

Kinematics [285 marks]

A particle moves along a straight line so that its velocity, $v \text{ m s}^{-1}$, after t seconds is given by $v(t) = 1.4^t - 2.7$, for $0 \leq t \leq 5$.

1a. Find when the particle is at rest. [2 marks]

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1b. Find the acceleration of the particle when $t = 2$. [2 marks]

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1c. Find the total distance travelled by the particle.

[3 marks]

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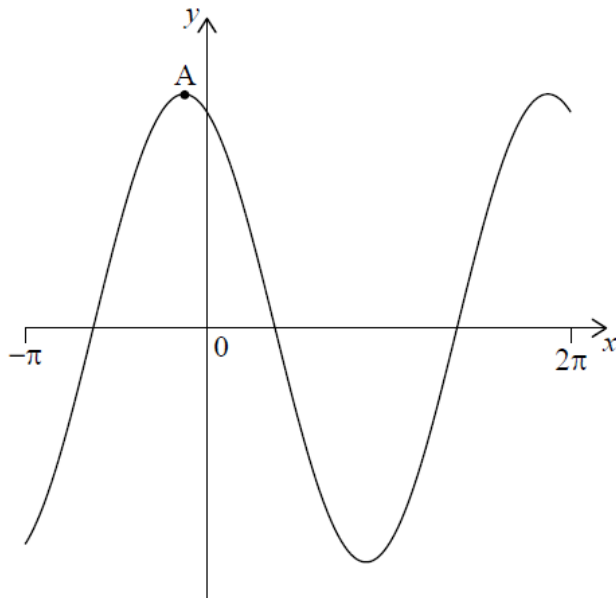
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Let $f(x) = 12 \cos x - 5 \sin x$, $-\pi \leq x \leq 2\pi$, be a periodic function with $f(x) = f(x + 2\pi)$

The following diagram shows the graph of f .



There is a maximum point at A. The minimum value of f is -13 .

2a. Find the coordinates of A.

[2 marks]

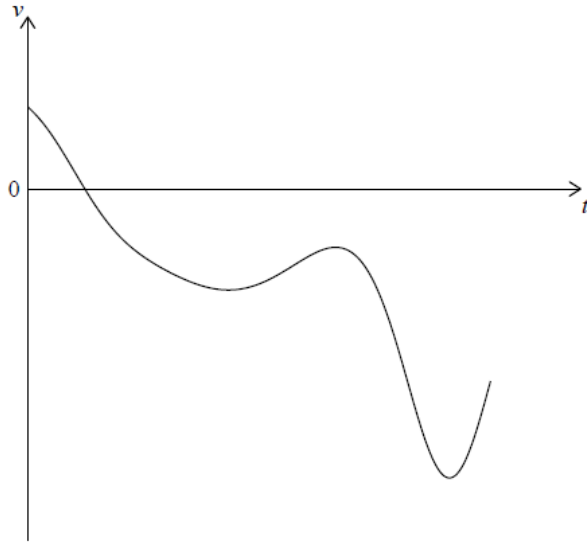
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A particle P moves along a straight line. The velocity $v \text{ m s}^{-1}$ of P after t seconds is given by $v(t) = 7 \cos t - 5t^{\cos t}$, for $0 \leq t \leq 7$.

The following diagram shows the graph of v .



3a. Find the initial velocity of P.

[2 marks]

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3b. Find the maximum speed of P.

[3 marks]

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3e. Find the total distance travelled by P.

[3 marks]

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Note: In this question, distance is in metres and time is in seconds.

A particle P moves in a straight line for five seconds. Its acceleration at time t is given by $a = 3t^2 - 14t + 8$, for $0 \leq t \leq 5$.

4a. Write down the values of t when $a = 0$.

[2 marks]

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4b. Hence or otherwise, find all possible values of t for which the velocity of P is decreasing.

[2 marks]

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When $t = 0$, the velocity of P is 3 ms^{-1} .

4c. Find an expression for the velocity of P at time t .

[6 marks]

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4d. Find the total distance travelled by P when its velocity is increasing.

[4 marks]

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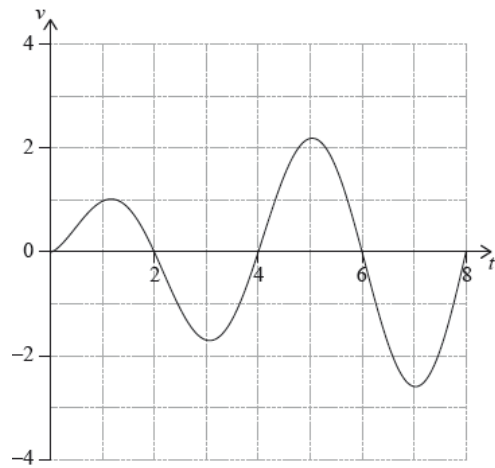
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A particle P moves along a straight line. Its velocity $v_P \text{ m s}^{-1}$ after t seconds is given by $v_P = \sqrt{t} \sin\left(\frac{\pi}{2}t\right)$, for $0 \leq t \leq 8$. The following diagram shows the graph of v_P .



5a. Write down the first value of t at which P changes direction. [1 mark]

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5b. Find the **total** distance travelled by P, for $0 \leq t \leq 8$. [2 marks]

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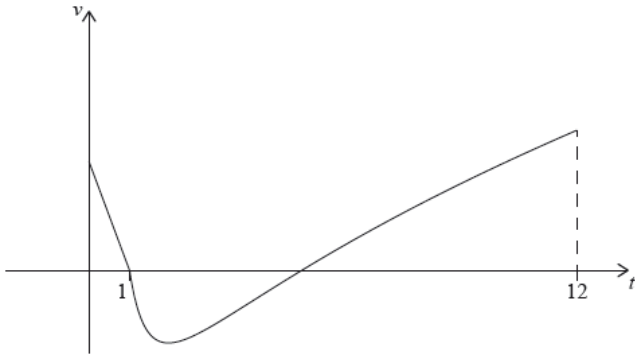
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A particle P starts from a point A and moves along a horizontal straight line. Its velocity $v \text{ cm s}^{-1}$ after t seconds is given by

$$v(t) = \begin{cases} -2t + 2, & \text{for } 0 \leq t \leq 1 \\ 3\sqrt{t} + \frac{4}{t^2} - 7, & \text{for } 1 \leq t \leq 12 \end{cases}$$

The following diagram shows the graph of v .



7a. Find the initial velocity of P .

[2 marks]

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P is at rest when $t = 1$ and $t = p$.

7b. Find the value of p .

[2 marks]

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When $t = q$, the acceleration of P is zero.

- 7c. (i) Find the value of q . [4 marks]
(ii) Hence, find the **speed** of P when $t = q$.

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- 7d. (i) Find the total distance travelled by P between $t = 1$ and $t = p$. [6 marks]
(ii) Hence or otherwise, find the displacement of P from A when $t = p$.

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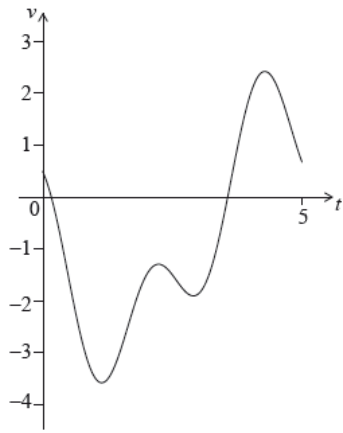
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The following sketch shows the graph of v .



8b. Find when P is first at rest.

[2 marks]

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8c. Write down the number of times P changes direction.

[2 marks]

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8d. Find the acceleration of P after 3 seconds.

[2 marks]

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8e. Find the maximum speed of P.

[3 marks]

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9. A particle moves in a straight line. Its velocity $v \text{ m s}^{-1}$ after t seconds is given by [7 marks]

$$v = 6t - 6, \text{ for } 0 \leq t \leq 2.$$

After p seconds, the particle is 2 m from its initial position. Find the possible values of p .

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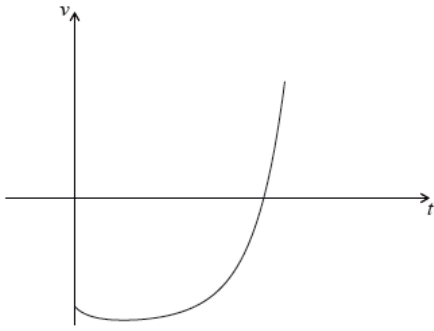
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The velocity $v \text{ ms}^{-1}$ of a particle after t seconds is given by

$$v(t) = (0.3t + 0.1)^t - 4, \text{ for } 0 \leq t \leq 5$$

The following diagram shows the graph of v .



10a. Find the value of t when the particle is at rest.

[3 marks]

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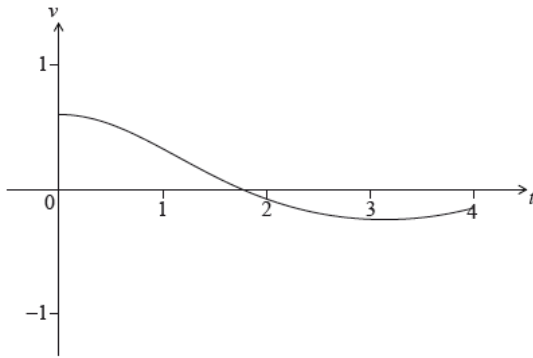
10b. Find the value of t when the acceleration of the particle is 0.

[3 marks]

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A particle starts from point A and moves along a straight line. Its velocity, $v \text{ ms}^{-1}$, after t seconds is given by $v(t) = e^{\frac{1}{2}\cos t} - 1$, for $0 \leq t \leq 4$. The particle is at rest when $t = \frac{\pi}{2}$.

The following diagram shows the graph of v .



11. Find the distance travelled by the particle for $0 \leq t \leq \frac{\pi}{2}$.

[2 marks]

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12. Ramiro and Lautaro are travelling from Buenos Aires to El Moro.

[8 marks]

Ramiro travels in a vehicle whose velocity in ms^{-1} is given by $V_R = 40 - t^2$, where t is in seconds.

Lautaro travels in a vehicle whose displacement from Buenos Aires in metres is given by $S_L = 2t^2 + 60$.

When $t = 0$, both vehicles are at the same point.

Find Ramiro's displacement from Buenos Aires when $t = 10$.

A particle moves in a straight line. Its velocity,
 $v \text{ ms}^{-1}$, at time
 t seconds, is given by

$$v = (t^2 - 4)^3, \text{ for } 0 \leq t \leq 3.$$

- 13a. Find the velocity of the particle when $t = 1$.

[2 marks]

- 13b. Find the value of t for which the particle is at rest.

[3 marks]

- 13c. Find the total distance the particle travels during the first three seconds.

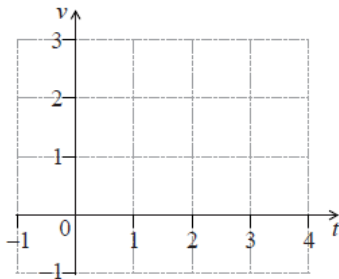
[3 marks]

13d. Show that the acceleration of the particle is given by $a = 6t(t^2 - 4)^2$. [3 marks]

13e. Find all possible values of t for which the velocity and acceleration are both positive or both negative. [4 marks]

A particle moves along a straight line such that its velocity,
 $v \text{ ms}^{-1}$, is given by
 $v(t) = 10te^{-1.7t}$, for
 $t \geq 0$.

14a. On the grid below, sketch the graph of v , for $0 \leq t \leq 4$. [3 marks]



14b. Find the distance travelled by the particle in the first three seconds. [2 marks]

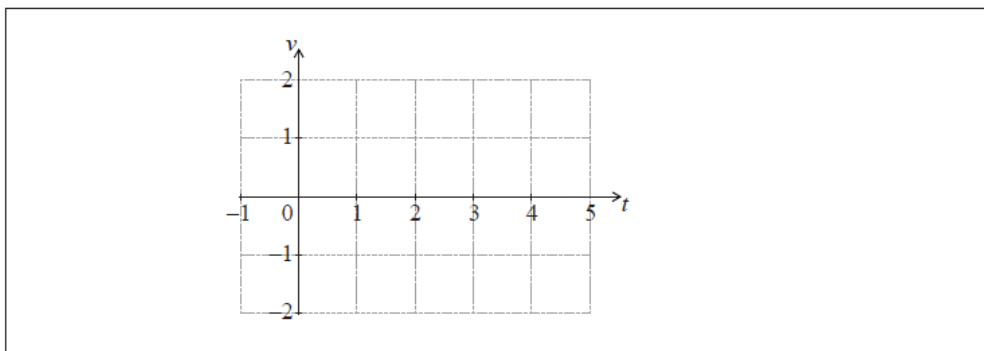
14c. Find the velocity of the particle when its acceleration is zero. [3 marks]

15. A rocket moving in a straight line has velocity $v \text{ km s}^{-1}$ and displacement $s \text{ km}$ at time t seconds. The velocity v is given by $v(t) = 6e^{2t} + t$. When $t = 0$, $s = 10$. [7 marks]

Find an expression for the displacement of the rocket in terms of t .

The velocity of a particle in ms^{-1} is given by
 $v = e^{\sin t} - 1$, for
 $0 \leq t \leq 5$.

16a. On the grid below, sketch the graph of v . [3 marks]



16b. Find the total distance travelled by the particle in the first five seconds. [1 mark]

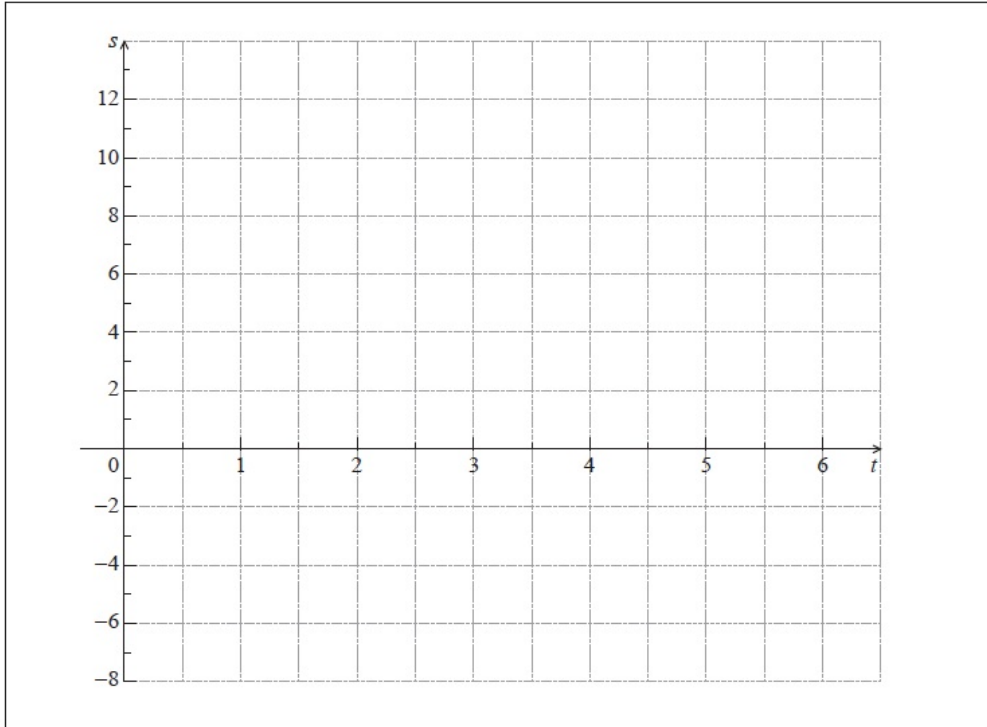
16c. Write down the positive t -intercept. [4 marks]

A particle's displacement, in metres, is given by

$$s(t) = 2t \cos t, \text{ for}$$

$$0 \leq t \leq 6, \text{ where } t \text{ is the time in seconds.}$$

17a. On the grid below, sketch the graph of s . [4 marks]



17b. Find the maximum velocity of the particle. [3 marks]

In this question, you are given that

$$\cos \frac{\pi}{3} = \frac{1}{2}, \text{ and}$$

$$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}.$$

The displacement of an object from a fixed point, O is given by

$$s(t) = t - \sin 2t \text{ for}$$

$$0 \leq t \leq \pi.$$

18a. Find $s'(t)$. [3 marks]

18b. In this interval, there are only two values of t for which the object is not moving. One value is $t = \frac{\pi}{6}$. [4 marks]

Find the other value.

18c. Show that $s'(t) > 0$ between these two values of t . [3 marks]

18d. Find the distance travelled between these two values of t . [5 marks]

A particle moves in a straight line with velocity
 $v = 12t - 2t^3 - 1$, for
 $t \geq 0$, where v is in centimetres per second and t is in seconds.

19a. Find the acceleration of the particle after 2.7 seconds. [3 marks]

19b. Find the displacement of the particle after 1.3 seconds. [3 marks]

Let
 $f(t) = 2t^2 + 7$, where
 $t > 0$. The function v is obtained when the graph of f is transformed by
a stretch by a scale factor of
 $\frac{1}{3}$ parallel to the y -axis,
followed by a translation by the vector
 $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$.

20a. Find $v(t)$, giving your answer in the form $a(t - b)^2 + c$. [4 marks]

20b. A particle moves along a straight line so that its velocity in ms^{-1} , at time t seconds, is given by v . Find the distance the particle travels between $t = 5.0$ and $t = 6.8$. [3 marks]

The velocity $v \text{ ms}^{-1}$ of a particle at time t seconds, is given by
 $v = 2t + \cos 2t$, for
 $0 \leq t \leq 2$.

21a. Write down the velocity of the particle when $t = 0$. [1 mark]

21b. When $t = k$, the acceleration is zero. [8 marks]

- (i) Show that $k = \frac{\pi}{4}$.
- (ii) Find the exact velocity when $t = \frac{\pi}{4}$.

21c. When $t < \frac{\pi}{4}$, $\frac{dv}{dt} > 0$ and when $t > \frac{\pi}{4}$, $\frac{dv}{dt} < 0$. [4 marks]

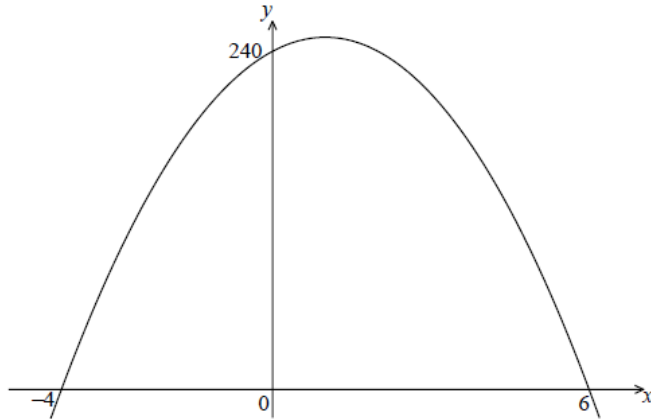
Sketch a graph of v against t .

21d. Let d be the distance travelled by the particle for $0 \leq t \leq 1$.

[3 marks]

- (i) Write down an expression for d .
- (ii) Represent d on your sketch.

The following diagram shows part of the graph of a quadratic function f .



The x -intercepts are at $(-4, 0)$ and $(6, 0)$, and the y -intercept is at $(0, 240)$.

22a. Write down $f(x)$ in the form $f(x) = -10(x - p)(x - q)$.

[2 marks]

22b. Find another expression for $f(x)$ in the form $f(x) = -10(x - h)^2 + k$.

[4 marks]

22c. Show that $f(x)$ can also be written in the form $f(x) = 240 + 20x - 10x^2$.

[2 marks]

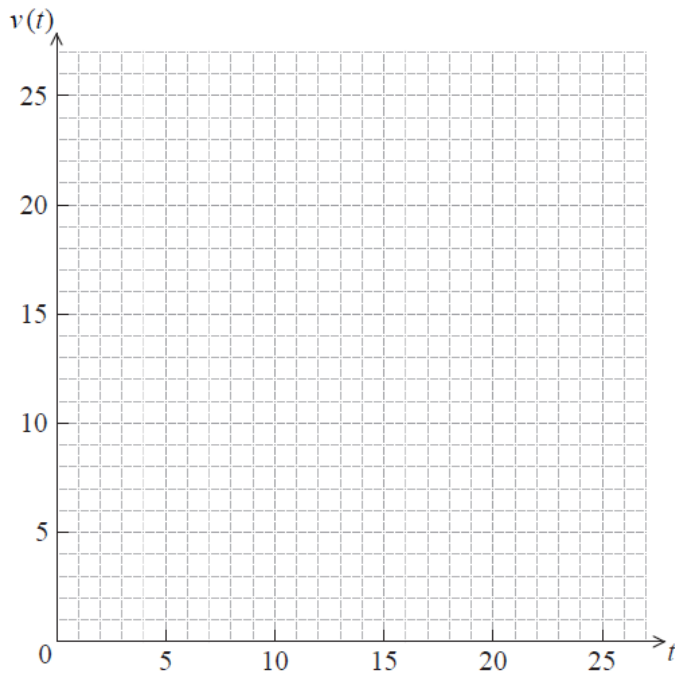
22d. A particle moves along a straight line so that its velocity, $v \text{ ms}^{-1}$, at time t seconds is given by $v = 240 + 20t - 10t^2$, for $0 \leq t \leq 6$.

[7 marks]

- (i) Find the value of t when the speed of the particle is greatest.
- (ii) Find the acceleration of the particle when its speed is zero.

The velocity $v \text{ ms}^{-1}$ of an object after t seconds is given by
 $v(t) = 15\sqrt{t} - 3t$, for
 $0 \leq t \leq 25$.

23a. On the grid below, sketch the graph of v , clearly indicating the maximum point. [3 marks]



23b. (i) Write down an expression for d . [4 marks]
(ii) Hence, write down the value of d .

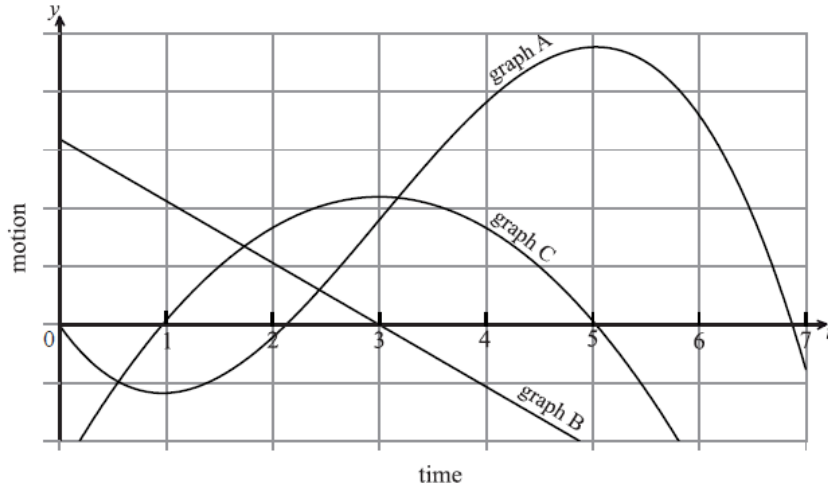
24. The acceleration, $a \text{ ms}^{-2}$, of a particle at time t seconds is given by [7 marks]

$$a = \frac{1}{t} + 3 \sin 2t, \text{ for } t \geq 1.$$

The particle is at rest when $t = 1$.

Find the velocity of the particle when $t = 5$.

The following diagram shows the graphs of the **displacement**, **velocity** and **acceleration** of a moving object as functions of time, t .



25a. Complete the following table by noting which graph A, B or C corresponds to each function. [4 marks]

Function	Graph
displacement	
acceleration	

25b. Write down the value of t when the velocity is greatest. [2 marks]

In this question s represents displacement in metres and t represents time in seconds.

The velocity v m s⁻¹ of a moving body is given by
 $v = 40 - at$ where a is a non-zero constant.

26a. (i) If $s = 100$ when $t = 0$, find an expression for s in terms of a and t . [6 marks]
 (ii) If $s = 0$ when $t = 0$, write down an expression for s in terms of a and t .

Trains approaching a station start to slow down when they pass a point P. As a train slows down, its velocity is given by
 $v = 40 - at$, where
 $t = 0$ at P. The station is 500 m from P.

26b. A train M slows down so that it comes to a stop at the station. [6 marks]
 (i) Find the time it takes train M to come to a stop, giving your answer in terms of a .
 (ii) Hence show that $a = \frac{8}{5}$.

26c. For a different train N, the value of a is 4. [5 marks]
 Show that this train will stop **before** it reaches the station.

The acceleration,
 $a \text{ ms}^{-2}$, of a particle at time t seconds is given by
 $a = 2t + \cos t$.

27a. Find the acceleration of the particle at $t = 0$. [2 marks]

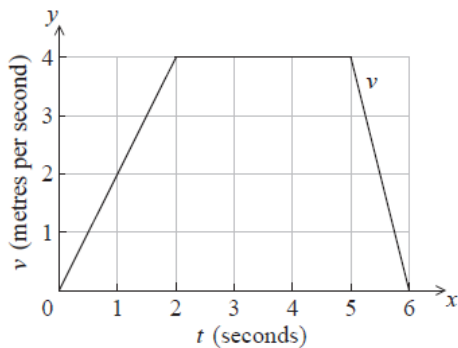
27b. Find the velocity, v , at time t , given that the initial velocity of the particle is 2 ms^{-1} . [5 marks]

27c. Find $\int_0^3 v dt$, giving your answer in the form $p - q \cos 3$. [7 marks]

27d. What information does the answer to part (c) give about the motion of the particle? [2 marks]

28. A particle moves along a straight line so that its velocity, $v \text{ ms}^{-1}$ at time t seconds is given by $v = 6e^{3t} + 4$. When $t = 0$, the displacement, s , of the particle is 7 metres. Find an expression for s in terms of t . [7 marks]

A toy car travels with velocity $v \text{ ms}^{-1}$ for six seconds. This is shown in the graph below.



29a. Write down the car's velocity at $t = 3$. [1 mark]

29b. Find the car's acceleration at $t = 1.5$. [2 marks]

29c. Find the total distance travelled. [3 marks]