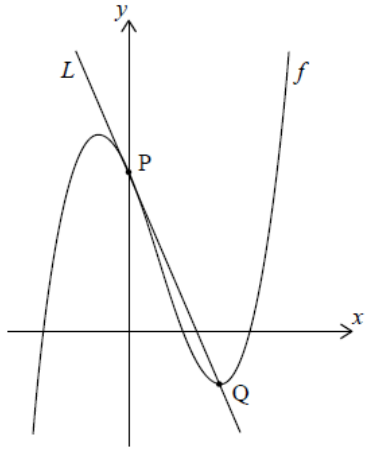


Ch2Review [254 marks]

Let $f(x) = x^3 - 2x^2 + ax + 6$. Part of the graph of f is shown in the following diagram.



The graph of f crosses the y -axis at the point P . The line L is tangent to the graph of f at P .

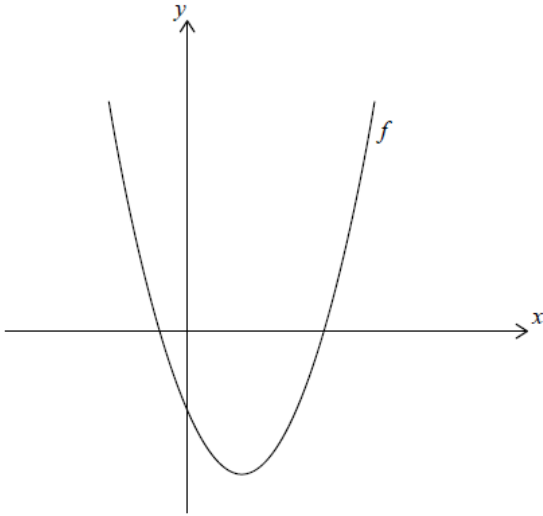
- 1a. Find the coordinates of P . [2 marks]

- 1b. Find $f'(x)$. [2 marks]

- 1c. Hence, find the equation of L in terms of a . [4 marks]

- 1d. The graph of f has a local minimum at the point Q . The line L passes through Q . [8 marks]
Find the value of a .

Let $f(x) = x^2 - 4x - 5$. The following diagram shows part of the graph of f .



2a. Find the x -intercepts of the graph of f . [5 marks]

2b. Find the equation of the axis of symmetry of the graph of f . [2 marks]

The function can be written in the form $f(x) = (x - h)^2 + k$.

2c. Write down the value of h . [1 mark]

2d. Find the value of k . [3 marks]

2e. The graph of a second function, g , is obtained by a reflection of the graph of f in the y -axis, followed by a translation of $\begin{pmatrix} -3 \\ 6 \end{pmatrix}$. [5 marks]

Find the coordinates of the vertex of the graph of g .

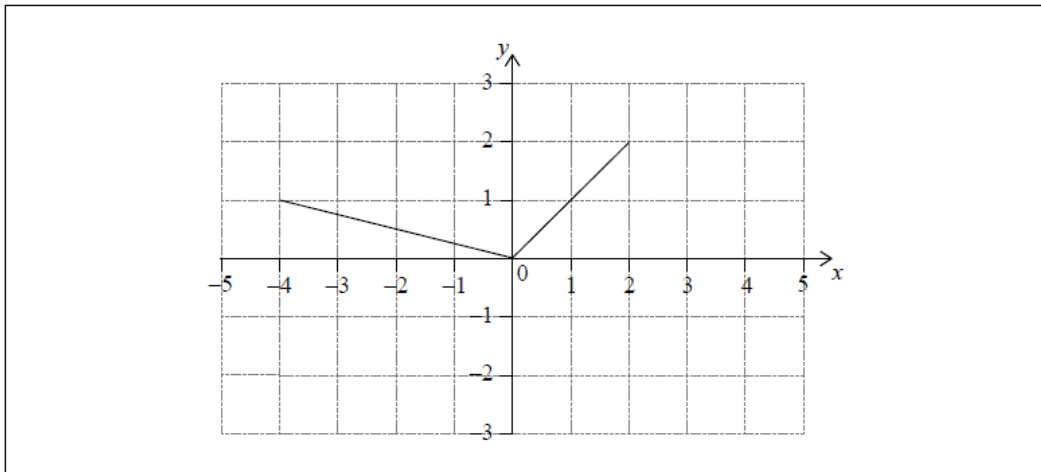
Let $f(x) = ax^2 - 4x - c$. A horizontal line, L , intersects the graph of f at $x = -1$ and $x = 3$.

3a. The equation of the axis of symmetry is $x = p$. Find p . [2 marks]

3b. Hence, show that $a = 2$. [2 marks]

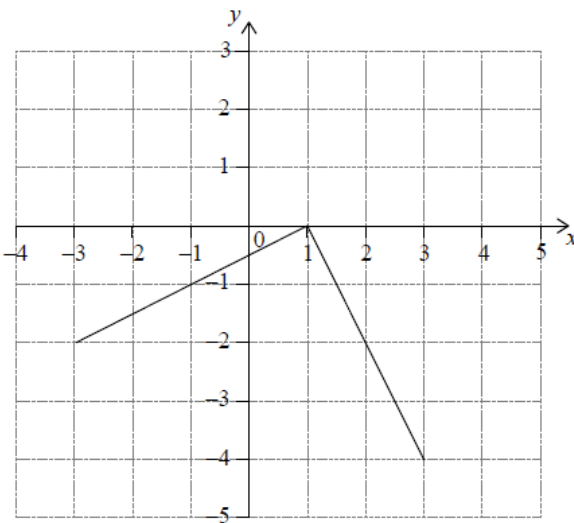
3c. The equation of L is $y = 5$. Find the value of c . [3 marks]

The following diagram shows the graph of a function f , for $-4 \leq x \leq 2$.



4a. On the same axes, sketch the graph of $f(-x)$. [2 marks]

4b. Another function, g , can be written in the form $g(x) = a \times f(x + b)$. The following diagram shows the graph of g . [4 marks]



Write down the value of a and of b .

5. Let $f(x) = px^2 + qx - 4p$, where $p \neq 0$. Find the number of roots for the equation $f(x) = 0$. [7 marks]

Justify your answer.

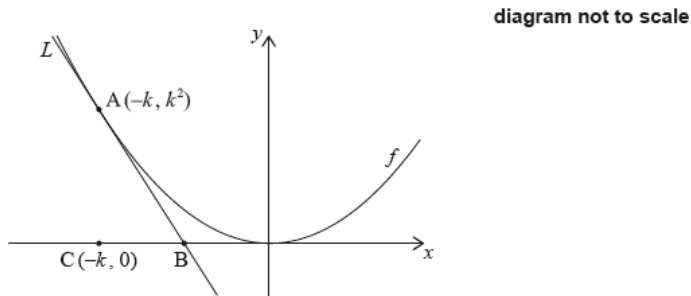
A quadratic function f can be written in the form $f(x) = a(x - p)(x - 3)$. The graph of f has axis of symmetry $x = 2.5$ and y -intercept at $(0, -6)$

6a. Find the value of p . [3 marks]

6b. Find the value of a . [3 marks]

- 6c. The line $y = kx - 5$ is a tangent to the curve of f . Find the values of k . [8 marks]

Let $f(x) = x^2$. The following diagram shows part of the graph of f .



The line L is the tangent to the graph of f at the point $A(-k, k^2)$, and intersects the x -axis at point B . The point C is $(-k, 0)$.

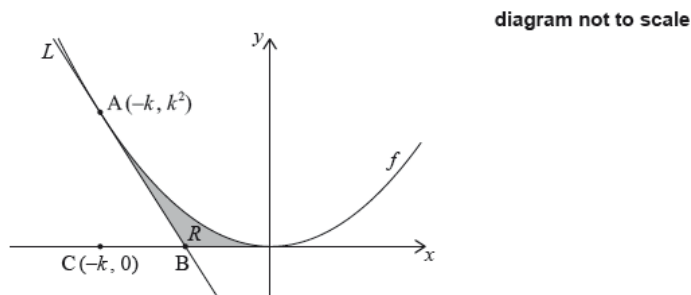
- 7a. Write down $f'(x)$. [1 mark]

- 7b. Find the gradient of L . [2 marks]

- 7c. Show that the x -coordinate of B is $-\frac{k}{2}$. [5 marks]

- 7d. Find the area of triangle ABC , giving your answer in terms of k . [2 marks]

The region R is enclosed by L , the graph of f , and the x -axis. This is shown in the following diagram.



- 7e. Given that the area of triangle ABC is p times the area of R , find the value of p . [7 marks]

8. Let $f(x) = m - \frac{1}{x}$, for $x \neq 0$. The line $y = x - m$ intersects the graph of f in two distinct points. Find the possible values of m . [7 marks]

Consider $f(x) = x^2 + qx + r$. The graph of f has a minimum value when $x = -1.5$.

The distance between the two zeros of f is 9.

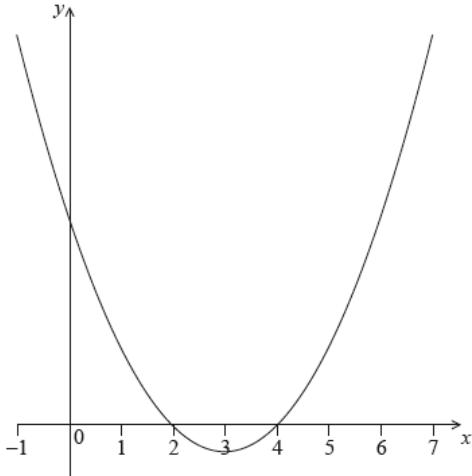
- 9a. Show that the two zeros are 3 and -6 . [2 marks]

9b. Find the value of q and of r .

[4 marks]

10. Let $f(x) = 3\tan^4 x + 2k$ and $g(x) = -\tan^4 x + 8k\tan^2 x + k$, for $0 \leq x \leq 1$, where [8 marks]
 $0 < k < 1$. The graphs of f and g intersect at exactly one point. Find the value of k .

The following diagram shows part of the graph of a quadratic function f .



The vertex is at $(3, -1)$ and the x -intercepts at 2 and 4.

The function f can be written in the form $f(x) = (x - h)^2 + k$.

11a. Write down the value of h and of k .

[2 marks]

The function can also be written in the form $f(x) = (x - a)(x - b)$.

11b. Write down the value of a and of b .

[2 marks]

11c. Find the y -intercept.

[2 marks]

$$\text{Let } f(x) = px^2 + (10 - p)x + \frac{5}{4}p - 5.$$

12a. Show that the discriminant of $f(x)$ is $100 - 4p^2$.

[3 marks]

12b. Find the values of p so that $f(x) = 0$ has two **equal** roots.

[3 marks]

$$\text{Let } f(x) = x^2 + x - 6.$$

13a. Write down the y -intercept of the graph of f .

[1 mark]

13b. Solve $f(x) = 0$.

[3 marks]

Let $f(x) = p + \frac{9}{x-q}$, for $x \neq q$. The line $x = 3$ is a vertical asymptote to the graph of f .

14a. Write down the value of q . [1 mark]

14b. The graph of f has a y -intercept at $(0, 4)$. [4 marks]
Find the value of p .

14c. The graph of f has a y -intercept at $(0, 4)$. [1 mark]

Write down the equation of the horizontal asymptote of the graph of f .

Let

$f(x) = a(x - h)^2 + k$. The vertex of the graph of

f is at

$(2, 3)$ and the graph passes through

$(1, 7)$.

15a. Write down the value of h and of k . [2 marks]

15b. Find the value of a . [3 marks]

Let

$f(x) = px^3 + px^2 + qx$.

16a. Find $f'(x)$. [2 marks]

16b. Given that $f'(x) \geq 0$, show that $p^2 \leq 3pq$. [5 marks]

Let

$f(x) = 3x^2 - 6x + p$. The equation

$f(x) = 0$ has two equal roots.

17a. Write down the **value** of the discriminant. [2 marks]

17b. Hence, show that $p = 3$. [1 mark]

17c. The graph of f has its vertex on the x -axis. [4 marks]

Find the coordinates of the vertex of the graph of f .

17d. The graph of f has its vertex on the x -axis. [1 mark]

Write down the solution of $f(x) = 0$.

17e. The graph of f has its vertex on the x -axis. [1 mark]

The function can be written in the form $f(x) = a(x - h)^2 + k$. Write down the value of a .

17f. The graph of f has its vertex on the x -axis. [1 mark]

The function can be written in the form $f(x) = a(x - h)^2 + k$. Write down the value of h .

17g. The graph of f has its vertex on the x -axis. [1 mark]

The function can be written in the form $f(x) = a(x - h)^2 + k$. Write down the value of k .

17h. The graph of f has its vertex on the x -axis. [4 marks]

The graph of a function g is obtained from the graph of f by a reflection of f in the x -axis,

followed by a translation by the vector $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$. Find g , giving your answer in the form

$$g(x) = Ax^2 + Bx + C.$$

18. The equation $x^2 + (k + 2)x + 2k = 0$ has two distinct real roots. [8 marks]

Find the possible values of k .

Let

$$f(x) = 3x - 2 \text{ and}$$

$$g(x) = \frac{5}{3x}, \text{ for}$$

$$x \neq 0.$$

19a. Find $f^{-1}(x)$. [2 marks]

19b. Show that $(g \circ f^{-1})(x) = \frac{5}{x+2}$. [2 marks]

Let

$$h(x) = \frac{5}{x+2}, \text{ for}$$

$x \geq 0$. The graph of h has a horizontal asymptote at

$$y = 0.$$

19c. Find the y -intercept of the graph of h . [2 marks]

19d. Hence, sketch the graph of h . [3 marks]

19e. For the graph of h^{-1} , write down the x -intercept; [1 mark]

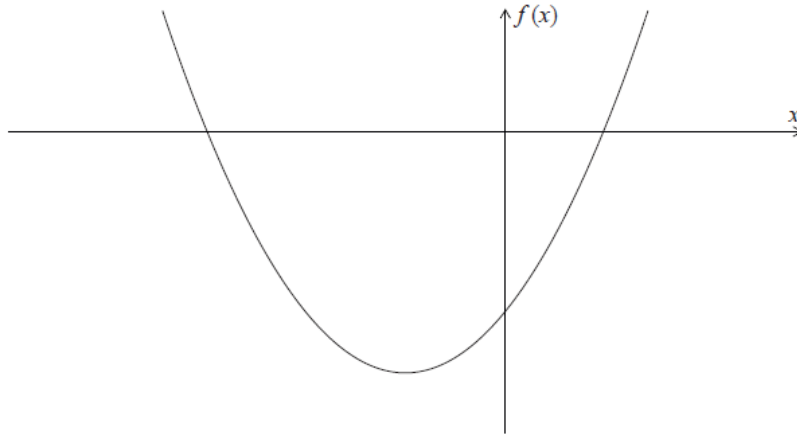
19f. For the graph of h^{-1} , write down the equation of the vertical asymptote.

[1 mark]

19g. Given that $h^{-1}(a) = 3$, find the value of a .

[3 marks]

The diagram below shows part of the graph of
 $f(x) = (x - 1)(x + 3)$.



20a. (a) Write down the x -intercepts of the graph of f .

[6 marks]

(b) Find the coordinates of the vertex of the graph of f .

20b. Write down the x -intercepts of the graph of f .

[2 marks]

20c. Find the coordinates of the vertex of the graph of f .

[4 marks]

21a. Find the value of $\log_2 40 - \log_2 5$.

[3 marks]

21b. Find the value of $8^{\log_2 5}$.

[4 marks]

Let

$$f(x) = \sin x + \frac{1}{2}x^2 - 2x, \text{ for}$$

$$0 \leq x \leq \pi.$$

22a. Find $f'(x)$.

[3 marks]

Let

g be a quadratic function such that

$g(0) = 5$. The line

$x = 2$ is the axis of symmetry of the graph of

g .

22b. Find $g(4)$.

[3 marks]

The function

g can be expressed in the form

$$g(x) = a(x - h)^2 + 3.$$

22c. (i) Write down the value of h .

[4 marks]

(ii) Find the value of a .

22d. Find the value of x for which the tangent to the graph of f is parallel to the tangent to the graph of g . [6 marks]

23. The equation

$x^2 - 3x + k^2 = 4$ has two distinct real roots. Find the possible values of k .

[6 marks]

Let

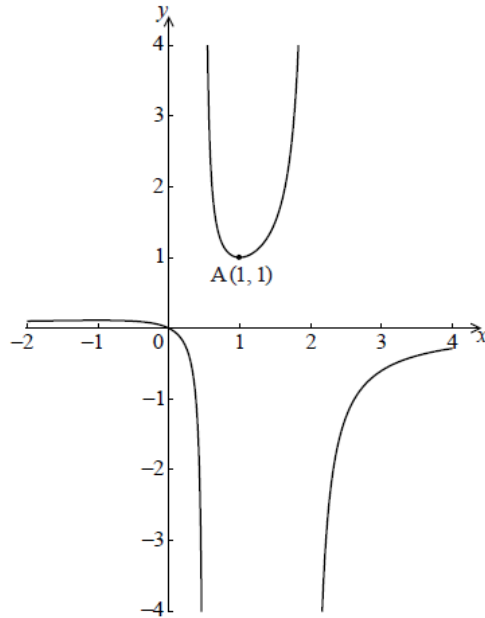
$$f(x) = \frac{x}{-2x^2+5x-2} \text{ for}$$

$$-2 \leq x \leq 4,$$

$$x \neq \frac{1}{2},$$

$$x \neq 2.$$

The graph of f is given below.



The graph of

f has a local minimum at A(

1,

1) and a local maximum at B.

24a. Use the quotient rule to show that $f'(x) = \frac{2x^2-2}{(-2x^2+5x-2)^2}$. [6 marks]

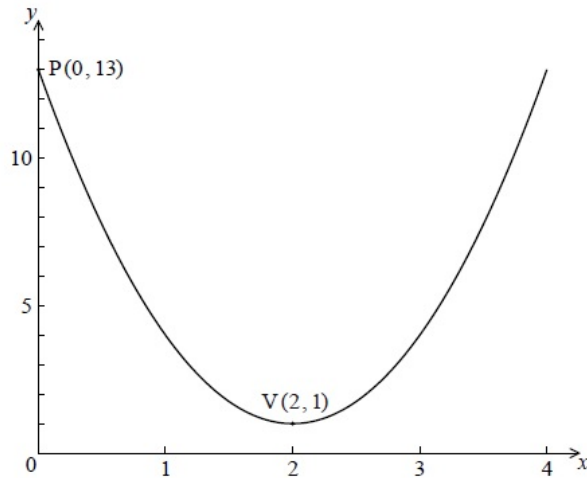
24b. Hence find the coordinates of B. [7 marks]

24c. Given that the line $y = k$ does not meet the graph of f , find the possible values of k . [3 marks]

25. Consider the equation $x^2 + (k - 1)x + 1 = 0$, where k is a real number. [7 marks]

Find the values of k for which the equation has two **equal** real solutions.

The following diagram shows the graph of a quadratic function f , for $0 \leq x \leq 4$.



The graph passes through the point $P(0, 13)$, and its vertex is the point $V(2, 1)$.

26a. The function can be written in the form $f(x) = a(x - h)^2 + k$. [4 marks]

- (i) Write down the value of h and of k .
- (ii) Show that $a = 3$.

26b. Find $f(x)$, giving your answer in the form $Ax^2 + Bx + C$. [3 marks]

26c. Calculate the area enclosed by the graph of f , the x -axis, and the lines $x = 2$ and $x = 4$. [8 marks]