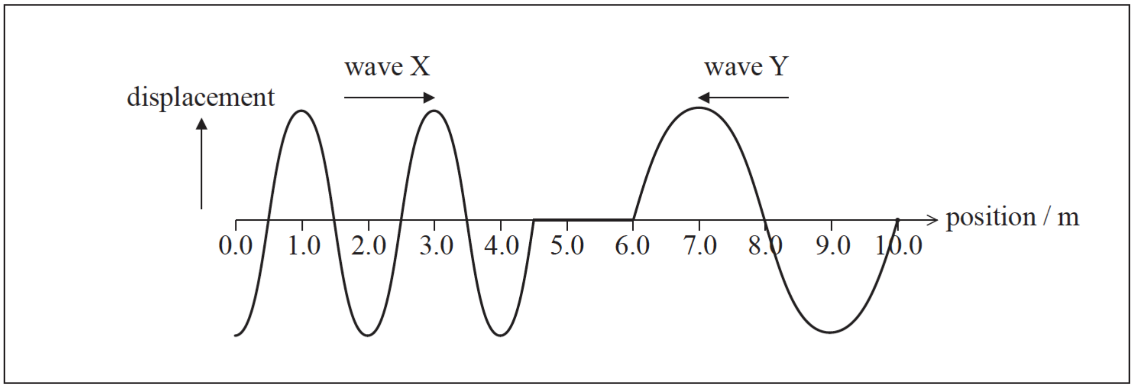
Markscheme

$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.20} = 31.42 \approx 31 \text{rads}^{-1}$;
$v_{\text{max}} = \omega x_0 = 31.42 \times 0.040$;
$v_{\text{max}} = 1.257 \approx 1.3 \text{ms}^{-1}$;

5e. A long spring is stretched so that it has a length of 10.0 m. Both ends are made to oscillate with simple harmonic motion so that transverse waves of equal amplitude but different frequency are generated. *[4 marks]*

Wave X, travelling from left to right, has wavelength 2.0 m, and wave Y, travelling from right to left, has wavelength 4.0 m. Both waves move along the spring at speed 10.0 m s^{-1}.

The diagram below shows the waves at an instant in time.



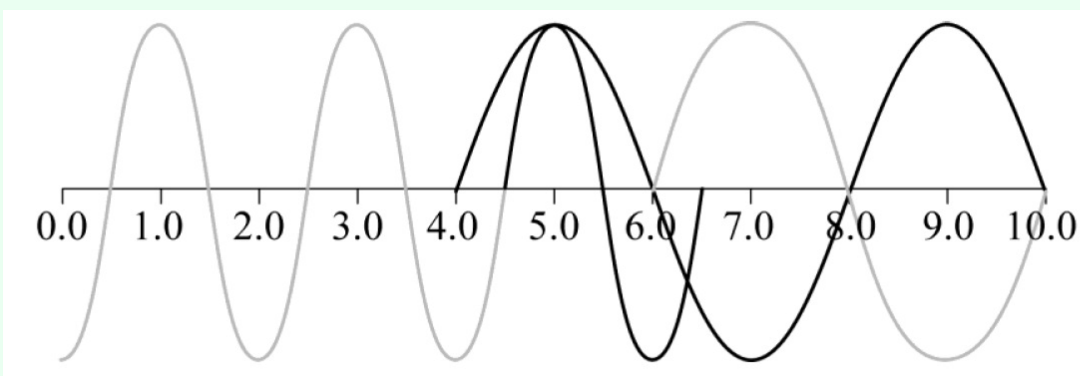
(i) State the principle of superposition as applied to waves.

(ii) By drawing on the diagram or otherwise, calculate the position at which the resultant wave will have maximum displacement 0.20 s later.

Markscheme

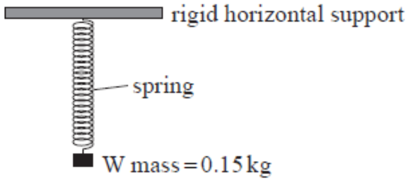
(i) if two or more waves overlap/meet/pass through the same point;
the resultant displacement at any point is found by adding the displacements produced by each individual wave;

(ii) 0.20 s later, wave X will have crests at 5.0, 3.0 and 1.0 m, wave Y will have crests at 5.0 and 9.0 m / each wave will have moved forward by 2.0 m in 0.20 s / wave profiles for 0.20 s later drawn on diagram;



maximum displacement where two crests meet, *i.e.* at 5.0 m;

6a. One end of a light spring is attached to a rigid horizontal support. *[8 marks]*



An object W of mass 0.15 kg is suspended from the other end of the spring. The extension x of the spring is proportional to the force F causing the extension. The force per unit extension of the spring k is 18 Nm^{-1}.

A student pulls W down such that the extension of the spring increases by 0.040 m. The student releases W and as a result W performs simple harmonic motion (SHM).

(i) State what is meant by the expression “W performs SHM”.

(ii) Determine the maximum acceleration of W.

(iii) Determine the period of oscillation of the spring.

(iv) Determine the maximum kinetic energy of W.

Markscheme

(i) the acceleration of (force acting on) W is proportional to its displacement
from equilibrium;
and directed towards equilibrium;

(ii) F=(18×0.04=) 0.72N;
acceleration=$\frac{0.72}{0.15}$=4.8 ms^{-2};

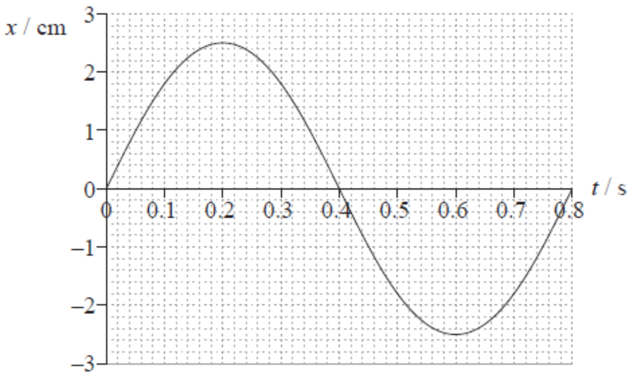
(iii) $\omega = \sqrt{\frac{a}{x}}$;
=10.95$rads^{-2}$;
$T = \left(\frac{2\pi}{\omega} =\right) \frac{6.28}{10.95} = 0.57\text{s}$;

(iv) =1.4×10^{-2}(J);

6b. A light spring is stretched horizontally and a longitudinal travelling wave is set up in the spring, travelling to the right. *[6 marks]*

(i) Describe, in terms of the propagation of energy, what is meant by a longitudinal travelling wave.

(ii) The graph shows how the displacement x of one coil C of the spring varies with time t.



The speed of the wave is 3.0 cms^{-1}. Determine the wavelength of the wave.

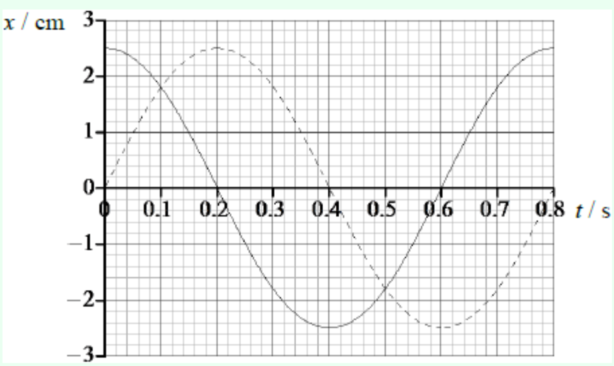
(iii) Draw, on the graph in (c)(ii), the displacement of a coil of the spring that is 1.8 cm away from C in the direction of travel of the wave, explaining your answer.

Markscheme

(i) the direction of oscillation of the particles of the medium;
(*must see "particles"*)
is in the direction of energy propagation;
Accept answer in terms of coils of spring in place of particles of medium.

(ii) frequency=$\left(\frac{1}{T}=\frac{1}{0.80}=\right)1.25\text{Hz}$;
wavelength=$\frac{v}{f}=\frac{3.0}{1.25}=2.4\text{cm}$ ***or*** 2.4×10^{-2}m;

(iii)



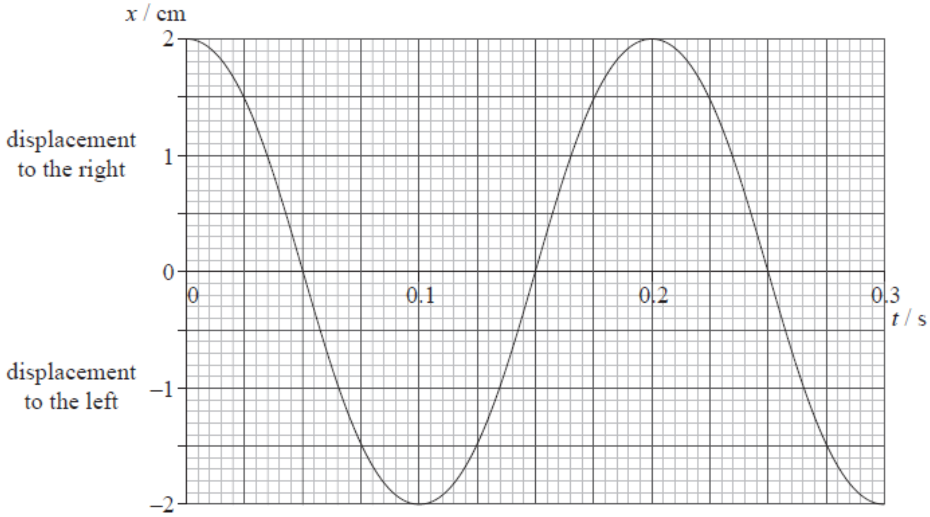
graph: positive cosine; *(line must cross axis at 0.2 and 0.6 as shown)*
explanation: 1.8 cm is ¾ of a wavelength;

Part 2 Simple harmonic oscillations

A longitudinal wave travels through a medium from left to right.

Graph 1 shows the variation with time *t* of the displacement *x* of a particle P in the medium.

Graph 1



7a. For particle P, *[6 marks]*

(i) state how graph 1 shows that its oscillations are not damped.

(ii) calculate the magnitude of its maximum acceleration.

(iii) calculate its speed at *t*=0.12 s.

(iv) state its direction of motion at *t*=0.12 s.

Markscheme

(i) the amplitude is constant;

(ii) period is 0.20s;

$$a_{\max} = \left(\left[\frac{2\pi}{T}\right]^2 x_0 = 31.4^2 \times 2.0 \times 10^{-2}\right) = 19.7 \approx 20\text{ms}^{-2}$$

*Award **[2]** for correct bald answer and ignore any negative signs in answer.*

(iii) displacement at *t* = 0.12cm is (–)1.62cm;

$$v\left(= \frac{2\pi}{T}\sqrt{x_0 - x^2}\right) = 31.4\sqrt{\left(2.0 \times 10^{-2}\right)^2 - \left(1.62 \times 10^{-2}\right)^2} = 0.37\text{ms}^{-1};$$

Accept displacement in range 1.60 to 1.70 cm for an answer in range 0.33ms $^{-1}$ *to 0.38ms* $^{-1}$.

or

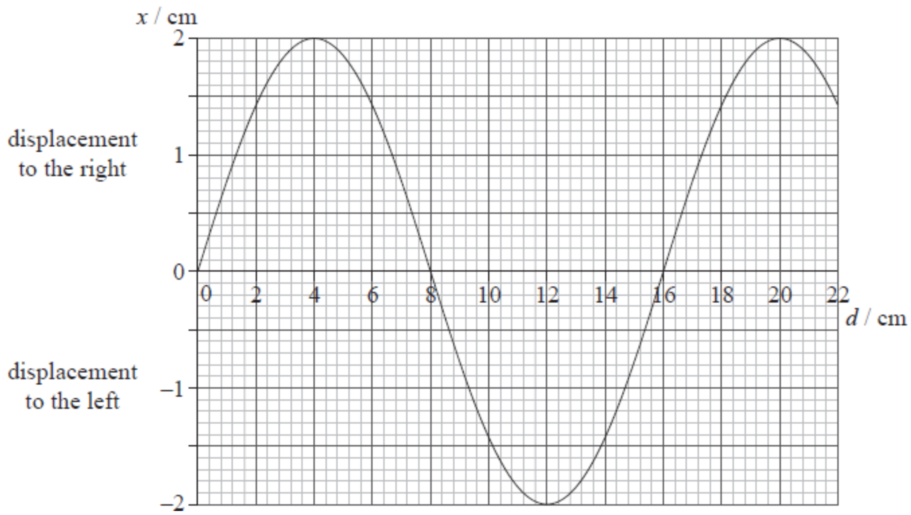
$$v_0 = \frac{2\pi}{T}x_0 = 0.628\text{ms}^{-1};$$
$$|v| = \left(\left|-v_0 \sin\left[\frac{2\pi}{T}t\right]\right| \Rightarrow |v| = |-0.628\sin[31.4 \times 0.12]| = |0.37|\right) = 0.37\text{ms}^{-1};$$

or

drawing a tangent at 0.12s;
measurement of slope of tangent;
Accept answer in range 0.33ms $^{-1}$ *to 0.38ms* $^{-1}$ *.*

7b. Graph 2 shows the variation with position *d* of the displacement *x* of particles in the medium at a particular instant of time. *[4 marks]*

Graph 2



Determine for the longitudinal wave, using graph 1 and graph 2,

(i) the frequency.

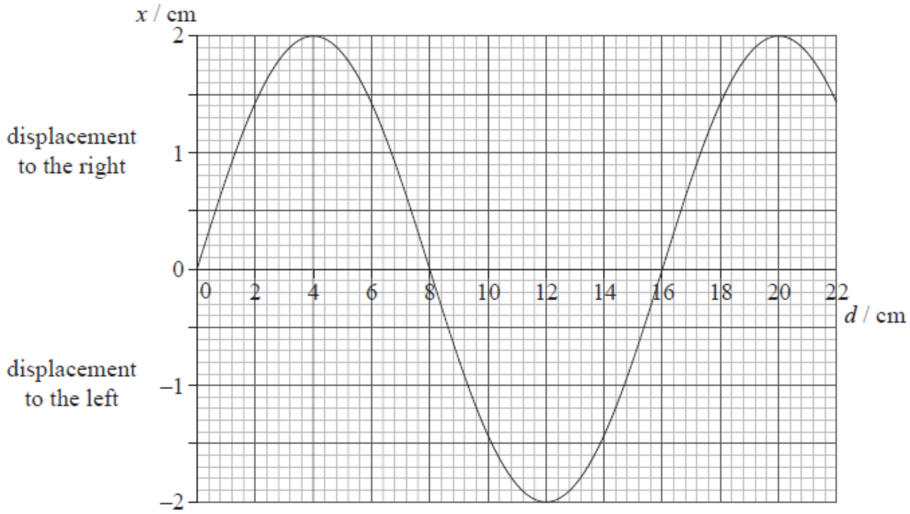
(ii) the speed.

Markscheme

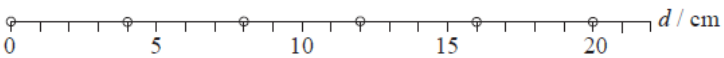
(i) use of $f = \frac{1}{T}$;
and so $f\left(= \frac{1}{0.20}\right) = 5.0\text{Hz}$;

(ii) wavelength is 16cm;
and so speed is $v(=f\lambda=5.0\times0.16)=0.80\text{ms}^{-1}$;

7c. **Graph 2** – reproduced to assist with answering (c)(i). *[4 marks]*



(c) The diagram shows the equilibrium positions of six particles in the medium.

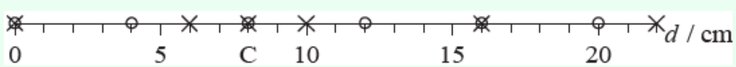


(i) On the diagram above, draw crosses to indicate the positions of these six particles at the instant of time when the displacement is given by graph 2.

(ii) On the diagram above, label with the letter C a particle that is at the centre of a compression.

Markscheme

(i) points at 0, 8 and 16 cm stay in the same place;
points at 4 and 20 cm move 2 cm to the right;
point at 12 cm moves 2 cm to the left;



(ii) the point at 8 cm;