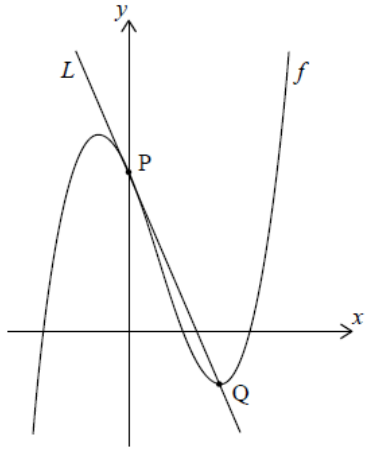


Ch2Review [254 marks]

Let $f(x) = x^3 - 2x^2 + ax + 6$. Part of the graph of f is shown in the following diagram.



The graph of f crosses the y -axis at the point P . The line L is tangent to the graph of f at P .

1a. Find the coordinates of P .

[2 marks]

Markscheme

valid approach (M1)

eg $f(0)$, $0^3 - 2(0)^2 + a(0) + 6$, $f(0) = 6$, $(0, y)$

$(0, 6)$ (accept $x = 0$ and $y = 6$) A1 N2

[2 marks]

1b. Find $f'(x)$.

[2 marks]

Markscheme

$f' = 3x^2 - 4x + a$ A2 N2

[2 marks]

1c. Hence, find the equation of L in terms of a .

[4 marks]

Markscheme

valid approach (M1)

eg $f'(0)$

correct working (A1)

eg $3(0)^2 - 4(0) + a$, slope = a , $f'(0) = a$

attempt to substitute gradient and coordinates into linear equation (M1)

eg $y - 6 = a(x - 0)$, $y - 0 = a(x - 6)$, $6 = a(0) + c$, $L = ax + 6$

correct equation A1 N3

eg $y = ax + 6$, $y - 6 = ax$, $y - 6 = a(x - 0)$

[4 marks]

- 1d. The graph of f has a local minimum at the point Q. The line L passes through Q. [8 marks]
Find the value of a .

Markscheme

valid approach to find intersection (M1)

eg $f(x) = L$

correct equation (A1)

eg $x^3 - 2x^2 + ax + 6 = ax + 6$

correct working (A1)

eg $x^3 - 2x^2 = 0$, $x^2(x - 2) = 0$

$x = 2$ at Q (A1)

valid approach to find minimum (M1)

eg $f'(x) = 0$

correct equation (A1)

eg $3x^2 - 4x + a = 0$

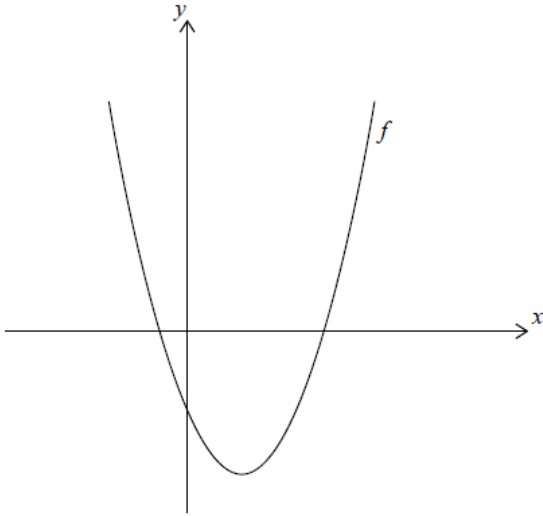
substitution of **their** value of x at Q into **their** $f'(x) = 0$ equation (M1)

eg $3(2)^2 - 4(2) + a = 0$, $12 - 8 + a = 0$

$a = -4$ A1 N0

[8 marks]

Let $f(x) = x^2 - 4x - 5$. The following diagram shows part of the graph of f .



2a. Find the x -intercepts of the graph of f .

[5 marks]

Markscheme

valid approach (M1)

eg $f(x) = 0$, $x^2 - 4x - 5 = 0$

valid attempt to solve quadratic equation (M1)

eg factorizing, formula, completing the square

evidence of correct working (A1)

eg $(x - 5)(x + 1)$, $x = \frac{4 \pm \sqrt{16 - 4(-5)}}{2}$

$x = -1$, $x = 5$ (accept $(-1, 0)$, $(5, 0)$) **A1A1 N3**

[5 marks]

2b. Find the equation of the axis of symmetry of the graph of f .

[2 marks]

Markscheme

correct working (A1)

eg $\frac{-(-4)}{2(1)}$, $\frac{-1+5}{2}$

$x = 2$ (must be an equation with $x =$) **A1 N2**

[2 marks]

The function can be written in the form $f(x) = (x - h)^2 + k$.

2c. Write down the value of h .

[1 mark]

Markscheme

$h = 2$ **A1 N1**

[1 mark]

2d. Find the value of k .

[3 marks]

Markscheme

METHOD 1

valid approach **(M1)**

eg $f(2)$

correct substitution **(A1)**

eg $(2)^2 - 4(2) - 5$

$k = -9$ **A1 N2**

METHOD 2

valid attempt to complete the square **(M1)**

eg $x^2 - 4x + 4$

correct working **(A1)**

eg $(x^2 - 4x + 4) - 4 - 5$, $(x - 2)^2 - 9$

$k = -9$ **A1 N2**

[3 marks]

2e. The graph of a second function, g , is obtained by a reflection of the graph of f in the y -axis, followed by a translation of $\begin{pmatrix} -3 \\ 6 \end{pmatrix}$. [5 marks]

Find the coordinates of the vertex of the graph of g .

Markscheme

METHOD 1 (working with vertex)

vertex of f is at $(2, -9)$ **(A1)**

correct horizontal reflection **(A1)**

eg $x = -2, (-2, -9)$

valid approach for translation of **their** x or y value **(M1)**

eg $x - 3, y + 6, \begin{pmatrix} -2 \\ -9 \end{pmatrix} + \begin{pmatrix} -3 \\ 6 \end{pmatrix}$, one correct coordinate for vertex

vertex of g is $(-5, -3)$ (accept $x = -5, y = -3$) **A1A1 N1N1**

METHOD 2 (working with function)

correct approach for horizontal reflection **(A1)**

eg $f(-x)$

correct horizontal reflection **(A1)**

eg $(-x)^2 - 4(-x) - 5, x^2 + 4x - 5, (-x - 2)^2 - 9$

valid approach for translation of **their** x or y value **(M1)**

eg $(x + 3)^2 + 4(x + 3) - 5 + 6, x^2 + 10x + 22, (x + 5)^2 - 3$, one correct coordinate for vertex

vertex of g is $(-5, -3)$ (accept $x = -5, y = -3$) **A1A1 N1N1**

[5 marks]

Let $f(x) = ax^2 - 4x - c$. A horizontal line, L , intersects the graph of f at $x = -1$ and $x = 3$.

3a. The equation of the axis of symmetry is $x = p$. Find p .

[2 marks]

Markscheme

METHOD 1 (using symmetry to find p)

valid approach **(M1)**

eg $\frac{-1+3}{2}$, 

$p = 1$ **A1 N2**

Note: Award no marks if they work backwards by substituting $a = 2$ into $-\frac{b}{2a}$ to find p .

Do not accept $p = \frac{2}{a}$.

METHOD 2 (calculating a first)

(i) & (ii) valid approach to calculate a **M1**

eg $a + 4 - c = a(3^2) - 4(3) - c$, $f(-1) = f(3)$

correct working **A1**

eg $8a = 16$

$a = 2$ **AG N0**

valid approach to find p **(M1)**

eg $-\frac{b}{2a}$, $\frac{4}{2(2)}$

$p = 1$ **A1 N2**

[2 marks]

3b. Hence, show that $a = 2$.

[2 marks]

Markscheme

METHOD 1

valid approach **M1**

eg $-\frac{b}{2a}$, $\frac{4}{2a}$ (might be seen in (i)), $f'(1) = 0$

correct equation **A1**

eg $\frac{4}{2a} = 1$, $2a(1) - 4 = 0$

$a = 2$ **AG NO**

METHOD 2 (calculating a first)

(i) & (ii) valid approach to calculate a **M1**

eg $a + 4 - c = a(3^2) - 4(3) - c$, $f(-1) = f(3)$

correct working **A1**

eg $8a = 16$

$a = 2$ **AG NO**

[2 marks]

3c. The equation of L is $y = 5$. Find the value of c .

[3 marks]

Markscheme

valid approach **(M1)**

eg $f(-1) = 5$, $f(3) = 5$

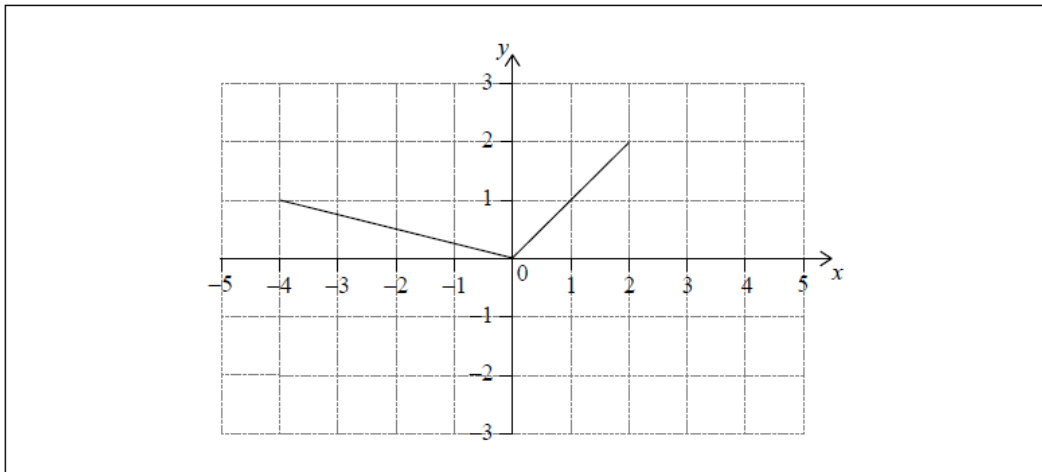
correct working **(A1)**

eg $2 + 4 - c = 5$, $18 - 12 - c = 5$

$c = 1$ **A1 N2**

[3 marks]

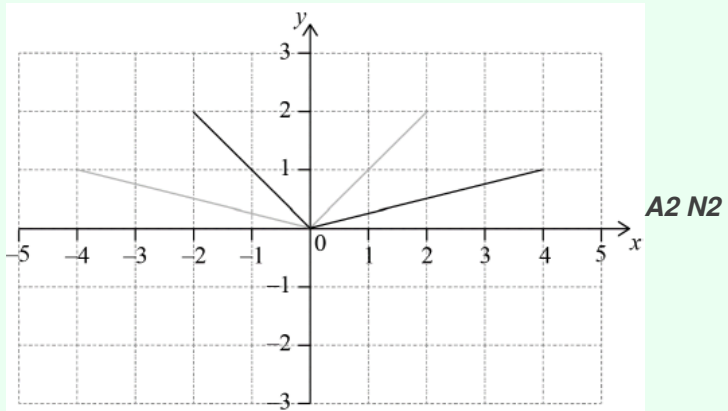
The following diagram shows the graph of a function f , for $-4 \leq x \leq 2$.



4a. On the same axes, sketch the graph of $f(-x)$.

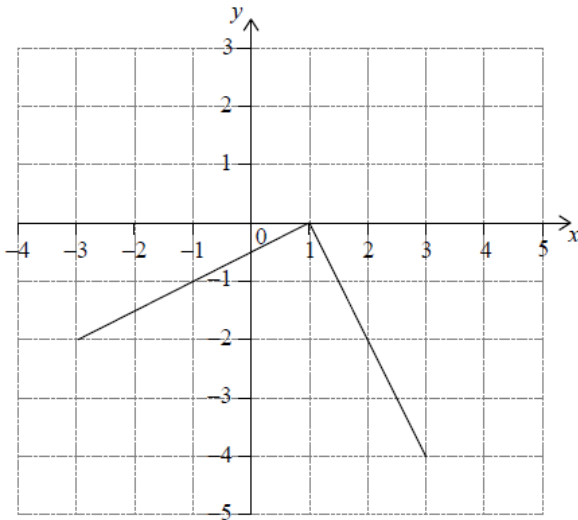
[2 marks]

Markscheme



[2 marks]

- 4b. Another function, g , can be written in the form $g(x) = a \times f(x + b)$. The following diagram shows the graph of g . [4 marks]



Write down the value of a and of b .

Markscheme

recognizing horizontal shift/translation of 1 unit (M1)

eg $b = 1$, moved 1 right

recognizing vertical stretch/dilation with scale factor 2 (M1)

eg $a = 2$, $y \times (-2)$

$a = -2$, $b = -1$ A1A1 N2N2

[4 marks]

5. Let $f(x) = px^2 + qx - 4p$, where $p \neq 0$. Find the number of roots for the equation $f(x) = 0$. [7 marks]

Justify your answer.

Markscheme

METHOD 1

evidence of discriminant (M1)
eg $b^2 - 4ac$, Δ

correct substitution into discriminant (A1)
eg $q^2 - 4p(-4p)$

correct discriminant A1
eg $q^2 + 16p^2$

$16p^2 > 0$ (accept $p^2 > 0$) A1

$q^2 \geq 0$ (do not accept $q^2 > 0$) A1

$q^2 + 16p^2 > 0$ A1

f has 2 roots A1 NO

METHOD 2

y -intercept = $-4p$ (seen anywhere) A1

if p is positive, then the y -intercept will be negative A1

an upward-opening parabola with a negative y -intercept R1
eg sketch that must indicate $p > 0$.

if p is negative, then the y -intercept will be positive A1

a downward-opening parabola with a positive y -intercept R1
eg sketch that must indicate $p > 0$.

f has 2 roots A2 NO

[7 marks]

A quadratic function f can be written in the form $f(x) = a(x - p)(x - 3)$. The graph of f has axis of symmetry $x = 2.5$ and y -intercept at $(0, -6)$


6a. Find the value of p .

[3 marks]

Markscheme

METHOD 1 (using x -intercept)

determining that 3 is an x -intercept (M1)

eg $x - 3 = 0$, 

valid approach (M1)

eg $3 - 2.5, \frac{p+3}{2} = 2.5$

$p = 2$ A1 N2

METHOD 2 (expanding $f(x)$)

correct expansion (accept absence of a) (A1)

eg $ax^2 - a(3+p)x + 3ap, x^2 - (3+p)x + 3p$

valid approach involving equation of axis of symmetry (M1)

eg $\frac{-b}{2a} = 2.5, \frac{a(3+p)}{2a} = \frac{5}{2}, \frac{3+p}{2} = \frac{5}{2}$

$p = 2$ A1 N2

METHOD 3 (using derivative)

correct derivative (accept absence of a) (A1)

eg $a(2x - 3 - p), 2x - 3 - p$

valid approach (M1)

eg $f'(2.5) = 0$

$p = 2$ A1 N2

[3 marks]

6b. Find the value of a .

[3 marks]

Markscheme

attempt to substitute $(0, -6)$ (M1)

eg $-6 = a(0 - 2)(0 - 3), 0 = a(-8)(-9), a(0)^2 - 5a(0) + 6a = -6$

correct working (A1)

eg $-6 = 6a$

$a = -1$ A1 N2

[3 marks]

6c. The line $y = kx - 5$ is a tangent to the curve of f . Find the values of k .

[8 marks]

Markscheme

METHOD 1 (using discriminant)

recognizing tangent intersects curve once (M1)

recognizing one solution when discriminant = 0 M1

attempt to set up equation (M1)

eg $g = f$, $kx - 5 = -x^2 + 5x - 6$

rearranging their equation to equal zero (M1)

eg $x^2 - 5x + kx + 1 = 0$

correct discriminant (if seen explicitly, not just in quadratic formula) A1

eg $(k - 5)^2 - 4$, $25 - 10k + k^2 - 4$

correct working (A1)

eg $k - 5 = \pm 2$, $(k - 3)(k - 7) = 0$, $\frac{10 \pm \sqrt{100 - 4 \times 21}}{2}$

$k = 3, 7$ A1A1 NO

METHOD 2 (using derivatives)

attempt to set up equation (M1)

eg $g = f$, $kx - 5 = -x^2 + 5x - 6$

recognizing derivative/slope are equal (M1)

eg $f' = m_T$, $f' = k$

correct derivative of f (A1)

eg $-2x + 5$

attempt to set up equation in terms of either x or k M1

eg $(-2x + 5)x - 5 = -x^2 + 5x - 6$, $k \left(\frac{5-k}{2} \right) - 5 = - \left(\frac{5-k}{2} \right)^2 + 5 \left(\frac{5-k}{2} \right) - 6$

rearranging their equation to equal zero (M1)

eg $x^2 - 1 = 0$, $k^2 - 10k + 21 = 0$

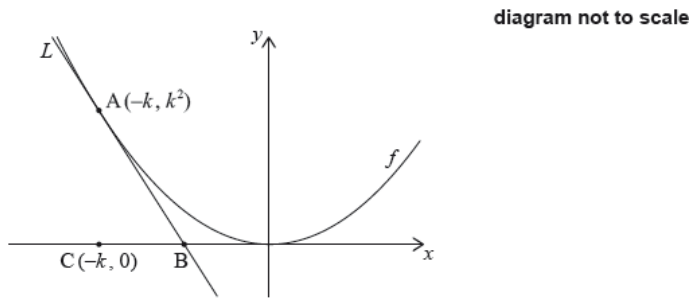
correct working (A1)

eg $x = \pm 1$, $(k - 3)(k - 7) = 0$, $\frac{10 \pm \sqrt{100 - 4 \times 21}}{2}$

$k = 3, 7$ A1A1 NO

[8 marks]

Let $f(x) = x^2$. The following diagram shows part of the graph of f .



The line L is the tangent to the graph of f at the point $A(-k, k^2)$, and intersects the x -axis at point B . The point C is $(-k, 0)$.

7a. Write down $f'(x)$.

[1 mark]

Markscheme

$$f'(x) = 2x \quad \mathbf{A1} \quad \mathbf{N1}$$

[1 mark]

7b. Find the gradient of L .

[2 marks]

Markscheme

attempt to substitute $x = -k$ into their derivative **(M1)**

gradient of L is $-2k$ **A1 N2**

[2 marks]

7c. Show that the x -coordinate of B is $-\frac{k}{2}$.

[5 marks]

Markscheme

METHOD 1

attempt to substitute coordinates of A and their gradient into equation of a line **(M1)**

eg $k^2 = -2k(-k) + b$

correct equation of L in any form **(A1)**

eg $y - k^2 = -2k(x + k)$, $y = -2kx - k^2$

valid approach **(M1)**

eg $y = 0$

correct substitution into L equation **A1**

eg $-k^2 = -2kx - 2k^2$, $0 = -2kx - k^2$

correct working **A1**

eg $2kx = -k^2$

$x = -\frac{k}{2}$ **AG NO**

METHOD 2

valid approach **(M1)**

eg gradient = $\frac{y_2 - y_1}{x_2 - x_1}$, $-2k = \frac{\text{rise}}{\text{run}}$

recognizing $y = 0$ at B **(A1)**

attempt to substitute coordinates of A and B into slope formula **(M1)**

eg $\frac{k^2 - 0}{-k - x}$, $\frac{-k^2}{x + k}$

correct equation **A1**

eg $\frac{k^2 - 0}{-k - x} = -2k$, $\frac{-k^2}{x + k} = -2k$, $-k^2 = -2k(x + k)$

correct working **A1**

eg $2kx = -k^2$

$x = -\frac{k}{2}$ **AG NO**

[5 marks]

7d. Find the area of triangle ABC, giving your answer in terms of k .

[2 marks]

Markscheme

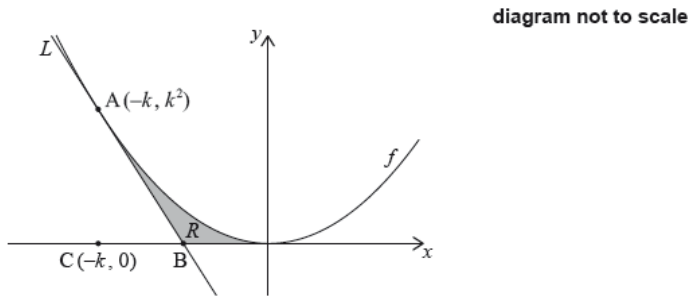
valid approach to find area of triangle **(M1)**

eg $\frac{1}{2}(k^2) \left(\frac{k}{2}\right)$

area of ABC = $\frac{k^3}{4}$ **A1 N2**

[2 marks]

The region R is enclosed by L , the graph of f , and the x -axis. This is shown in the following diagram.



7e. Given that the area of triangle ABC is p times the area of R , find the value of p . [7 marks]

Markscheme

METHOD 1 ($\int f$ – triangle)

valid approach to find area from $-k$ to 0 (M1)

eg $\int_{-k}^0 x^2 dx$, $\int_0^{-k} f$

correct integration (seen anywhere, even if **MO** awarded) **A1**

eg $\frac{x^3}{3}$, $\left[\frac{1}{3}x^3\right]_{-k}^0$

substituting **their** limits into **their** integrated function and subtracting (M1)

eg $0 - \frac{(-k)^3}{3}$, area from $-k$ to 0 is $\frac{k^3}{3}$

Note: Award **MO** for substituting into original or differentiated function.

attempt to find area of R (M1)

eg $\int_{-k}^0 f(x) dx$ – triangle

correct working for R (A1)

eg $\frac{k^3}{3} - \frac{k^3}{4}$, $R = \frac{k^3}{12}$

correct substitution into triangle = pR (A1)

eg $\frac{k^3}{4} = p \left(\frac{k^3}{3} - \frac{k^3}{4} \right)$, $\frac{k^3}{4} = p \left(\frac{k^3}{12} \right)$

$p = 3$ **A1 N2**

METHOD 2 ($\int (f - L)$)

valid approach to find area of R (M1)

eg $\int_{-k}^{-\frac{k}{2}} x^2 - (-2kx - k^2) dx + \int_{-\frac{k}{2}}^0 x^2 dx$, $\int_{-k}^{-\frac{k}{2}} (f - L) + \int_{-\frac{k}{2}}^0 f$

correct integration (seen anywhere, even if **MO** awarded) **A2**

eg $\frac{x^3}{3} + kx^2 + k^2x$, $\left[\frac{x^3}{3} + kx^2 + k^2x\right]_{-k}^{-\frac{k}{2}} + \left[\frac{x^3}{3}\right]_{-\frac{k}{2}}^0$

substituting **their** limits into **their** integrated function and subtracting (M1)

eg

$$\left(\left(\frac{-k}{3} \right)^3 + k \left(\frac{-k}{2} \right)^2 + k^2 \left(\frac{-k}{2} \right) \right) - \left(\frac{(-k)^3}{3} + k(-k)^2 + k^2(-k) \right) + (0) - \left(\frac{\left(\frac{-k}{2} \right)^3}{3} \right)$$

Note: Award **M0** for substituting into original or differentiated function.

correct working for R **(A1)**

eg $\frac{k^3}{24} + \frac{k^3}{24}, -\frac{k^3}{24} + \frac{k^3}{4} - \frac{k^3}{2} + \frac{k^3}{3} - k^3 + k^3 + \frac{k^3}{24}, R = \frac{k^3}{12}$

correct substitution into triangle = pR **(A1)**

eg $\frac{k^3}{4} = p \left(\frac{k^3}{24} + \frac{k^3}{24} \right), \frac{k^3}{4} = p \left(\frac{k^3}{12} \right)$

$p = 3$ **A1 N2**

[7 marks]

8. Let $f(x) = m - \frac{1}{x}$, for $x \neq 0$. The line $y = x - m$ intersects the graph of f in two distinct points. Find the possible values of m . **[7 marks]**

Markscheme

valid approach **(M1)**

eg $f = y, m - \frac{1}{x} = x - m$

correct working to eliminate denominator **(A1)**

eg $mx - 1 = x(x - m), mx - 1 = x^2 - mx$

correct quadratic equal to zero **A1**

eg $x^2 - 2mx + 1 = 0$

correct reasoning **R1**

eg for two solutions, $b^2 - 4ac > 0$

correct substitution into the discriminant formula **(A1)**

eg $(-2m)^2 - 4$

correct working **(A1)**

eg $4m^2 > 4, m^2 = 1$, sketch of positive parabola on the x -axis

correct interval **A1 N4**

eg $|m| > 1, m < -1$ or $m > 1$

[7 marks]

Consider $f(x) = x^2 + qx + r$. The graph of f has a minimum value when $x = -1.5$.

The distance between the two zeros of f is 9.

- 9a. Show that the two zeros are 3 and -6 .

[2 marks]

Markscheme

recognition that the x -coordinate of the vertex is -1.5 (seen anywhere) **(M1)**

eg axis of symmetry is -1.5 , sketch, $f'(-1.5) = 0$

correct working to find the zeroes **A1**

eg -1.5 ± 4.5

$x = -6$ and $x = 3$ **AG N0**

[2 marks]

9b. Find the value of q and of r .

[4 marks]

Markscheme

METHOD 1 (using factors)

attempt to write factors **(M1)**

eg $(x - 6)(x + 3)$

correct factors **A1**

eg $(x - 3)(x + 6)$

$q = 3, r = -18$ **A1A1 N3**

METHOD 2 (using derivative or vertex)

valid approach to find q **(M1)**

eg $f'(-1.5) = 0, -\frac{q}{2a} = -1.5$

$q = 3$ **A1**

correct substitution **A1**

eg $3^2 + 3(3) + r = 0, (-6)^2 + 3(-6) + r = 0$

$r = -18$ **A1**

$q = 3, r = -18$ **N3**

METHOD 3 (solving simultaneously)

valid approach setting up system of two equations **(M1)**

eg $9 + 3q + r = 0, 36 - 6q + r = 0$

one correct value

eg $q = 3, r = -18$ **A1**

correct substitution **A1**

eg $3^2 + 3(3) + r = 0, (-6)^2 + 3(-6) + r = 0, 3^2 + 3q - 18 = 0, 36 - 6q - 18 = 0$

second correct value **A1**

eg $q = 3, r = -18$

$q = 3, r = -18$ **N3**

[4 marks]

10. Let $f(x) = 3\tan^4x + 2k$ and $g(x) = -\tan^4x + 8k\tan^2x + k$, for $0 \leq x \leq 1$, where [8 marks]
 $0 < k < 1$. The graphs of f and g intersect at exactly one point. Find the value of k .

Markscheme

discriminant = 0 (seen anywhere) **M1**

valid approach **(M1)**

eg $f = g$, $3\tan^4x + 2k = -\tan^4x + 8k\tan^2x + k$

rearranging their equation (to equal zero) **(M1)**

eg $4\tan^4x - 8k\tan^2x + k = 0$, $4m^4 - 8km^2 + k = 0$

recognizing LHS is quadratic **(M1)**

eg $4(\tan^2x)^2 - 8k\tan^2x + k = 0$, $4m^2 - 8km + k = 0$

correct substitution into discriminant **A1**

eg $(-8k)^2 - 4(4)(k)$

correct working to find discriminant or solve discriminant = 0 **(A1)**

eg $64k^2 - 16k$, $\frac{-(-16) \pm \sqrt{16^2}}{2 \times 64}$

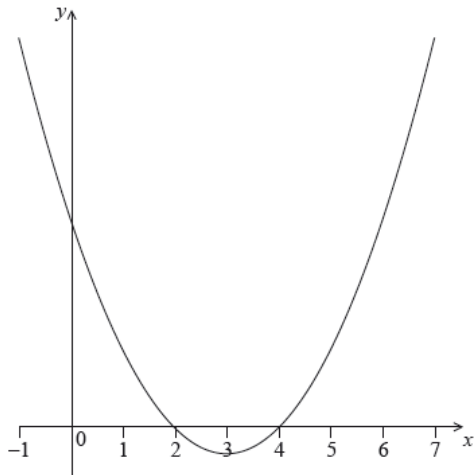
correct simplification **(A1)**

egx $16k(4k - 1)$, $\frac{32}{2 \times 64}$

$k = \frac{1}{4}$ **A1 N2**

[8 marks]

The following diagram shows part of the graph of a quadratic function f .



The vertex is at $(3, -1)$ and the x -intercepts at 2 and 4.

The function f can be written in the form $f(x) = (x - h)^2 + k$.

- 11a. Write down the value of h and of k .

[2 marks]

Markscheme

$$h = 3, k = -1 \quad \mathbf{A1A1} \quad \mathbf{N2}$$

[2 marks]

The function can also be written in the form $f(x) = (x - a)(x - b)$.

11b. Write down the value of a and of b .

[2 marks]

Markscheme

$$a = 2, b = 4 \text{ (or } a = 4, b = 2) \quad \mathbf{A1A1} \quad \mathbf{N2}$$

[2 marks]

11c. Find the y -intercept.

[2 marks]

Markscheme

attempt to substitute $x = 0$ into their f (M1)

$$\text{eg } (0 - 3)^2 - 1, (0 - 2)(0 - 4)$$

$$y = 8 \quad \mathbf{A1} \quad \mathbf{N2}$$

[2 marks]

$$\text{Let } f(x) = px^2 + (10 - p)x + \frac{5}{4}p - 5.$$

12a. Show that the discriminant of $f(x)$ is $100 - 4p^2$.

[3 marks]

Markscheme

correct substitution into $b^2 - 4ac$ A1

$$\text{eg } (10 - p)^2 - 4(p)\left(\frac{5}{4}p - 5\right)$$

correct expansion of each term A1A1

$$\text{eg } 100 - 20p + p^2 - 5p^2 + 20p, 100 - 20p + p^2 - (5p^2 - 20p)$$

$$100 - 4p^2 \quad \mathbf{AG} \quad \mathbf{N0}$$

[3 marks]

12b. Find the values of p so that $f(x) = 0$ has two **equal** roots.

[3 marks]

Markscheme

recognizing discriminant is zero for equal roots **(R1)**

eg $D = 0$, $4p^2 = 100$

correct working **(A1)**

eg $p^2 = 25$, 1 correct value of p

both correct values $p = \pm 5$ **A1 N2**

[3 marks]

Total [6 marks]

Let $f(x) = x^2 + x - 6$.

13a. Write down the y -intercept of the graph of f .

[1 mark]

Markscheme

y -intercept is -6 , $(0, -6)$, $y = -6$ **A1**

[1 mark]

13b. Solve $f(x) = 0$.

[3 marks]

Markscheme

valid attempt to solve **(M1)**

eg $(x - 2)(x + 3) = 0$, $x = \frac{-1 \pm \sqrt{1+24}}{2}$, one correct answer

$x = 2$, $x = -3$ **A1A1 N3**

[3 marks]

Let $f(x) = p + \frac{9}{x-q}$, for $x \neq q$. The line $x = 3$ is a vertical asymptote to the graph of f .

14a. Write down the value of q .

[1 mark]

Markscheme

$q = 3$ **A1 N1**

[1 mark]

14b. The graph of f has a y -intercept at $(0, 4)$.

[4 marks]

Find the value of p .

Markscheme

correct expression for $f(0)$ (A1)

eg $p + \frac{9}{0-3}$, $4 = p + \frac{9}{-3}$

recognizing that $f(0) = 4$ (may be seen in equation) (M1)

correct working (A1)

eg $4 = p - 3$

$p = 7$ A1 N3

[3 marks]

14c. The graph of f has a y -intercept at $(0, 4)$.

[1 mark]

Write down the equation of the horizontal asymptote of the graph of f .

Markscheme

$y = 7$ (must be an equation, do not accept $p = 7$ A1 N1)

[1 mark]

Total [6 marks]

Let

$f(x) = a(x - h)^2 + k$. The vertex of the graph of f is at

$(2, 3)$ and the graph passes through

$(1, 7)$.

15a. Write down the value of h and of k .

[2 marks]

Markscheme

$h = 2$, $k = 3$ A1A1 N2

[2 marks]

15b. Find the value of a .

[3 marks]

Markscheme

attempt to substitute
(1, 7) in any order into **their**

$$f(x) \quad (\mathbf{M1})$$

$$\text{eg } 7 = a(1 - 2)^2 + 3, 7 = a(1 - 3)^2 + 2, 1 = a(7 - 2)^2 + 3$$

correct equation **(A1)**

$$\text{eg } 7 = a + 3$$

$$a = 4 \quad \mathbf{A1} \quad \mathbf{N2}$$

[3 marks]

Let

$$f(x) = px^3 + px^2 + qx.$$

16a. Find $f'(x)$.

[2 marks]

Markscheme

$$f'(x) = 3px^2 + 2px + q \quad \mathbf{A2} \quad \mathbf{N2}$$

Note: Award **A1** if only 1 error.

[2 marks]

16b. Given that $f'(x) \geq 0$, show that $p^2 \leq 3pq$.

[5 marks]

Markscheme

evidence of discriminant (must be seen explicitly, not in quadratic formula) **(M1)**

eg $b^2 - 4ac$

correct substitution into discriminant (may be seen in inequality) **A1**

eg $(2p)^2 - 4 \times 3p \times q, 4p^2 - 12pq$

$f'(x) \geq 0$ then f' has two equal roots or no roots **(R1)**

recognizing discriminant less or equal than zero **R1**

eg $\Delta \leq 0, 4p^2 - 12pq \leq 0$

correct working that clearly leads to the required answer **A1**

eg $p^2 - 3pq \leq 0, 4p^2 \leq 12pq$

$p^2 \leq 3pq$ **AG NO**

[5 marks]

Let

$f(x) = 3x^2 - 6x + p$. The equation

$f(x) = 0$ has two equal roots.

17a. Write down the **value** of the discriminant.

[2 marks]

Markscheme

correct value 0, or $36 - 12p$ **A2 N2**

[2 marks]

17b. Hence, show that $p = 3$.

[1 mark]

Markscheme

correct equation which clearly leads to $p = 3$ **A1**

eg $36 - 12p = 0, 36 = 12p$

$p = 3$ **AG NO**

[1 mark]

17c. The graph of f has its vertex on the x -axis.

[4 marks]

Find the coordinates of the vertex of the graph of f .

Markscheme

METHOD 1

valid approach **(M1)**

eg $x = -\frac{b}{2a}$

correct working **A1**

eg $-\frac{(-6)}{2(3)}, x = \frac{6}{6}$

correct answers **A1A1 N2**

eg $x = 1, y = 0; (1, 0)$

METHOD 2

valid approach **(M1)**

eg $f(x) = 0$, factorisation, completing the square

correct working **A1**

eg $x^2 - 2x + 1 = 0, (3x - 3)(x - 1), f(x) = 3(x - 1)^2$

correct answers **A1A1 N2**

eg $x = 1, y = 0; (1, 0)$

METHOD 3

valid approach using derivative **(M1)**

eg $f'(x) = 0, 6x - 6$

correct equation **A1**

eg $6x - 6 = 0$

correct answers **A1A1 N2**

eg $x = 1, y = 0; (1, 0)$

[4 marks]

17d. The graph of f has its vertex on the x -axis.

[1 mark]

Write down the solution of $f(x) = 0$.

Markscheme

$x = 1$ **A1 N1**

[1 mark]

17e. The graph of f has its vertex on the x -axis.

[1 mark]

The function can be written in the form $f(x) = a(x - h)^2 + k$. Write down the value of a .

Markscheme

$$a = 3 \quad \mathbf{A1} \quad \mathbf{N1}$$

[1 mark]

17f. The graph of f has its vertex on the x -axis.

[1 mark]

The function can be written in the form $f(x) = a(x - h)^2 + k$. Write down the value of h .

Markscheme

$$h = 1 \quad \mathbf{A1} \quad \mathbf{N1}$$

[1 mark]

17g. The graph of f has its vertex on the x -axis.

[1 mark]

The function can be written in the form $f(x) = a(x - h)^2 + k$. Write down the value of k .

Markscheme

$$k = 0 \quad \mathbf{A1} \quad \mathbf{N1}$$

[1 mark]

17h. The graph of f has its vertex on the x -axis.

[4 marks]

The graph of a function g is obtained from the graph of f by a reflection of f in the x -axis,

followed by a translation by the vector $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$. Find g , giving your answer in the form

$$g(x) = Ax^2 + Bx + C.$$

Markscheme

attempt to apply vertical reflection (M1)

eg $-f(x)$, $-3(x-1)^2$, sketch

attempt to apply vertical shift 6 units up (M1)

eg $-f(x) + 6$, vertex (1,6)

transformations performed correctly (in correct order) (A1)

eg $-3(x-1)^2 + 6$, $-3x^2 + 6x - 3 + 6$

$g(x) = -3x^2 + 6x + 3$ A1 N3

[4 marks]

18. The equation $x^2 + (k+2)x + 2k = 0$ has two distinct real roots.

[8 marks]

Find the possible values of k .

Markscheme

evidence of discriminant (M1)

eg $b^2 - 4ac$, $\Delta = 0$

correct substitution into discriminant (A1)

eg $(k+2)^2 - 4(2k)$, $k^2 + 4k + 4 - 8k$

correct discriminant A1

eg $k^2 - 4k + 4$, $(k-2)^2$

recognizing discriminant is positive R1

eg $\Delta > 0$, $(k+2)^2 - 4(2k) > 0$

attempt to solve their quadratic in k (M1)

eg factorizing, $k = \frac{4 \pm \sqrt{16-16}}{2}$

correct working A1

eg $(k-2)^2 > 0$, $k = 2$, sketch of positive parabola on the x -axis

correct values A2 N4

eg $k \in \mathbb{R}$ and $k \neq 2$, $\mathbb{R} \setminus \{2\}$, $]-\infty, 2[\cup]2, \infty[$

[8 marks]

Let

$$f(x) = 3x - 2 \text{ and}$$

$$g(x) = \frac{5}{3x}, \text{ for}$$

$$x \neq 0.$$

19a. Find $f^{-1}(x)$.

[2 marks]

Markscheme

interchanging x and y (M1)

eg $x = 3y - 2$

$$f^{-1}(x) = \frac{x+2}{3} \text{ (accept } y = \frac{x+2}{3}, \frac{x+2}{3}) \quad \mathbf{A1} \quad \mathbf{N2}$$

[2 marks]

19b. Show that $(g \circ f^{-1})(x) = \frac{5}{x+2}$.

[2 marks]

Markscheme

attempt to form composite (in any order) (M1)

eg $g\left(\frac{x+2}{3}\right), \frac{\frac{5}{3}+2}{3}$

correct substitution **A1**

eg $\frac{5}{3\left(\frac{x+2}{3}\right)}$

$$(g \circ f^{-1})(x) = \frac{5}{x+2} \quad \mathbf{AG} \quad \mathbf{N0}$$

[2 marks]

Let

$$h(x) = \frac{5}{x+2}, \text{ for}$$

$x \geq 0$. The graph of h has a horizontal asymptote at

$$y = 0.$$

19c. Find the y -intercept of the graph of h .

[2 marks]

Markscheme

valid approach (M1)

eg $h(0), \frac{5}{0+2}$

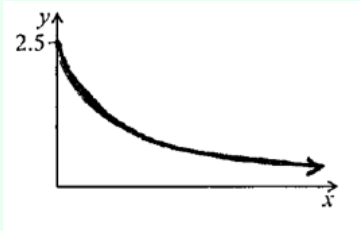
$$y = \frac{5}{2} \text{ (accept } (0, 2.5)) \quad \mathbf{A1} \quad \mathbf{N2}$$

[2 marks]

19d. Hence, sketch the graph of h .

[3 marks]

Markscheme



A1A2 N3

Notes: Award **A1** for approximately correct shape (reciprocal, decreasing, concave up).

Only if this **A1** is awarded, award **A2** for all the following approximately correct features: y -intercept at $(0, 2.5)$, asymptotic to x -axis, correct domain $x \geq 0$.

If only two of these features are correct, award **A1**.

[3 marks]

19e. For the graph of h^{-1} , write down the x -intercept;

[1 mark]

Markscheme

$x = \frac{5}{2}$ (accept $(2.5, 0)$) **A1 N1**

[1 mark]

19f. For the graph of h^{-1} , write down the equation of the vertical asymptote.

[1 mark]

Markscheme

$x = 0$ (must be an equation) **A1 N1**

[1 mark]

19g. Given that $h^{-1}(a) = 3$, find the value of a .

[3 marks]

Markscheme

METHOD 1

attempt to substitute 3 into h (seen anywhere) **(M1)**

eg $h(3), \frac{5}{3+2}$

correct equation **(A1)**

eg $a = \frac{5}{3+2}, h(3) = a$

$a = 1$ **A1 N2**

[3 marks]

METHOD 2

attempt to find inverse (may be seen in (d)) **(M1)**

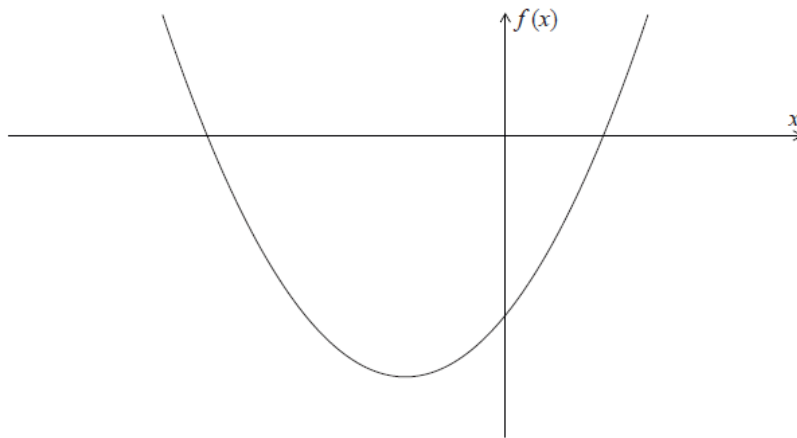
eg $x = \frac{5}{y+2}, h^{-1} = \frac{5}{x} - 2, \frac{5}{x} + 2$

correct equation, $\frac{5}{x} - 2 = 3$ **(A1)**

$a = 1$ **A1 N2**

[3 marks]

The diagram below shows part of the graph of
 $f(x) = (x - 1)(x + 3)$.



20a. (a) Write down the x -intercepts of the graph of f .

[6 marks]

(b) Find the coordinates of the vertex of the graph of f .

Markscheme

(a) $x = 1$, $x = -3$ (accept $(1, 0)$, $(-3, 0)$) **A1A1 N2**

[2 marks]

(b) **METHOD 1**

attempt to find x -coordinate **(M1)**

eg $\frac{1+3}{2}$, $x = \frac{-b}{2a}$, $f'(x) = 0$

correct value, $x = -1$ (may be seen as a coordinate in the answer) **A1**

attempt to find **their** y -coordinate **(M1)**

eg $f(-1)$, -2×2 , $y = \frac{-D}{4a}$

$y = -4$ **A1**

vertex $(-1, -4)$ **N3**

METHOD 2

attempt to complete the square **(M1)**

eg $x^2 + 2x + 1 - 1 - 3$

attempt to put into vertex form **(M1)**

eg $(x + 1)^2 - 4$, $(x - 1)^2 + 4$

vertex $(-1, -4)$ **A1A1 N3**

[4 marks]

20b. Write down the x -intercepts of the graph of f .

[2 marks]

Markscheme

$x = 1$, $x = -3$ (accept $(1, 0)$, $(-3, 0)$) **A1A1 N2**

[2 marks]

20c. Find the coordinates of the vertex of the graph of f .

[4 marks]

Markscheme

METHOD 1

attempt to find x -coordinate (M1)

$$\text{eg } \frac{1+3}{2}, x = \frac{-b}{2a}, f'(x) = 0$$

correct value, $x = -1$ (may be seen as a coordinate in the answer) **A1**

attempt to find **their** y -coordinate (M1)

$$\text{eg } f(-1), -2 \times 2, y = \frac{-D}{4a}$$

$$y = -4 \quad \mathbf{A1}$$

vertex $(-1, -4)$ **N3**

METHOD 2

attempt to complete the square (M1)

$$\text{eg } x^2 + 2x + 1 - 1 - 3$$

attempt to put into vertex form (M1)

$$\text{eg } (x + 1)^2 - 4, (x - 1)^2 + 4$$

vertex $(-1, -4)$ **A1A1 N3**

[4 marks]

21a. Find the value of $\log_2 40 - \log_2 5$.

[3 marks]

Markscheme

evidence of correct formula (M1)

$$\text{eg } \log a - \log b = \log \frac{a}{b}, \log\left(\frac{40}{5}\right), \log 8 + \log 5 - \log 5$$

Note: Ignore missing or incorrect base.

correct working (A1)

$$\text{eg } \log_2 8, 2^3 = 8$$

$$\log_2 40 - \log_2 5 = 3 \quad \mathbf{A1 \quad N2}$$

[3 marks]

21b. Find the value of $8^{\log_2 5}$.

[4 marks]

Markscheme

attempt to write 8 as a power of 2 (seen anywhere) **(M1)**

eg $(2^3)^{\log_2 5}$, $2^3 = 8$, 2^a

multiplying powers **(M1)**

eg $2^{3\log_2 5}$, $a\log_2 5$

correct working **(A1)**

eg $2^{\log_2 125}$, $\log_2 5^3$, $(2^{\log_2 5})^3$

$8^{\log_2 5} = 125$ **A1 N3**

[4 marks]

Let

$$f(x) = \sin x + \frac{1}{2}x^2 - 2x, \text{ for}$$

$$0 \leq x \leq \pi.$$

22a. Find $f'(x)$.

[3 marks]

Markscheme

$$f'(x) = \cos x + x - 2 \quad \mathbf{A1A1A1} \quad \mathbf{N3}$$

Note: Award **A1** for each term.

[3 marks]

Let

g be a quadratic function such that

$$g(0) = 5. \text{ The line}$$

$x = 2$ is the axis of symmetry of the graph of

g .

22b. Find $g(4)$.

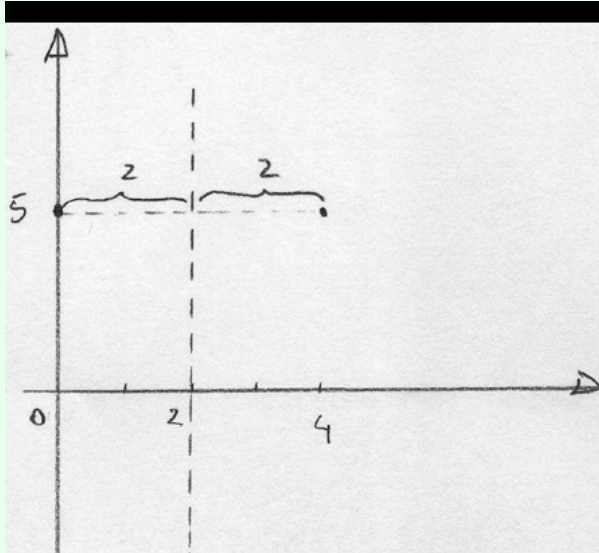
[3 marks]

Markscheme

recognizing $g(0) = 5$ gives the point $(0, 5)$ **(R1)**

recognize symmetry **(M1)**

eg vertex, sketch



$g(4) = 5$ **A1 N3**

[3 marks]

The function

g can be expressed in the form

$$g(x) = a(x - h)^2 + 3.$$

22c. (i) Write down the value of h .

[4 marks]

(ii) Find the value of a .

Markscheme

(i) $h = 2$ **A1 N1**

(ii) substituting into $g(x) = a(x - 2)^2 + 3$ (not the vertex) **(M1)**

eg $5 = a(0 - 2)^2 + 3$, $5 = a(4 - 2)^2 + 3$

working towards solution **(A1)**

eg $5 = 4a + 3$, $4a = 2$

$a = \frac{1}{2}$ **A1 N2**

[4 marks]

22d. Find the value of x for which the tangent to the graph of f is parallel to the tangent to [6 marks]
the graph of g .

Markscheme

$$g(x) = \frac{1}{2}(x-2)^2 + 3 = \frac{1}{2}x^2 - 2x + 5$$

correct derivative of g **A1A1**

eg $2 \times \frac{1}{2}(x-2), x-2$

evidence of equating both derivatives **(M1)**

eg $f' = g'$

correct equation **(A1)**

eg $\cos x + x - 2 = x - 2$

working towards a solution **(A1)**

eg $\cos x = 0$, combining like terms

$$x = \frac{\pi}{2} \quad \mathbf{A1} \quad \mathbf{NO}$$

Note: Do not award final **A1** if additional values are given.

[6 marks]

23. The equation $x^2 - 3x + k^2 = 4$ has two distinct real roots. Find the possible values of k .

[6 marks]

Markscheme

evidence of rearranged quadratic equation (may be seen in working) **A1**

e.g. $x^2 - 3x + k^2 - 4 = 0$, $k^2 - 4$

evidence of discriminant (must be seen explicitly, not in quadratic formula) **(M1)**

e.g. $b^2 - 4ac$, $\Delta = (-3)^2 - 4(1)(k^2 - 4)$

recognizing that discriminant is greater than zero (seen anywhere, including answer) **R1**

e.g. $b^2 - 4ac > 0$, $9 + 16 - 4k^2 > 0$

correct working (accept equality) **A1**

e.g. $25 - 4k^2 > 0$, $4k^2 < 25$, $k^2 = \frac{25}{4}$

both correct values (even if inequality never seen) **(A1)**

e.g. $\pm\sqrt{\frac{25}{4}}$, ± 2.5

correct interval **A1 N3**

e.g. $-\frac{5}{2} < k < \frac{5}{2}$, $-2.5 < k < 2.5$

Note: Do not award the final mark for unfinished values, or for incorrect or reversed inequalities, including \leq , $k > -2.5$, $k < 2.5$.

Special cases:

If working shown, and candidates attempt to rearrange the quadratic equation to equal zero, but find an incorrect value of c , award **A1M1R1A0A0A0**.

If working shown, and candidates do not rearrange the quadratic equation to equal zero, but find $c = k^2$ or $c = \pm 4$, award **A0M1R1A0A0A0**.

[6 marks]

Let

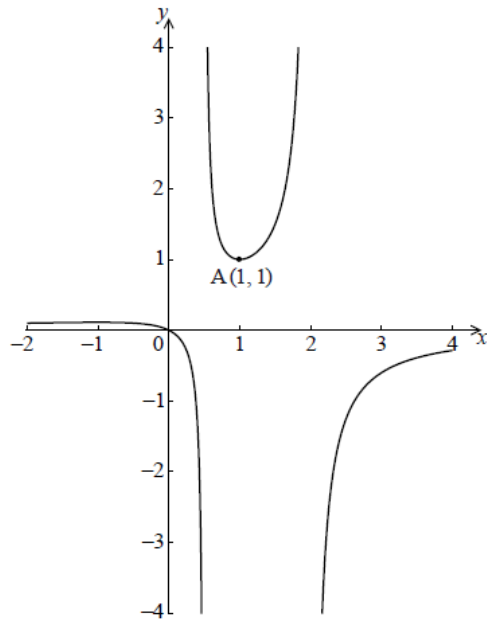
$$f(x) = \frac{x}{-2x^2+5x-2} \text{ for}$$

$$-2 \leq x \leq 4,$$

$$x \neq \frac{1}{2},$$

$$x \neq 2.$$

The graph of f is given below.



The graph of

f has a local minimum at A(

1,

1) and a local maximum at B.

24a. Use the quotient rule to show that $f'(x) = \frac{2x^2-2}{(-2x^2+5x-2)^2}$.

[6 marks]

Markscheme

correct derivatives **applied** in quotient rule (A1)A1A1

$$1, -4x + 5$$

Note: Award (A1) for 1, A1 for $-4x$ and A1 for 5, **only** if it is clear candidates are using the quotient rule.

correct substitution into quotient rule A1

$$\text{e.g. } \frac{1 \times (-2x^2 + 5x - 2) - x(-4x + 5)}{(-2x^2 + 5x - 2)^2}, \frac{-2x^2 + 5x - 2 - x(-4x + 5)}{(-2x^2 + 5x - 2)^2}$$

correct working (A1)

$$\text{e.g. } \frac{-2x^2 + 5x - 2 - (-4x^2 + 5x)}{(-2x^2 + 5x - 2)^2}$$

expression clearly leading to the answer A1

$$\text{e.g. } \frac{-2x^2 + 5x - 2 + 4x^2 - 5x}{(-2x^2 + 5x - 2)^2}$$

$$f'(x) = \frac{2x^2 - 2}{(-2x^2 + 5x - 2)^2} \quad \text{AG} \quad \text{NO}$$

[6 marks]

24b. Hence find the coordinates of B.

[7 marks]

Markscheme

evidence of attempting to solve $f'(x) = 0$ (M1)

$$\text{e.g. } 2x^2 - 2 = 0$$

evidence of correct working A1

$$\text{e.g. } x^2 = 1, \frac{\pm\sqrt{16}}{4}, 2(x - 1)(x + 1)$$

correct solution to quadratic (A1)

$$\text{e.g. } x = \pm 1$$

correct x-coordinate $x = -1$ (may be seen in coordinate form $(-1, \frac{1}{9})$) A1 N2

attempt to substitute -1 into f (do not accept any other value) (M1)

$$\text{e.g. } f(-1) = \frac{-1}{-2 \times (-1)^2 + 5 \times (-1) - 2}$$

correct working

$$\text{e.g. } \frac{-1}{-2 - 5 - 2} \quad \text{A1}$$

correct y-coordinate $y = \frac{1}{9}$ (may be seen in coordinate form $(-1, \frac{1}{9})$) A1 N2

[7 marks]

24c. Given that the line $y = k$ does not meet the graph of f , find the possible values of k . [3 marks]

Markscheme

recognizing values between max and min (R1)

$$\frac{1}{9} < k < 1 \quad \mathbf{A2} \quad \mathbf{N3}$$

[3 marks]

25. Consider the equation $x^2 + (k - 1)x + 1 = 0$, where k is a real number.

[7 marks]

Find the values of k for which the equation has two **equal** real solutions.

Markscheme

METHOD 1

evidence of valid approach (M1)

e.g. $b^2 - 4ac$, quadratic formula

correct substitution into $b^2 - 4ac$ (may be seen in formula) (A1)

e.g. $(k - 1)^2 - 4 \times 1 \times 1$, $(k - 1)^2 - 4$, $k^2 - 2k - 3$

setting **their** discriminant equal to zero M1

e.g. $\Delta = 0$, $(k - 1)^2 - 4 = 0$

attempt to solve the quadratic (M1)

e.g. $(k - 1)^2 = 4$, factorizing

correct working A1

e.g. $(k - 1) = \pm 2$, $(k - 3)(k + 1)$

$k = -1$, $k = 3$ (do not accept inequalities) A1A1 N2

[7 marks]

METHOD 2

recognizing perfect square (M1)

e.g. $(x + 1)^2 = 0$, $(x - 1)^2$

correct expansion (A1)(A1)

e.g. $x^2 + 2x + 1 = 0$, $x^2 - 2x + 1$

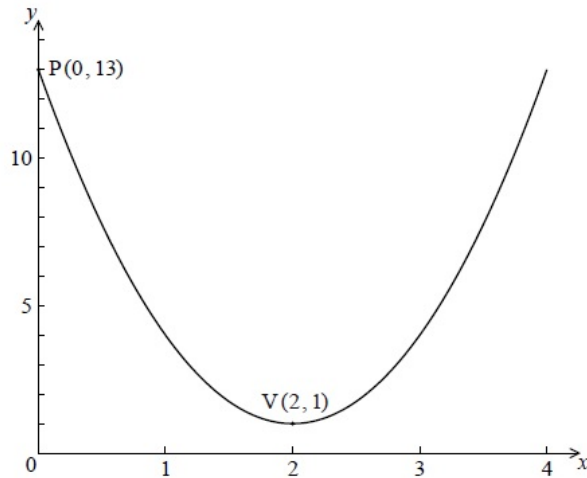
equating coefficients of x A1A1

e.g. $k - 1 = -2$, $k - 1 = 2$

$k = -1$, $k = 3$ A1A1 N2

[7 marks]

The following diagram shows the graph of a quadratic function f , for $0 \leq x \leq 4$.



The graph passes through the point $P(0, 13)$, and its vertex is the point $V(2, 1)$.

26a. The function can be written in the form $f(x) = a(x - h)^2 + k$. [4 marks]

- (i) Write down the value of h and of k .
- (ii) Show that $a = 3$.

Markscheme

(i) $h = 2, k = 1$ **A1A1 N2**

(ii) attempt to substitute coordinates of any point (except the vertex) on the graph into f
M1

e.g. $13 = a(0 - 2)^2 + 1$

working towards solution **A1**

e.g. $13 = 4a + 1$

$a = 3$ **AG NO**

[4 marks]

26b. Find $f(x)$, giving your answer in the form $Ax^2 + Bx + C$. [3 marks]

Markscheme

attempting to expand **their** binomial (M1)

e.g. $f(x) = 3(x^2 - 2 \times 2x + 4) + 1$, $(x - 2)^2 = x^2 - 4x + 4$

correct working (A1)

e.g. $f(x) = 3x^2 - 12x + 12 + 1$

$f(x) = 3x^2 - 12x + 13$ (accept $A = 3$, $B = -12$, $C = 13$) A1 N2

[3 marks]

26c. Calculate the area enclosed by the graph of f , the x -axis, and the lines $x = 2$ and $x = 4$. [8 marks]

Markscheme

METHOD 1

integral expression (A1)

e.g. $\int_2^4 (3x^2 - 12x + 13)$, $\int f dx$

Area = $[x^3 - 6x^2 + 13x]_2^4$ A1A1A1

Note: Award A1 for x^3 , A1 for $-6x^2$, A1 for $13x$.

correct substitution of **correct** limits into **their** expression A1A1

e.g. $(4^3 - 6 \times 4^2 + 13 \times 4) - (2^3 - 6 \times 2^2 + 13 \times 2)$, $64 - 96 + 52 - (8 - 24 + 26)$

Note: Award A1 for substituting 4, A1 for substituting 2.

correct working (A1)

e.g. $64 - 96 + 52 - 8 + 24 - 26$, $20 - 10$

Area = 10 A1 N3

[8 marks]

METHOD 2

integral expression (A1)

e.g. $\int_2^4 (3(x - 2)^2 + 1)$, $\int f dx$

Area = $[(x - 2)^3 + x]_2^4$ A2A1

Note: Award A2 for $(x - 2)^3$, A1 for x .

correct substitution of **correct** limits into **their** expression A1A1

e.g. $(4 - 2)^3 + 4 - [(2 - 2)^3 + 2]$, $2^3 + 4 - (0^3 + 2)$, $2^3 + 4 - 2$

Note: Award A1 for substituting 4, A1 for substituting 2.

correct working (A1)

e.g. $8 + 4 - 2$

Area = 10 A1 N3

[8 marks]

METHOD 3

recognizing area from 0 to 2 is same as area from 2 to 4 (R1)

e.g. sketch, $\int_2^4 f = \int_0^2 f$

integral expression (A1)

e.g. $\int_0^2 (3x^2 - 12x + 13) , \int f dx$

Area = $[x^3 - 6x^2 + 13x]_0^2$ A1A1A1

Note: Award A1 for x^3 , A1 for $-6x^2$, A1 for $13x$.

correct substitution of correct limits into their expression A1(A1)

e.g. $(2^3 - 6 \times 2^2 + 13 \times 2) - (0^3 - 6 \times 0^2 + 13 \times 0) , 8 - 24 + 26$

Note: Award A1 for substituting 2, (A1) for substituting 0.

Area = 10 A1 N3

[8 marks]