Kinematics *[285 marks]*

A particle moves along a straight line so that its velocity, v m s^{-1}, after t seconds is given by $v(t) = 1.4^t - 2.7$, for $0 \le t \le 5$.

1a. Find when the particle is at rest. *[2 marks]*

Markscheme

valid approach ***(M1)***

eg $v(t) = 0$, sketch of graph

2.95195

$t = \log_{1.4} 2.7$ (exact), $t = 2.95$ (s) ***A1 N2***

[2 marks]

1b. Find the acceleration of the particle when $t = 2$. *[2 marks]*

Markscheme

valid approach ***(M1)***

eg $a(t) = v'(t)$, $v'(2)$

0.659485

$a(2) = 1.96 \ln 1.4$ (exact), $a(2) = 0.659$ (m s^{-2}) ***A1 N2***

[2 marks]

1c. Find the total distance travelled by the particle. *[3 marks]*

Markscheme

correct approach *(A1)*

eg $\int_0^5 |v(t)|\,\mathrm{d}t,\ \int_0^{2.95} (-v(t))\,\mathrm{d}t + \int_{295}^5 v(t)\,\mathrm{d}t$

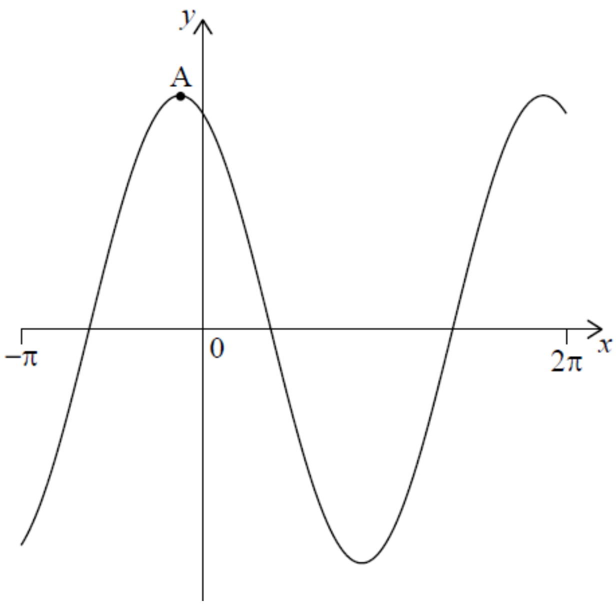
5.3479

distance = 5.35 (m) ***A2 N3***

[3 marks]

Let $f(x) = 12\cos x - 5\sin x,\ -\pi \leqslant x \leqslant 2\pi$, be a periodic function with $f(x) = f(x + 2\pi)$

The following diagram shows the graph of f.



There is a maximum point at A. The minimum value of f is –13 .

2a. Find the coordinates of A. *[2 marks]*

Markscheme

–0.394791,13

A(–0.395, 13) ***A1A1 N2***

[2 marks]

2b. For the graph of f, write down the amplitude. *[1 mark]*

Markscheme

13 ***A1 N1***

[1 mark]

2c. For the graph of f, write down the period. *[1 mark]*

Markscheme

2π, 6.28 ***A1 N1***

[1 mark]

2d. Hence, write $f(x)$ in the form $p \cos(x + r)$. *[3 marks]*

Markscheme

valid approach ***(M1)***

eg recognizing that amplitude is *p* or shift is *r*

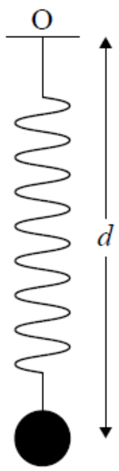
$f(x) = 13 \cos(x + 0.395)$ (accept *p* = 13, *r* = 0.395) ***A1A1 N3***

Note: Accept any value of *r* of the form $0.395 + 2\pi k,\ k \in \mathbb{Z}$

[3 marks]

A ball on a spring is attached to a fixed point O. The ball is then pulled down and released, so that it moves back and forth vertically.

diagram not to scale



The distance, d centimetres, of the centre of the ball from O at time t seconds, is given by

$d(t) = f(t) + 17,\ 0 \leqslant t \leqslant 5.$

2e. Find the maximum speed of the ball. *[3 marks]*

Markscheme

recognizing need for $d'(t)$ ***(M1)***

eg $-12\sin(t) - 5\cos(t)$

correct approach (accept any variable for t) ***(A1)***

eg $-13\sin(t + 0.395)$, sketch of d', $(1.18, -13)$, $t = 4.32$

maximum speed = 13 (cms^{-1}) ***A1 N2***

[3 marks]

2f. Find the first time when the ball's speed is changing at a rate of 2 cm s $^{-2}$. *[5 marks]*

Markscheme

recognizing that acceleration is needed ***(M1)***

eg a(t), $d''(t)$

correct equation (accept any variable for t) ***(A1)***

eg $a(t) = -2,\ \left|\frac{\mathrm{d}}{\mathrm{d}t}(d'(t))\right| = 2,\ -12\cos(t) + 5\sin(t) = -2$

valid attempt to solve **their** equation ***(M1)***

eg sketch, 1.33

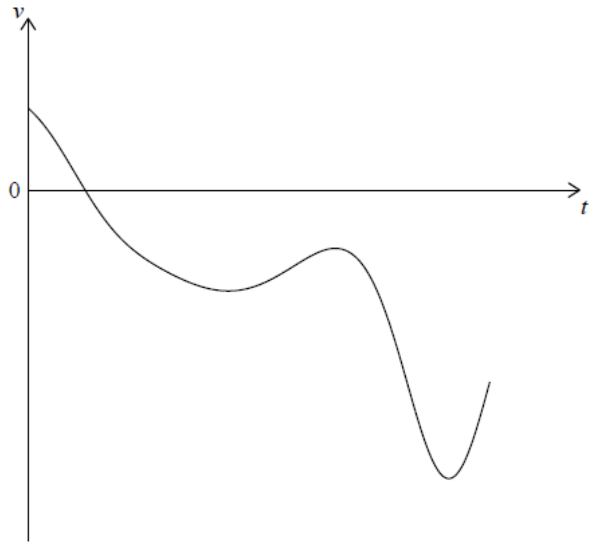
1.02154

1.02 ***A2 N3***

[5 marks]

A particle P moves along a straight line. The velocity v m s^{-1} of P after t seconds is given by $v(t) = 7\cos t - 5t^{\cos t}$, for $0 \le t \le 7$.

The following diagram shows the graph of v.



3a. Find the initial velocity of P. *[2 marks]*

Markscheme

initial velocity when $t = 0$ ***(M1)***

eg $v(0)$

$v = 17$ (m s^{-1}) ***A1 N2***

[2 marks]

3b. Find the maximum speed of P. *[3 marks]*

Markscheme

recognizing maximum speed when $|v|$ is greatest ***(M1)***

eg minimum, maximum, $v' = 0$

one correct coordinate for minimum ***(A1)***

eg 6.37896, –24.6571

24.7 (ms^{-1}) ***A1 N2***

[3 marks]

3c. Write down the number of times that the acceleration of P is 0 m s^{-2}. *[3 marks]*

Markscheme

recognizing a = v' ***(M1)***

eg $a = \frac{\mathrm{d}v}{\mathrm{d}t}$, correct derivative of first term

identifying when a = 0 ***(M1)***

eg turning points of v, t-intercepts of v'

3 ***A1 N3***

[3 marks]

3d. Find the acceleration of P when it changes direction. *[4 marks]*

Markscheme

recognizing P changes direction when $v = 0$ ***(M1)***

$t = 0.863851$ ***(A1)***

–9.24689

$a = -9.25$ (ms^{-2}) ***A2 N3***

[4 marks]

3e. Find the total distance travelled by P. *[3 marks]*

Markscheme

correct substitution of limits or function into formula ***(A1)***

eg $\int_0^7 |v|,\ \int_0^{0.8638} v\mathrm{d}t - \int_{0.8638}^{7} v\mathrm{d}t,\ \int |7\cos x - 5x^{\cos x}|\,dx,\ 3.32 = 60.6$

63.8874

63.9 (metres) ***A2 N3***

[3 marks]

Note: In this question, distance is in metres and time is in seconds.

A particle P moves in a straight line for five seconds. Its acceleration at time t is given by $a = 3t^2 - 14t + 8$, for $0 \leqslant t \leqslant 5$.

4a. Write down the values of t when $a = 0$. *[2 marks]*

Markscheme

$t = \frac{2}{3}$ (exact), 0.667, $t = 4$ ***A1A1*** ***N2***

[2 marks]

4b. Hence or otherwise, find all possible values of t for which the velocity of P is decreasing. *[2 marks]*

Markscheme

recognizing that v is decreasing when a is negative ***(M1)***

eg $a < 0,\ 3t^2 - 14t + 8 \leqslant 0$, sketch of a

correct interval ***A1*** ***N2***

eg $\frac{2}{3} < t < 4$

[2 marks]

When $t = 0$, the velocity of P is $3\ \mathrm{m\,s^{-1}}$.

4c. Find an expression for the velocity of P at time t. *[6 marks]*

Markscheme

valid approach (do not accept a definite integral) ***(M1)***

eg $v \int a$

correct integration (accept missing c) ***(A1)(A1)(A1)***

$t^3 - 7t^2 + 8t + c$

substituting $t = 0,\ v = 3$, (must have c) ***(M1)***

eg $3 = 0^3 - 7(0^2) + 8(0) + c,\ c = 3$

$v = t^3 - 7t^2 + 8t + 3$ ***A1*** ***N6***

[6 marks]

4d. Find the total distance travelled by P when its velocity is increasing. *[4 marks]*

Markscheme

recognizing that v increases outside the interval found in part (b) ***(M1)***

eg $0 < t < \frac{2}{3}$, $4 < t < 5$, diagram

one correct substitution into distance formula ***(A1)***

eg $\int_0^{\frac{2}{3}} |v|$, $\int_4^5 |v|$, $\int_{\frac{2}{3}}^4 |v|$, $\int_0^5 |v|$

one correct pair ***(A1)***

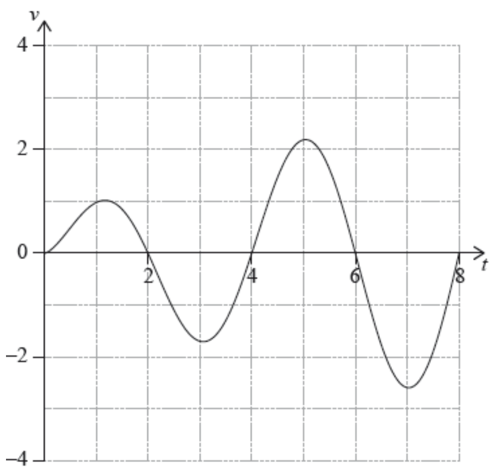
eg 3.13580 and 11.0833, 20.9906 and 35.2097

14.2191 ***A1 N2***

$d = 14.2$ (m)

[4 marks]

A particle P moves along a straight line. Its velocity v_P $\mathrm{m\,s^{-1}}$ after t seconds is given by $v_P = \sqrt{t}\sin\left(\frac{\pi}{2}t\right)$, for $0 \leqslant t \leqslant 8$. The following diagram shows the graph of v_P.



5a. Write down the first value of t at which P changes direction. *[1 mark]*

Markscheme

$t = 2$ ***A1 N1***

[1 mark]

5b. Find the **total** distance travelled by P, for $0 \leqslant t \leqslant 8$. *[2 marks]*

Markscheme

substitution of limits or function into formula or correct sum ***(A1)***

eg $\int_0^8 |v|\,\mathrm{d}t,\ \int |v_Q|\,\mathrm{d}t,\ \int_0^2 v\mathrm{d}t - \int_2^4 v\mathrm{d}t + \int_4^6 v\mathrm{d}t - \int_6^8 v\mathrm{d}t$

9.64782

distance $= 9.65\ (\text{metres})$ ***A1*** ***N2***

[2 marks]

5c. A second particle Q also moves along a straight line. Its velocity, $v_{\mathrm{Q}}\ \mathrm{m\,s^{-1}}$ after t seconds is given by $v_{\mathrm{Q}} = \sqrt{t}$ for $0 \leqslant t \leqslant 8$. After k seconds Q has travelled the same total distance as P. *[4 marks]*

Find k.

Markscheme

correct approach ***(A1)***

eg $s = \int \sqrt{t},\ \int_0^k \sqrt{t}\mathrm{d}t,\ \int_0^k |v_{\mathrm{Q}}|\,\mathrm{d}t$

correct integration ***(A1)***

eg $\int \sqrt{t} = \frac{2}{3}t^{\frac{3}{2}} + c,\ \left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^k,\ \frac{2}{3}k^{\frac{3}{2}}$

equating their expression to the distance travelled by their P ***(M1)***

eg $\frac{2}{3}k^{\frac{3}{2}} = 9.65,\ \int_0^k \sqrt{t}\mathrm{d}t = 9.65$

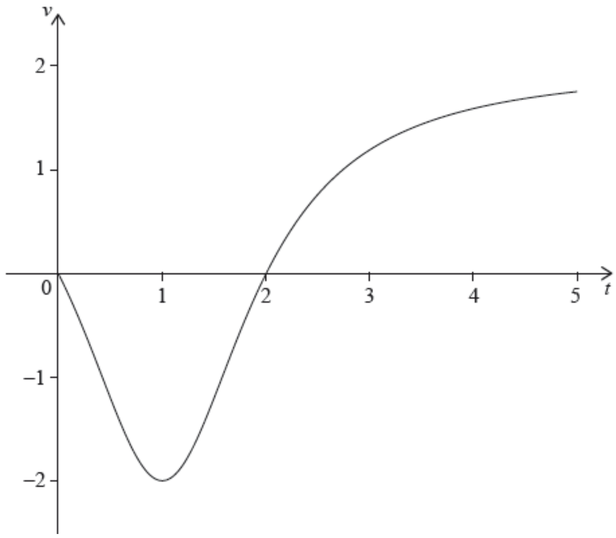
5.93855

5.94 (seconds) ***A1*** ***N3***

[4 marks]

6. **Note: In this question, distance is in metres and time is in seconds.** *[6 marks]*

A particle moves along a horizontal line starting at a fixed point A. The velocity v of the particle, at time t, is given by $v(t) = \frac{2t^2-4t}{t^2-2t+2}$, for $0 \leqslant t \leqslant 5$. The following diagram shows the graph of v



There are t-intercepts at $(0,\ 0)$ and $(2,\ 0)$.

Find the maximum distance of the particle from A during the time $0 \leqslant t \leqslant 5$ and justify your answer.

Markscheme

METHOD 1 (displacement)

recognizing $s = \int v \mathrm{d}t$ ***(M1)***

consideration of displacement at $t = 2$ **and** $t = 5$ (seen anywhere) ***M1***

eg $\int_0^2 v$ and $\int_0^5 v$

Note: Must have both for any further marks.

correct displacement at $t = 2$ and $t = 5$ (seen anywhere) ***A1A1***

-2.28318 (accept 2.28318), 1.55513

valid reasoning comparing correct displacements ***R1***

eg $|-2.28| > |1.56|$, more left than right

2.28 (m) ***A1*** ***N1***

Note: Do not award the final ***A1*** without the ***R1***.

METHOD 2 (distance travelled)

recognizing distance $= \int |v| \, \mathrm{d}t$ ***(M1)***

consideration of distance travelled from $t = 0$ to 2 **and** $t = 2$ to 5 (seen anywhere) ***M1***

eg $\int_0^2 v$ and $\int_2^5 v$

Note: Must have both for any further marks

correct distances travelled (seen anywhere) ***A1A1***

2.28318, (accept -2.28318), 3.83832

valid reasoning comparing correct distance values ***R1***

eg $3.84 - 2.28 < 2.28, \ 3.84 < 2 \times 2.28$

2.28 (m) ***A1*** ***N1***

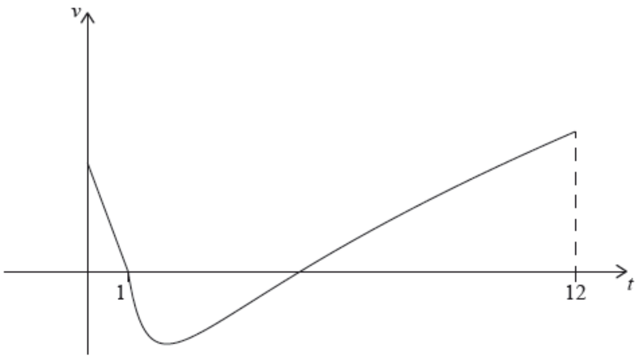
Note: Do not award the final ***A1*** without the ***R1***.

[6 marks]

A particle P starts from a point A and moves along a horizontal straight line. Its velocity $v \text{ cm s}^{-1}$ after t seconds is given by

$$v(t) = \begin{cases} -2t+2, & \text{for } 0 \leqslant t \leqslant 1 \\ 3\sqrt{t} + \frac{4}{t^2} - 7, & \text{for } 1 \leqslant t \leqslant 12 \end{cases}$$

The following diagram shows the graph of v.



7a. Find the initial velocity of P. *[2 marks]*

Markscheme

valid attempt to substitute $t = 0$ into the correct function ***(M1)***

eg $-2(0)+2$

2 ***A1 N2***

[2 marks]

P is at rest when $t = 1$ and $t = p$.

7b. Find the value of p. *[2 marks]*

Markscheme

recognizing $v = 0$ when P is at rest ***(M1)***

5.21834

$p = 5.22 \text{ (seconds)}$ ***A1 N2***

[2 marks]

When $t = q$, the acceleration of P is zero.

7c. (i) Find the value of q. *[4 marks]*

(ii) Hence, find the **speed** of P when $t = q$.

Markscheme

(i) recognizing that $a = v'$ ***(M1)***

eg $v' = 0$, minimum on graph

1.95343

$q = 1.95$ ***A1*** ***N2***

(ii) valid approach to find **their** minimum ***(M1)***

eg $v(q), \ -1.75879$, reference to min on graph

1.75879

speed $= 1.76 \ (\mathrm{cm\,s}^{-1})$ ***A1*** ***N2***

[4 marks]

7d. (i) Find the total distance travelled by P between $t = 1$ and $t = p$. *[6 marks]*

(ii) Hence or otherwise, find the displacement of P from A when $t = p$.

Markscheme

(i) substitution of **correct** $v(t)$ into distance formula, ***(A1)***

eg $\int_1^p \left|3\sqrt{t} + \frac{4}{t^2} - 7\right| \mathrm{d}t, \ \left|\int 3\sqrt{t} + \frac{4}{t^2} - 7\mathrm{d}t\right|$

4.45368

distance $= 4.45 \ (\mathrm{cm})$ ***A1*** ***N2***

(ii) displacement from $t = 1$ to $t = p$ (seen anywhere) ***(A1)***

eg $-4.45368, \ \int_1^p \left(3\sqrt{t} + \frac{4}{t^2} - 7\right) \mathrm{d}t$

displacement from $t = 0$ to $t = 1$ ***(A1)***

eg $\int_0^1 (-2t + 2)\mathrm{d}t, \ 0.5 \times 1 \times 2, \ 1$

valid approach to find displacement for $0 \leqslant t \leqslant p$ ***M1***

eg $\int_0^1 (-2t + 2)\mathrm{d}t + \int_1^p \left(3\sqrt{t} + \frac{4}{t^2} - 7\right) \mathrm{d}t, \ \int_0^1 (-2t + 2)\mathrm{d}t - 4.45$

-3.45368

displacement $= -3.45 \ (\mathrm{cm})$ ***A1*** ***N2***

[6 marks]

A particle P moves along a straight line so that its velocity, $v \,\mathrm{m\,s}^{-1}$, after t seconds, is given by $v = \cos 3t - 2\sin t - 0.5$, for $0 \leqslant t \leqslant 5$. The initial displacement of P from a fixed point O is 4 metres.

8a. Find the displacement of P from O after 5 seconds. *[5 marks]*

Markscheme

METHOD 1

recognizing $s = \int v$ ***(M1)***

recognizing displacement of P in first 5 seconds (seen anywhere) ***A1***

(accept missing $\mathrm{d}t$)

eg $\int_0^5 v\mathrm{d}t,\ -3.71591$

valid approach to find total displacement ***(M1)***

eg $4 + (-3.7159),\ s = 4 + \int_0^5 v$

0.284086

0.284 (m) ***A2*** ***N3***

METHOD 2

recognizing $s = \int v$ ***(M1)***

correct integration ***A1***

eg $\frac{1}{3}\sin 3t + 2\cos t - \frac{t}{2} + c$ (do not penalize missing “c”)

attempt to find c ***(M1)***

eg $4 = \frac{1}{3}\sin(0) + 2\cos(0) - -\frac{0}{2} + c,\ 4 = \frac{1}{3}\sin 3t + 2\cos t - \frac{t}{2} + c,\ 2 + c = 4$

attempt to substitute $t = 5$ into their expression with c ***(M1)***

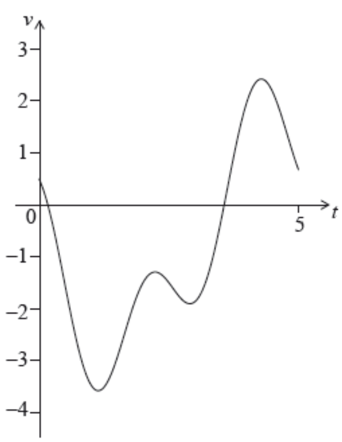
eg $s(5),\ \frac{1}{3}\sin(15) + 2\cos(5)5 - -\frac{5}{2} + 2$

0.284086

0.284 (m) ***A1*** ***N3***

[5 marks]

The following sketch shows the graph of v.



8b. Find when P is first at rest. *[2 marks]*

Markscheme

recognizing that at rest, $v = 0$ ***(M1)***

$t = 0.179900$

$t = 0.180 \text{ (secs)}$ ***A1*** ***N2***

[2 marks]

8c. Write down the number of times P changes direction. *[2 marks]*

Markscheme

recognizing when change of direction occurs ***(M1)***

eg v crosses t axis

2 (times) ***A1*** ***N2***

[2 marks]

8d. Find the acceleration of P after 3 seconds. *[2 marks]*

Markscheme

acceleration is v' (seen anywhere) ***(M1)***

eg $v'(3)$

0.743631

$0.744 \text{ (ms}^{-2}\text{)}$ ***A1*** ***N2***

[2 marks]

8e. Find the maximum speed of P. *[3 marks]*

Markscheme

valid approach involving max or min of v ***(M1)***

eg $v\prime = 0,\ a = 0$, graph

one correct co-ordinate for min ***(A1)***

eg $1.14102,\ -3.27876$

$3.28 \text{ (ms}^{-1}\text{)}$ ***A1*** ***N2***

[3 marks]

9. A particle moves in a straight line. Its velocity $v \text{ m s}^{-1}$ after t seconds is given by *[7 marks]*

$$v = 6t - 6, \text{ for } 0 \leqslant t \leqslant 2.$$

After p seconds, the particle is 2 m from its initial position. Find the possible values of p.

Markscheme

correct approach ***(A1)***

eg $s = \int v, \int_0^p 6t - 6\mathrm{d}t$

correct integration ***(A1)***

eg $\int 6t - 6\mathrm{d}t = 3t^2 - 6t + C, \left[3t^2 - 6t\right]_0^p$

recognizing that there are two possibilities ***(M1)***

eg 2 correct answers, $s = \pm 2, c \pm 2$

two correct equations in p ***A1A1***

eg $3p^2 - 6p = 2, 3p^2 - 6p = -2$

0.42265, 1.57735

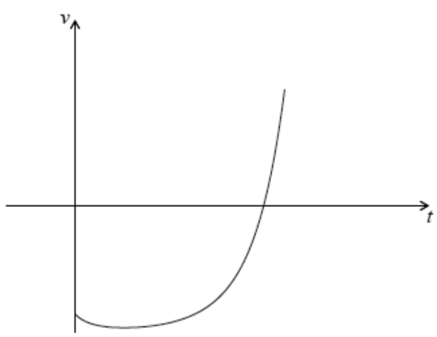
$p = 0.423 \text{ or } p = 1.58$ ***A1A1 N3***

[7 marks]

The velocity $v \text{ ms}^{-1}$ of a particle after t seconds is given by

$v(t) = (0.3t + 0.1)^t - 4$, for $0 \leq t \leq 5$

The following diagram shows the graph of v.



10a. Find the value of t when the particle is at rest. *[3 marks]*

Markscheme

recognizing particle at rest when $v = 0$ ***(M1)***

eg $(0.3t + 0.1)^t - 4 = 0$, x-intercept on graph of v

$t = 4.27631$

$t = 4.28 \text{ (seconds)}$ ***A2 N3***

[3 marks]

10b. Find the value of t when the acceleration of the particle is 0. *[3 marks]*

Markscheme

valid approach to find t when a is 0 ***(M1)***

eg $v'(t) = 0$, v minimum

$t = 1.19236$

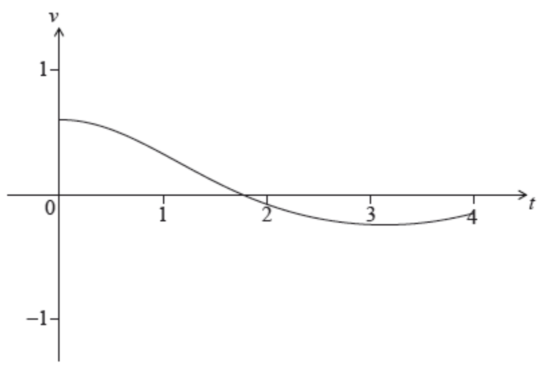
$t = 1.19 \ (\text{seconds})$ ***A2 N3***

[3 marks]

Total [6 marks]

A particle starts from point A and moves along a straight line. Its velocity, $v \text{ ms}^{-1}$, after t seconds is given by $v(t) = \text{e}^{\frac{1}{2}\cos t} - 1$, for $0 \le t \le 4$. The particle is at rest when $t = \frac{\pi}{2}$.

The following diagram shows the graph of v.



11. Find the distance travelled by the particle for $0 \le t \le \frac{\pi}{2}$. *[2 marks]*

Markscheme

correct substitution of function and/or limits into formula ***(A1)***

(accept absence of dt, but do not accept any errors)

eg $\int_0^{\frac{\pi}{2}} v$, $\int \left|\text{e}^{\frac{1}{2}\cos t} - 1\right| \text{d}t$, $\int \left(\text{e}^{\frac{1}{2}\cos t} - 1\right)$

0.613747

distance is $0.614 \ [0.613, \ 0.614] \ (\text{m})$ ***A1 N2***

[2 marks]

12. Ramiro and Lautaro are travelling from Buenos Aires to El Moro. *[8 marks]*

Ramiro travels in a vehicle whose velocity in ms^{-1} is given by $V_R = 40 - t^2$, where t is in seconds.

Lautaro travels in a vehicle whose displacement from Buenos Aires in metres is given by $S_L = 2t^2 + 60$.

When $t = 0$, both vehicles are at the same point.

Find Ramiro's displacement from Buenos Aires when $t = 10$.

Markscheme

METHOD 1

$S_L(0) = 60$ (seen anywhere) ***(A1)***

recognizing need to integrate V_R ***(M1)***

eg $S_R(t) \int V_R \mathrm{d}t$

correct expression ***A1A1***

eg $40t - \frac{1}{3}t^3 + C$

Note: Award ***A1*** for $40t$, and ***A1*** for $-\frac{1}{3}t^3$.

equate displacements to find *C* ***(R1)***

eg $40(0) - \frac{1}{3}(0)^3 + C = 60,\ S_L(0) = S_R(0)$

$C = 60$ ***A1***

attempt to find displacement ***(M1)***

eg $S_R(10),\ 40(10) - \frac{1}{3}(10)^3 + 60$

126.666

$126\frac{2}{3}$ (exact), 127 (m) ***A1*** ***N5***

METHOD 2

recognizing need to integrate V_R ***(M1)***

eg $S_R(t) = \int V_R \mathrm{d}t$

valid approach involving a definite integral ***(M1)***

eg $\int_a^b V_R \mathrm{d}t$

correct expression with limits ***(A1)***

eg $\int_0^{10} \left(40 - t^2\right) \mathrm{d}t,\ \int_0^{10} V_R \mathrm{d}t,\ \left[40t - \frac{1}{3}t^3\right]_0^{10}$

66.6666 ***A2***

$S_L(0) = 60$ (seen anywhere) ***(A1)***

valid approach to find total displacement ***(M1)***

eg $60 + 66.666$

126.666

$126\frac{2}{3}$ (exact), 127 (m) ***A1*** ***N5***

METHOD 3

$S_L(0) = 60$ (seen anywhere) ***(A1)***

recognizing need to integrate V_R ***(M1)***

eg $S_R(t) = \int V_R \mathrm{d}t$

correct expression ***A1A1***

eg $40t - \frac{1}{3}t^3 + C$

Note: Award ***A1*** for $40t$, and ***A1*** for $-\frac{1}{3}t^3$.

correct expression for Ramiro displacement ***A1***

eg $S_R(10) - S_R(0)$, $\left[40t - \frac{1}{3}t^3 + C\right]_0^{10}$

66.6666 ***A1***

valid approach to find total displacement ***(M1)***

eg $60 + 66.6666$

$126\frac{2}{3}$ (exact), 127 (m) ***A1*** ***N5***

[8 marks]

A particle moves in a straight line. Its velocity,
$v\ \mathrm{ms}^{-1}$, at time
t seconds, is given by

$$v = \left(t^2 - 4\right)^3, \text{ for } 0 \leqslant t \leqslant 3.$$

13a. Find the velocity of the particle when $t = 1$. *[2 marks]*

Markscheme

substituting $t = 1$ into v ***(M1)***

eg $v(1)$, $\left(1^2 - 4\right)^3$

velocity $= -27\ \left(\mathrm{ms}^{-1}\right)$ ***A1*** ***N2***

[2 marks]

13b. Find the value of t for which the particle is at rest. *[3 marks]*

Markscheme

valid reasoning ***(R1)***

eg $v = 0,\ \left(t^2 - 4\right)^3 = 0$

correct working ***(A1)***

eg $t^2 - 4 = 0,\ t = \pm 2$, sketch

$t = 2$ ***A1 N2***

[3 marks]

13c. Find the total distance the particle travels during the first three seconds. *[3 marks]*

Markscheme

correct integral expression for distance ***(A1)***

eg $\int_0^3 |v|,\ \int \left|\left(t^2 - 4\right)^3\right|,\ -\int_0^2 v\mathrm{d}t + \int_2^3 v\mathrm{d}t,$

$\int_0^2 \left(4 - t^2\right)^3 \mathrm{d}t + \int_2^3 \left(t^2 - 4\right)^3 \mathrm{d}t$ (do not accept $\int_0^3 v\mathrm{d}t$)

86.2571

$\text{distance} = 86.3\ (\text{m})$ ***A2 N3***

[3 marks]

13d. Show that the acceleration of the particle is given by $a = 6t(t^2 - 4)^2$. *[3 marks]*

Markscheme

evidence of differentiating velocity ***(M1)***

eg $v'(t)$

$a = 3\left(t^2 - 4\right)^2 (2t)$ ***A2***

$a = 6t\left(t^2 - 4\right)^2$ ***AG N0***

[3 marks]

13e. Find all possible values of t for which the velocity and acceleration are both positive or both negative. *[4 marks]*

Markscheme

METHOD 1

valid approach ***M1***

eg graphs of v and a

correct working ***(A1)***

eg areas of same sign indicated on graph

$2 < t \leqslant 3$ (accept $t > 2$) ***A2 N2***

METHOD 2

recognizing that $a \geqslant 0$ (accept a *is* always positive) (seen anywhere) ***R1***

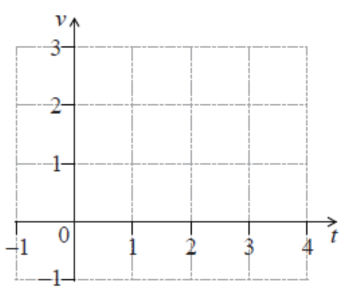
recognizing that v is positive when $t > 2$ (seen anywhere) ***(R1)***

$2 < t \leqslant 3$ (accept $t > 2$) ***A2 N2***

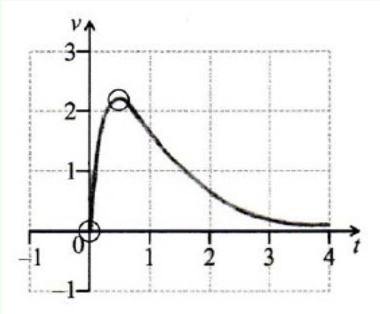
[4 marks]

A particle moves along a straight line such that its velocity, $v\ \mathrm{ms}^{-1}$, is given by $v(t) = 10te^{-1.7t}$, for $t \geqslant 0$.

14a. On the grid below, sketch the graph of v, for $0 \leqslant t \leqslant 4$. *[3 marks]*



Markscheme



A1A2 N3

Notes: Award ***A1*** for approximately correct domain $0 \leqslant t \leqslant 4$.

The shape must be approximately correct, with maximum skewed left. **Only** if the shape is approximately correct, award ***A2*** for all the following approximately correct features, in circle of tolerance where drawn (accept seeing correct coordinates for the maximum, even if point outside circle):

Maximum point, passes through origin, asymptotic to t-axis (but must not touch the axis).

If only two of these features are correct, award ***A1***.

[3 marks]

14b. Find the distance travelled by the particle in the first three seconds. *[2 marks]*

Markscheme

valid approach (including 0 and 3) ***(M1)***

eg $\int_0^3 10t\mathrm{e}^{-1.7t}\mathrm{d}t,\ \int_0^3 f(x)$, area from 0 to 3 (may be shaded in diagram)

$\text{distance} = 3.33\ (\text{m})$ ***A1 N2***

[2 marks]

14c. Find the velocity of the particle when its acceleration is zero. *[3 marks]*

Markscheme

recognizing acceleration is derivative of velocity ***(R1)***

eg $a = \frac{\mathrm{d}v}{\mathrm{d}t}$, attempt to find $\frac{\mathrm{d}v}{\mathrm{d}t}$, reference to maximum on the graph of v

valid approach to find v when $a = 0$ (may be seen on graph) ***(M1)***

eg $\frac{\mathrm{d}v}{\mathrm{d}t} = 0,\ 10\mathrm{e}^{-1.7t} - 17t\mathrm{e}^{-1.7t} = 0,\ t = 0.588$

$\text{velocity} = 2.16\ (\text{ms}^{-1})$ ***A1 N3***

Note: Award ***R1M1A0*** for $(0.588, 216)$ if velocity is not identified as final answer

[3 marks]

15. A rocket moving in a straight line has velocity v km s^{-1} and displacement s km at time t seconds. The velocity v is given by $v(t) = 6\mathrm{e}^{2t} + t$. When $t = 0$, $s = 10$. *[7 marks]*

Find an expression for the displacement of the rocket in terms of t.

Markscheme

evidence of anti-differentiation ***(M1)***

eg $\int (6\mathrm{e}^{2t} + t)$

$s = 3\mathrm{e}^{2t} + \frac{t^2}{2} + C$ ***A2A1***

Note: Award ***A2*** for $3\mathrm{e}^{2t}$, ***A1*** for $\frac{t^2}{2}$.

attempt to substitute $(0,\ 10)$ into **their** integrated expression (even if C is missing) ***(M1)***

correct working ***(A1)***

eg $10 = 3 + C$, $C = 7$

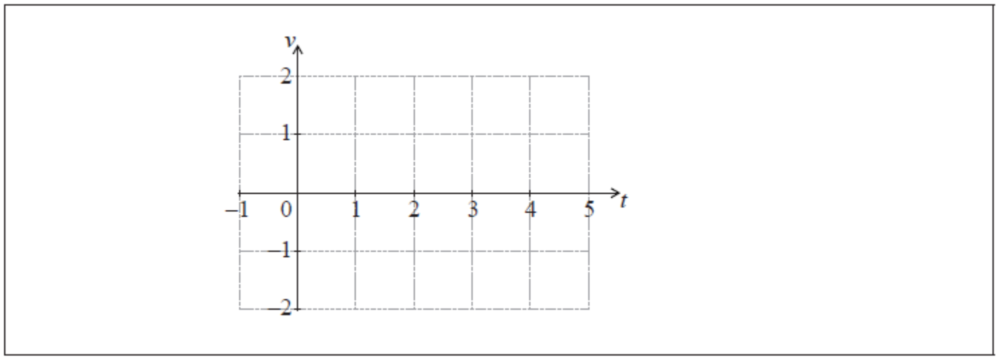
$s = 3\mathrm{e}^{2t} + \frac{t^2}{2} + 7$ ***A1 N6***

Note: Exception to the ***FT*** rule. If working shown, allow full ***FT*** on incorrect integration which must involve a power of e.

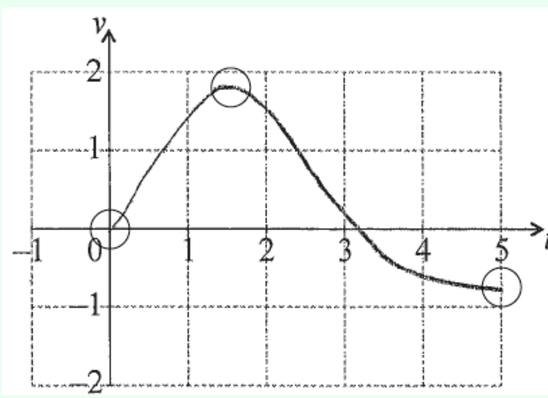
[7 marks]

The velocity of a particle in ms^{-1} is given by
$v = \text{e}^{\sin t} - 1$, for
$0 \leq t \leq 5$.

16a. On the grid below, sketch the graph of v . *[3 marks]*



Markscheme



A1A1A1 ***N3***

Note: Award ***A1*** for approximately correct shape crossing *x*-axis with $3 < x < 3.5$.

Only if this ***A1*** is awarded, award the following:

A1 for maximum in circle, ***A1*** for endpoints in circle.

[3 marks]

16b. Find the total distance travelled by the particle in the first five seconds. *[1 mark]*

Markscheme

$t = \pi$ (exact), 3.14 ***A1*** ***N1***

[1 mark]

16c. Write down the positive t-intercept. *[4 marks]*

Markscheme

recognizing distance is area under velocity curve **(M1)**

eg $s = \int v$, shading on diagram, attempt to integrate

valid approach to find the total area **(M1)**

eg $\text{area A} + \text{area B}$, $\int v\text{d}t - \int v\text{d}t$, $\int_0^{3.14} v\text{d}t + \int_{3.14}^{5} v\text{d}t$, $\int |v|$

correct working with integration and limits (accept $\text{d}x$ or missing $\text{d}t$) **(A1)**

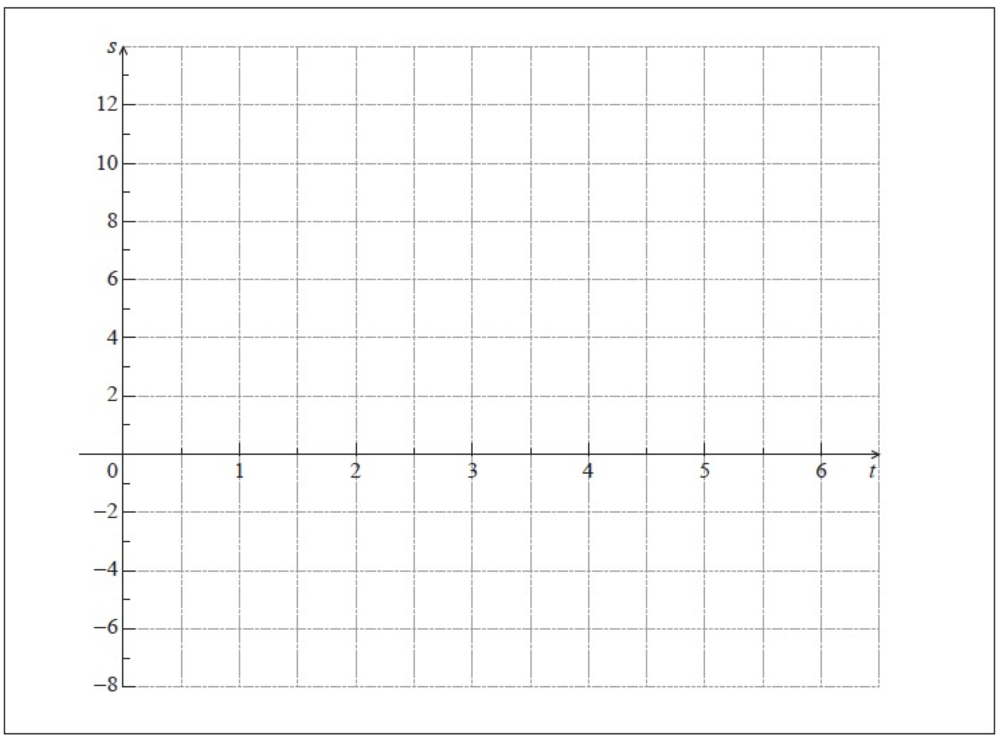
eg $\int_0^{3.14} v\text{d}t + \int_5^{3.14} v\text{d}t$, $3.067\ldots + 0.878\ldots$, $\int_0^5 |\text{e}^{\sin t} - 1|$

distance $= 3.95$ (m) **A1** **N3**

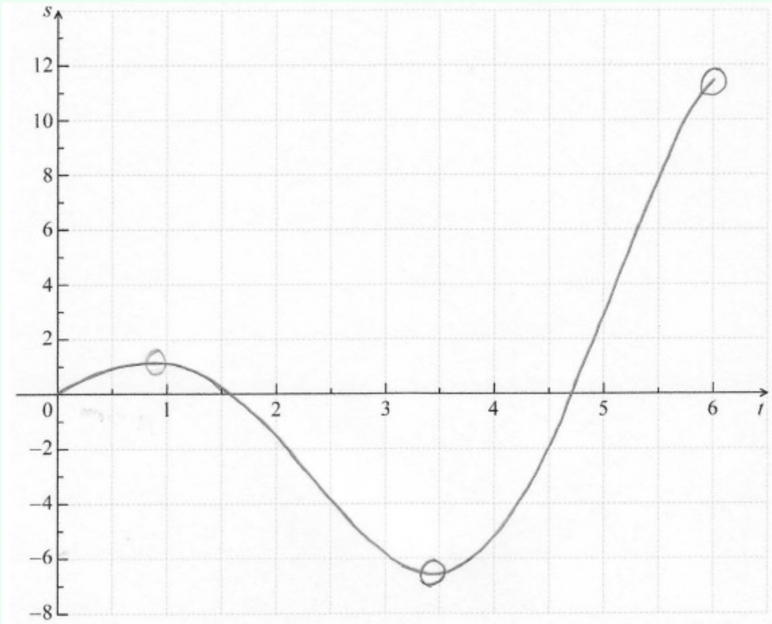
[4 marks]

A particle's displacement, in metres, is given by
$s(t) = 2t\cos t$, for
$0 \le t \le 6$, where t is the time in seconds.

17a. On the grid below, sketch the graph of s . *[4 marks]*



Markscheme



A1A1A1A1 ***N4***

Note: Award ***A1*** for approximately correct shape (do not accept line segments).

Only if this ***A1*** is awarded, award the following:

A1 for maximum and minimum within circles,

A1 for *x*-intercepts between 1 and 2 **and** between 4 and 5,

A1 for left endpoint at $(0, 0)$ and right endpoint within circle.

[4 marks]

17b. Find the maximum velocity of the particle. *[3 marks]*

Markscheme

appropriate approach ***(M1)***

e.g. recognizing that $v = s'$, finding derivative, $a = s''$

valid method to find maximum ***(M1)***

e.g. sketch of v , $v'(t) = 0$, $t = 5.08698\ldots$

$v = 10.20025\ldots$

$v = 10.2$ $[10.2, 10.3]$ ***A1*** ***N2***

[3 marks]

In this question, you are given that

$\cos\frac{\pi}{3} = \frac{1}{2}$, and

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$.

The displacement of an object from a fixed point, O is given by

$s(t) = t - \sin 2t$ for

$0 \le t \le \pi$.

18a. Find $s'(t)$. *[3 marks]*

Markscheme

$s'(t) = 1 - 2\cos 2t$ ***A1A2*** ***N3***

Note: Award ***A1*** for 1, ***A2*** for $-2\cos 2t$.

[3 marks]

18b. In this interval, there are only two values of t for which the object is not moving. One value is $t = \frac{\pi}{6}$. *[4 marks]*

Find the other value.

Markscheme

evidence of valid approach ***(M1)***

e.g. setting $s'(t) = 0$

correct working ***A1***

e.g. $2\cos 2t = 1$, $\cos 2t = \frac{1}{2}$

$2t = \frac{\pi}{3}, \frac{5\pi}{3}, \ldots$ ***(A1)***

$t = \frac{5\pi}{6}$ ***A1*** ***N3***

[4 marks]

18c. Show that $s'(t) > 0$ between these two values of t . *[3 marks]*

Markscheme

evidence of valid approach ***(M1)***

e.g. choosing a value in the interval $\frac{\pi}{6} < t < \frac{5\pi}{6}$

correct substitution ***A1***

e.g. $s'\left(\frac{\pi}{2}\right) = 1 - 2\cos\pi$

$s'\left(\frac{\pi}{2}\right) = 3$ ***A1***

$s'(t) > 0$ ***AG*** ***N0***

[3 marks]

18d. Find the distance travelled between these two values of t. *[5 marks]*

Markscheme

evidence of approach using *s* or integral of s' ***(M1)***

e.g. $\int s'(t)\mathrm{d}t$; $s\left(\frac{5\pi}{6}\right)$, $s\left(\frac{\pi}{6}\right)$; $[t - \sin 2t]_{\frac{\pi}{6}}^{\frac{5\pi}{6}}$

substituting values and subtracting ***(M1)***

e.g. $s\left(\frac{5\pi}{6}\right) - s\left(\frac{\pi}{6}\right)$, $\left(\frac{\pi}{6} - \frac{\sqrt{3}}{2}\right) - \left(\frac{5\pi}{6} - \left(-\frac{\sqrt{3}}{2}\right)\right)$

correct substitution ***A1***

e.g. $\frac{5\pi}{6} - \sin\frac{5\pi}{3} - \left[\frac{\pi}{6} - \sin\frac{\pi}{3}\right]$, $\left(\frac{5\pi}{6} - \left(-\frac{\sqrt{3}}{2}\right)\right) - \left(\frac{\pi}{6} - \frac{\sqrt{3}}{2}\right)$

distance is $\frac{2\pi}{3} + \sqrt{3}$ ***A1A1*** ***N3***

Note: Award ***A1*** for $\frac{2\pi}{3}$, ***A1*** for $\sqrt{3}$.

[5 marks]

A particle moves in a straight line with velocity
$v = 12t - 2t^3 - 1$, for
$t \geq 0$, where *v* is in centimetres per second and *t* is in seconds.

19a. Find the acceleration of the particle after 2.7 seconds. *[3 marks]*

Markscheme

recognizing that acceleration is the derivative of velocity (seen anywhere) ***(R1)***

e.g. $a = \frac{d^2s}{dt^2}, v', 12 - 6t^2$

correctly substituting 2.7 into their expression for *a* (not into *v*) ***(A1)***

e.g. $s''(2.7)$

$\text{acceleration} = -31.74$ (exact), -31.7 ***A1*** ***N3***

[3 marks]

19b. Find the displacement of the particle after 1.3 seconds. *[3 marks]*

Markscheme

recognizing that displacement is the integral of velocity ***R1***

e.g. $s = \int v$

correctly substituting 1.3 ***(A1)***

e.g. $\int_0^{1.3} v dt$

$\text{displacement} = 7.41195$ (exact), 7.41 (cm) ***A1*** ***N2***

[3 marks]

Let $f(t) = 2t^2 + 7$, where $t > 0$. The function *v* is obtained when the graph of *f* is transformed by

a stretch by a scale factor of $\frac{1}{3}$ parallel to the *y*-axis,

followed by a translation by the vector $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$.

20a. Find $v(t)$, giving your answer in the form $a(t - b)^2 + c$. *[4 marks]*

Markscheme

applies vertical stretch parallel to the *y*-axis factor of $\frac{1}{3}$ ***(M1)***

e.g. multiply by $\frac{1}{3}$, $\frac{1}{3}f(t)$, $\frac{1}{3} \times 2$

applies horizontal shift 2 units to the right ***(M1)***

e.g. $f(t-2)$, $t-2$

applies a vertical shift 4 units down ***(M1)***

e.g. subtracting 4, $f(t)-4$, $\frac{7}{3}-4$

$v(t) = \frac{2}{3}(t-2)^2 - \frac{5}{3}$ ***A1 N4***

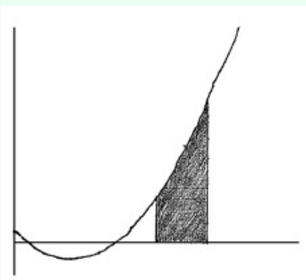
[4 marks]

20b. A particle moves along a straight line so that its velocity in ms $^{-1}$, at time *t* seconds, is given by *v* . Find the distance the particle travels between $t = 5.0$ and $t = 6.8$. *[3 marks]*

Markscheme

recognizing that distance travelled is area under the curve ***M1***

e.g. $\int v, \frac{2}{9}(t-2)^3 - \frac{5}{3}t$, sketch



distance = 15.576 (accept 15.6) ***A2 N2***

[3 marks]

The velocity *v* ms^{-1} of a particle at time *t* seconds, is given by
$v = 2t + \cos 2t$, for
$0 \le t \le 2$.

21a. Write down the velocity of the particle when $t = 0$. *[1 mark]*

Markscheme

$v = 1$ ***A1 N1***

[1 mark]

21b. When $t = k$, the acceleration is zero. *[8 marks]*

(i) Show that $k = \frac{\pi}{4}$.

(ii) Find the exact velocity when $t = \frac{\pi}{4}$.

Markscheme

(i) $\frac{\mathrm{d}}{\mathrm{d}t}(2t) = 2$ ***A1***

$\frac{\mathrm{d}}{\mathrm{d}t}(\cos 2t) = -2\sin 2t$ ***A1A1***

Note: Award ***A1*** for coefficient 2 and ***A1*** for $-\sin 2t$.

evidence of considering acceleration = 0 ***(M1)***

e.g. $\frac{\mathrm{d}v}{\mathrm{d}t} = 0$, $2 - 2\sin 2t = 0$

correct manipulation ***A1***

e.g. $\sin 2k = 1$, $\sin 2t = 1$

$2k = \frac{\pi}{2}$ (accept $2t = \frac{\pi}{2}$) ***A1***

$k = \frac{\pi}{4}$ ***AG*** ***N0***

(ii) attempt to substitute $t = \frac{\pi}{4}$ into *v* ***(M1)***

e.g. $2\left(\frac{\pi}{4}\right) + \cos\left(\frac{2\pi}{4}\right)$

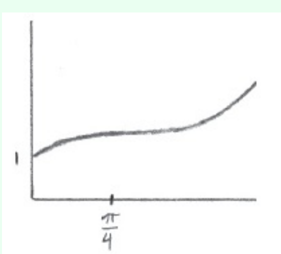
$v = \frac{\pi}{2}$ ***A1*** ***N2***

[8 marks]

21c. When $t < \frac{\pi}{4}$, $\frac{\mathrm{d}v}{\mathrm{d}t} > 0$ and when $t > \frac{\pi}{4}$, $\frac{\mathrm{d}v}{\mathrm{d}t} > 0$. *[4 marks]*

Sketch a graph of *v* against *t* .

Markscheme



A1A1A2 ***N4***

Notes: Award ***A1*** for *y*-intercept at $(0, 1)$, ***A1*** for curve having zero gradient at $t = \frac{\pi}{4}$, ***A2*** for shape that is concave down to the left of $\frac{\pi}{4}$ and concave up to the right of $\frac{\pi}{4}$. If a correct curve is drawn without indicating $t = \frac{\pi}{4}$, do not award the second ***A1*** for the zero gradient, but award the final ***A2*** if appropriate. Sketch need not be drawn to scale. Only essential features need to be clear.

[4 marks]

21d. Let *d* be the distance travelled by the particle for $0 \le t \le 1$. *[3 marks]*

(i) Write down an expression for *d* .

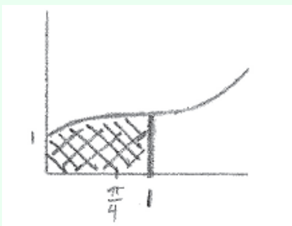
(ii) Represent *d* on your sketch.

Markscheme

(i) correct expression ***A2***

e.g. $\int_0^1 (2t + \cos 2t)\mathrm{d}t$, $\left[t^2 + \frac{\sin 2t}{2}\right]_0^1$, $1 + \frac{\sin 2}{2}$, $\int_0^1 v\mathrm{d}t$

(ii)

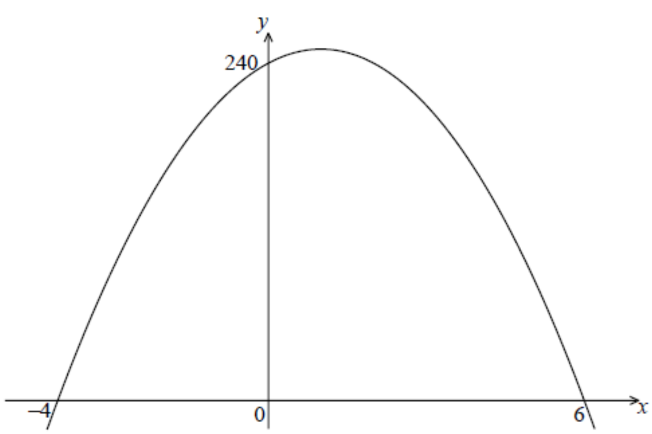


A1

Note: The line at $t = 1$ needs to be clearly after $t = \frac{\pi}{4}$.

[3 marks]

The following diagram shows part of the graph of a quadratic function *f* .



The *x*-intercepts are at $(-4, 0)$ and $(6, 0)$, and the *y*-intercept is at $(0, 240)$.

22a. Write down $f(x)$ in the form $f(x) = -10(x - p)(x - q)$. *[2 marks]*

Markscheme

$f(x) = -10(x+4)(x-6)$ ***A1A1*** ***N2***

[2 marks]

22b. Find another expression for $f(x)$ in the form $f(x) = -10(x-h)^2 + k$. *[4 marks]*

Markscheme

METHOD 1

attempting to find the *x*-coordinate of maximum point ***(M1)***

e.g. averaging the *x*-intercepts, sketch, $y' = 0$, axis of symmetry

attempting to find the *y*-coordinate of maximum point ***(M1)***

e.g. $k = -10(1+4)(1-6)$

$f(x) = -10(x-1)^2 + 250$ ***A1A1*** ***N4***

METHOD 2

attempt to expand $f(x)$ ***(M1)***

e.g. $-10(x^2 - 2x - 24)$

attempt to complete the square ***(M1)***

e.g. $-10((x-1)^2 - 1 - 24)$

$f(x) = -10(x-1)^2 + 250$ ***A1A1*** ***N4***

[4 marks]

22c. Show that $f(x)$ can also be written in the form $f(x) = 240 + 20x - 10x^2$. *[2 marks]*

Markscheme

attempt to simplify ***(M1)***

e.g. distributive property, $-10(x-1)(x-1) + 250$

correct simplification ***A1***

e.g. $-10(x^2 - 6x + 4x - 24)$, $-10(x^2 - 2x + 1) + 250$

$f(x) = 240 + 20x - 10x^2$ ***AG*** ***N0***

[2 marks]

22d. A particle moves along a straight line so that its velocity, $v\ \mathrm{ms}^{-1}$, at time t seconds is given by $v = 240 + 20t - 10t^2$, for $0 \le t \le 6$. *[7 marks]*

(i) Find the value of t when the speed of the particle is greatest.

(ii) Find the acceleration of the particle when its speed is zero.

Markscheme

(i) valid approach ***(M1)***

e.g. vertex of parabola, $v'(t) = 0$

$t = 1$ ***A1 N2***

(ii) recognizing $a(t) = v'(t)$ ***(M1)***

$a(t) = 20 - 20t$ ***A1A1***

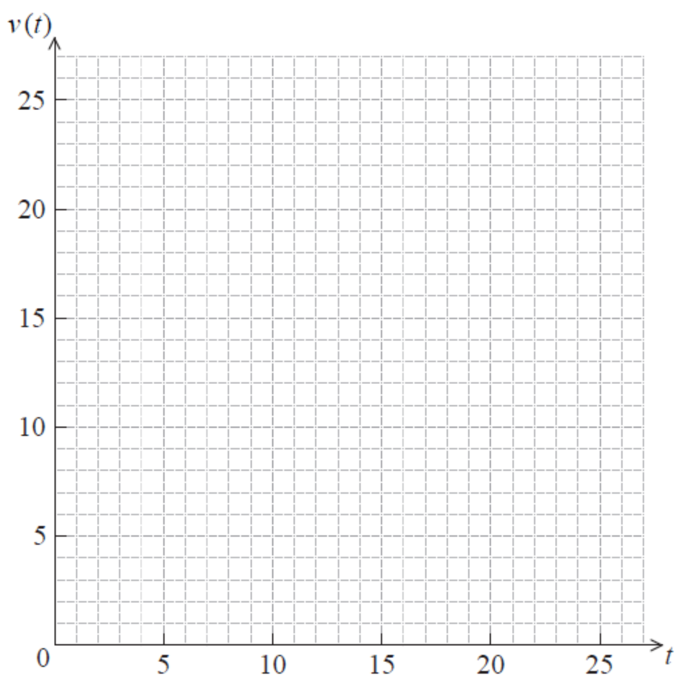
speed is zero $\Rightarrow t = 6$ ***(A1)***

$a(6) = -100\ (\mathrm{ms}^{-2})$ ***A1 N3***

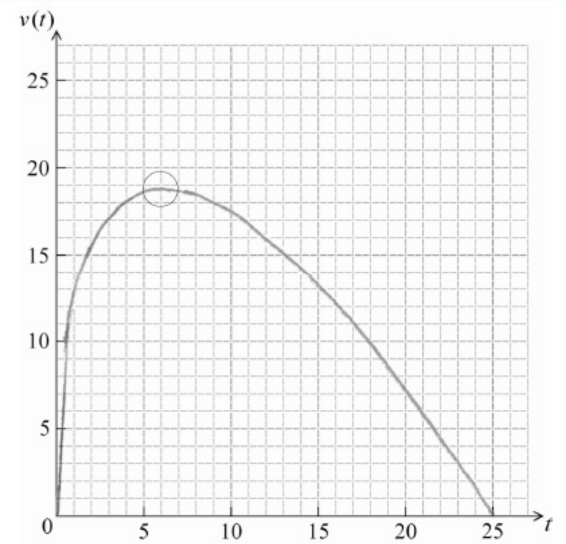
[7 marks]

The velocity v ms^{-1} of an object after t seconds is given by
$v(t) = 15\sqrt{t} - 3t$, for
$0 \le t \le 25$.

23a. On the grid below, sketch the graph of v, clearly indicating the maximum point. *[3 marks]*



Markscheme



A1A1A1 ***N3***

Note: Award ***A1*** for approximately correct shape, ***A1*** for right endpoint at $(25, 0)$ and ***A1*** for maximum point in circle.

[3 marks]

23b. (i) Write down an expression for *d* . *[4 marks]*

(ii) Hence, write down the value of *d* .

Markscheme

(i) recognizing that *d* is the area under the curve ***(M1)***

e.g. $\int v(t)$

correct expression in terms of *t*, with correct limits ***A2*** ***N3***

e.g. $d = \int_0^9 (15\sqrt{t} - 3t)\mathrm{d}t$, $d = \int_0^9 v\mathrm{d}t$

(ii) $d = 148.5$ (m) (accept 149 to 3 sf) ***A1*** ***N1***

[4 marks]

24. The acceleration, $a \text{ ms}^{-2}$, of a particle at time *t* seconds is given by *[7 marks]*

$$a = \frac{1}{t} + 3\sin 2t, \text{ for } t \geq 1.$$

The particle is at rest when $t = 1$.

Find the velocity of the particle when $t = 5$.

Markscheme

evidence of integrating the acceleration function ***(M1)***

e.g. $\int \left(\frac{1}{t} + 3\sin 2t\right)\mathrm{d}t$

correct expression $\ln t - \frac{3}{2}\cos 2t + c$ ***A1A1***

evidence of substituting (1, 0) ***(M1)***

e.g. $0 = \ln 1 - \frac{3}{2}\cos 2 + c$

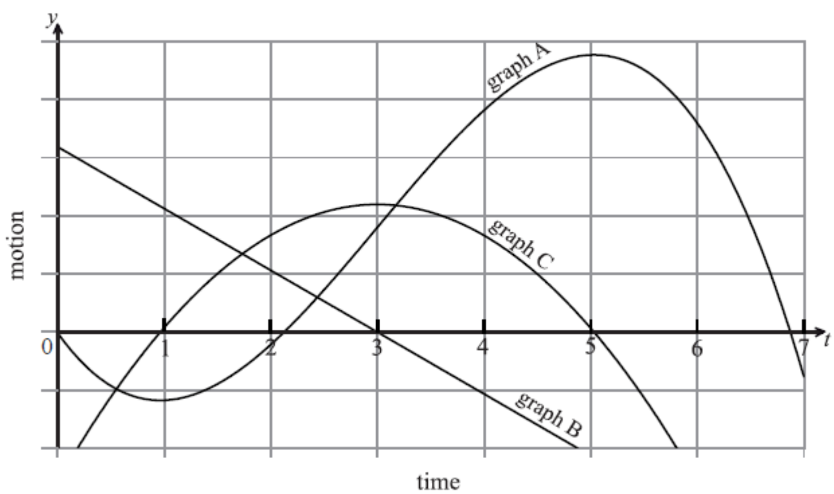
$c = -0.624 \left(= \frac{3}{2}\cos 2 - \ln 1 \text{ or } \frac{3}{2}\cos 2\right)$ ***(A1)***

$v = \ln t - \frac{3}{2}\cos 2t - 0.624$
$\left(= \ln t - \frac{3}{2}\cos 2t + \frac{3}{2}\cos 2 \text{ or } \ln t - \frac{3}{2}\cos 2t + \frac{3}{2}\cos 2 - \ln 1\right)$ ***(A1)***

$v(5) = 2.24$ (accept the exact answer $\ln 5 - 1.5\cos 10 + 1.5\cos 2$) ***A1 N3***

[7 marks]

The following diagram shows the graphs of the **displacement**, **velocity** and **acceleration** of a moving object as functions of time, *t*.



25a. Complete the following table by noting which graph A, B or C corresponds to each function. *[4 marks]*

Function	Graph
displacement	
acceleration	

Markscheme

Function	Graph
displacement	A
acceleration	B

A2A2 ***N4***

[4 marks]

25b. Write down the value of t when the velocity is greatest. *[2 marks]*

Markscheme

$t = 3$ ***A2*** ***N2***

[2 marks]

In this question *s* represents displacement in metres and *t* represents time in seconds.

The velocity *v* m s^{-1} of a moving body is given by
$v = 40 - at$ where *a* is a non-zero constant.

26a. (i) If $s = 100$ when $t = 0$, find an expression for *s* in terms of *a* and *t*. *[6 marks]*

(ii) If $s = 0$ when $t = 0$, write down an expression for *s* in terms of *a* and *t*.

Markscheme

Note: In this question, do not penalize absence of units.

(i) $s = \int (40 - at)\mathrm{d}t$ ***(M1)***

$s = 40t - \frac{1}{2}at^2 + c$ ***(A1)(A1)***

substituting $s = 100$ when $t = 0$ ($c = 100$) ***(M1)***

$s = 40t - \frac{1}{2}at^2 + 100$ ***A1*** ***N5***

(ii) $s = 40t - \frac{1}{2}at^2$ ***A1*** ***N1***

[6 marks]

Trains approaching a station start to slow down when they pass a point P. As a train slows down, its velocity is given by
$v = 40 - at$, where
$t = 0$ at P. The station is 500 m from P.

26b. A train M slows down so that it comes to a stop at the station. *[6 marks]*

(i) Find the time it takes train M to come to a stop, giving your answer in terms of *a*.

(ii) Hence show that $a = \frac{8}{5}$.

Markscheme

(i) stops at station, so $v = 0$ ***(M1)***

$t = \frac{40}{a}$ (seconds) ***A1*** ***N2***

(ii) evidence of choosing formula for *s* from (a) (ii) ***(M1)***

substituting $t = \frac{40}{a}$ ***(M1)***

e.g. $40 \times \frac{40}{a} - \frac{1}{2}a \times \frac{40^2}{a^2}$

setting up equation ***M1***

e.g. $500 = s$, $500 = 40 \times \frac{40}{a} - \frac{1}{2}a \times \frac{40^2}{a^2}$, $500 = \frac{1600}{a} - \frac{800}{a}$

evidence of simplification to an expression which obviously leads to $a = \frac{8}{5}$ ***A1***

e.g. $500a = 800$, $5 = \frac{8}{a}$, $1000a = 3200 - 1600$

$a = \frac{8}{5}$ ***AG*** ***N0***

[6 marks]

26c. For a different train N, the value of *a* is 4. *[5 marks]*

Show that this train will stop **before** it reaches the station.

Markscheme

METHOD 1

$v = 40 - 4t$, stops when $v = 0$

$40 - 4t = 0$ ***(A1)***

$t = 10$ ***A1***

substituting into expression for *s* ***M1***

$s = 40 \times 10 - \frac{1}{2} \times 4 \times 10^2$

$s = 200$ ***A1***

since $200 < 500$ (allow ***FT*** on their *s*, if $s < 500$) ***R1***

train stops before the station ***AG*** ***N0***

METHOD 2

from (b) $t = \frac{40}{4} = 10$ ***A2***

substituting into expression for ***s***

e.g. $s = 40 \times 10 - \frac{1}{2} \times 4 \times 10^2$ ***M1***

$s = 200$ ***A1***

since $200 < 500$ ***R1***

train stops before the station ***AG*** ***N0***

METHOD 3

a is deceleration ***A2***

$4 > \frac{8}{5}$ ***A1***

so stops in shorter time ***(A1)***

so less distance travelled ***R1***

so stops before station ***AG*** ***N0***

[5 marks]

The acceleration,
a ms^{-2}, of a particle at time *t* seconds is given by
$a = 2t + \cos t$.

27a. Find the acceleration of the particle at $t = 0$. *[2 marks]*

Markscheme

substituting $t = 0$ ***(M1)***

e.g. $a(0) = 0 + \cos 0$

$a(0) = 1$ ***A1*** ***N2***

[2 marks]

27b. Find the velocity, *v*, at time *t*, given that the initial velocity of the particle is $2\ \mathrm{ms}^{-1}$. *[5 marks]*

Markscheme

evidence of integrating the acceleration function ***(M1)***

e.g. $\int (2t + \cos t)\mathrm{d}t$

correct expression $t^2 + \sin t + c$ ***A1A1***

Note: If "$+c$" is omitted, award no further marks.

evidence of substituting (2,0) into indefinite integral ***(M1)***

e.g. $2 = 0 + \sin 0 + c$, $c = 2$

$v(t) = t^2 + \sin t + 2$ ***A1 N3***

[5 marks]

27c. Find $\int_0^3 v\mathrm{d}t$, giving your answer in the form $p - q\cos 3$. *[7 marks]*

Markscheme

$\int (t^2 + \sin t + 2)\mathrm{d}t = \frac{t^3}{3} - \cos t + 2t$ ***A1A1A1***

Note: Award ***A1*** for each correct term.

evidence of using $v(3) - v(0)$ ***(M1)***

correct substitution ***A1***

e.g. $(9 - \cos 3 + 6) - (0 - \cos 0 + 0)$, $(15 - \cos 3) - (-1)$

$16 - \cos 3$ (accept $p = 16$, $q = -1$) ***A1A1 N3***

[7 marks]

27d. What information does the answer to part (c) give about the motion of the particle? *[2 marks]*

Markscheme

reference to motion, reference to first 3 seconds ***R1R1 N2***

e.g. displacement in 3 seconds, distance travelled in 3 seconds

[2 marks]

28. A particle moves along a straight line so that its velocity, $v\ \mathrm{ms}^{-1}$at time *t* seconds is given by $v = 6\mathrm{e}^{3t} + 4$. When $t = 0$, the displacement, *s*, of the particle is 7 metres. Find an expression for *s* in terms of *t*. *[7 marks]*

Markscheme

evidence of anti-differentiation ***(M1)***

e.g. $s = \int (6e^{3x} + 4)dx$

$s = 2e^{3t} + 4t + C$ ***A2A1***

substituting $t = 0$, ***(M1)***

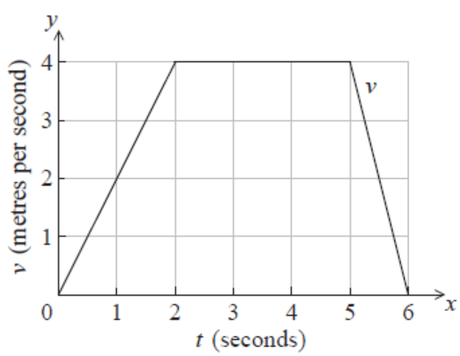
$7 = 2 + C$ ***A1***

$C = 5$

$s = 2e^{3t} + 4t + 5$ ***A1*** ***N3***

[7 marks]

A toy car travels with velocity v ms^{-1} for six seconds. This is shown in thegraph below.



29a. Write down the car's velocity at $t = 3$. *[1 mark]*

Markscheme

$4\ (ms^{-1})$ ***A1*** ***N1***

[1 mark]

29b. Find the car's acceleration at $t = 1.5$. *[2 marks]*

Markscheme

recognizing that acceleration is the gradient ***M1***

e.g. $a(1.5) = \frac{4-0}{2-0}$

$a = 2\ (ms^{-2})$ ***A1*** ***N1***

[2 marks]

29c. Find the total distance travelled. *[3 marks]*

Markscheme

recognizing area under curve ***M1***

e.g. trapezium, triangles, integration

correct substitution ***A1***

e.g. $\frac{1}{2}(3+6)4$, $\int_0^6 |v(t)|\mathrm{d}t$

distance 18 (m) ***A1*** ***N2***

[3 marks]