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IB Physics HL

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May 2016

Internal Assessment:

Is Internal Resistance Negligible?

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Abstract

The purpose of this investigation is to evaluate to what extent the internal resistances of power supplies are indeed negligible, as is often assumed to be true in high school problems. For this investigation, an internal resistance is considered negligible if it makes up less than 5% of the total resistance. This resistance can be found through the use of the equation: $V = \varepsilon - R_{int}I$, which is derived using the basic principles of circuitry, namely Ohm's Law. The terminal potential difference of the circuit is measured at different values of current and then plotted on a graph. The graph produced then displays a linear regression with y-intercept ε and a gradient of -R_{int}, which makes it possible to solve for the internal resistance. On top of that, the theoretical maximum current produced by each power supply is found, to demonstrate the impact of the internal resistance, by using: $I_{max} = \frac{\varepsilon}{R_{int}}$.

This is done for three different power supplies, giving rise to the following results:

	Internal Resistance (Ω)	Maximum Current (A)	Threshold External Resistance (Ω)
Supply A (13.8V)	1.0 ± 20%	13.5 ± 20%	19.0
Supply B (20V)	0.4 ± 25%	45.5 ± 25%	7.6
Supply C (30V)	1.5 ± 20%	20.1 ± 20%	28.5

Table 1 The results of the investigation.

This shows that the internal resistances of the power supplies vary quite a lot and can reach relatively large values. This, then, leads to the conclusion that for circuits with low external resistances (below the threshold external resistance), the internal resistance can become a large portion of the total resistance, making it invalid to declare such internal resistance to be negligible.

Introduction

When we started studying the physics of electric circuits in class, I noticed that almost all problems described power supplies as having negligible *internal resistance*. This immediately sparked my interest since I have always been interested in the underlying assumptions of problem solving in physics. I then asked my teacher and learned it had to do with the resistance within the power supply itself. This resistance is due to the electrical components of the power supply, which cannot have zero resistance. I was even more delighted when I found out that the value of this resistance was something that I could actually investigate myself. The basis of the investigation relies on the fundamental equation of circuitry, Ohm's Law: $V = RI$.

Logically, the total potential difference is equal to the electromotive force (emf) of the power supply (ε), and the total resistance is equal to the sum of the external and internal resistance. The equation can then be rewritten as: $\varepsilon = (R_{ext} + R_{int})I = R_{ext}I + R_{int}I$. Now, it is known that $R_{ext}I$ is equal to the terminal potential difference (V) of the circuit. That is, the potential drop across the two terminals of the power supply. We can then derive the following equation: $V = \varepsilon - R_{int}I$. It is interesting to note that the voltage produced by a power supply will effectively never be equal to its emf, due to the loss of energy in the supply itself. If we were to plot a graph of the terminal potential difference versus the current, we would expect a linear relationship, with the emf as its y-intercept and the -R_{int} as its slope, allowing us to find the internal resistance of the power supply, which I personally find extremely intriguing.

I then decided to investigate the internal resistances of three different power supplies, using the above equation, to find out whether or not internal resistance is indeed negligible for typical high school physics problems. To do this, a definition of *negligible* internal resistance must be established. Since I could not clearly find such a definition, I decided to define internal resistance to be negligible if it makes up less than 5% of the total resistance, as this seems like a reasonable percentage to me. This definition will be used throughout this investigation.

Research Question

To what extent are the internal resistances of different power supplies indeed negligible?

Variables

Independent: The current generated (I), which is varied through adjusting the value of the external variable resistor (R).

Dependent: The terminal potential difference (V).

Controlled:

- The same external variable resistors are used.
- The same wires are used, so that their resistance remains constant.
- The same voltmeter is used, so that its readings remain consistent with itself.
- The same ammeter is used, so that its readings remain consistent with itself.

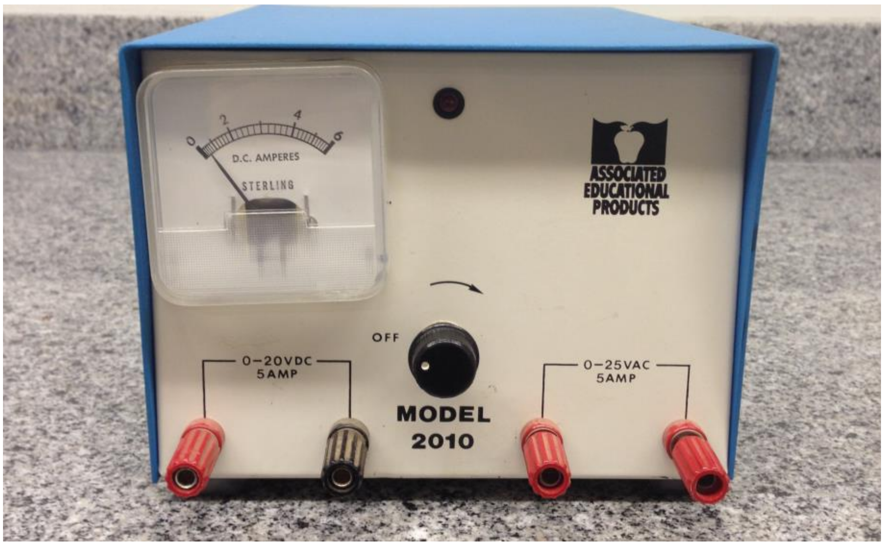
Materials

- Three different power supplies
- Two variable resistors (0-10Ω).
- A voltmeter.
- An ammeter.
- A set of wires to set up the circuit.

The following power supplies, with their respective emf, are used for the experiment, which will be referred to as supply A, B, and C accordingly:

Supply A (13.8V)

Supply B (20V)



Supply C (30V)

Setup

The materials mentioned above should be set up according to the diagram below, which I created myself using *CircuitLab*[1]. Note that the elements enclosed by the dashed box represent the entire power supply. One should ensure that no wires are forming a short circuit by fixing the wires with tape. Two resistors are used so that a greater range of values for the current can be obtained, which improves the precision of the linear regression.

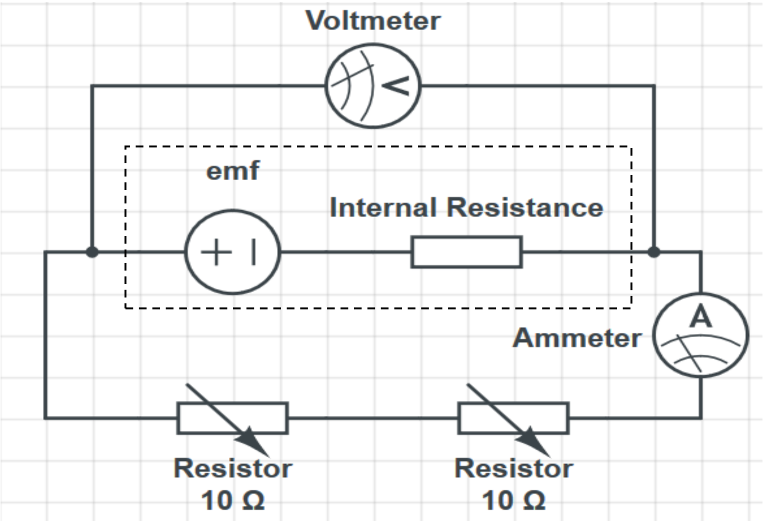


Figure 1 A diagram displaying the setup of the investigation.

[1] https://www.circuitlab.com/

Procedure

1. Make sure the power supply is set to the right voltage, depending on the outlet (220V or 110V).
2. Set the variable resistors to their maximum value to ensure no overheating occurs.
3. Check if the circuit is set up exactly like the diagram above to prevent a short circuit.
4. Turn on the power supply.
5. Record the current (I) and terminal potential difference (V).
6. Vary the external resistance and record the new V-I reading.
7. Keep varying the external resistance until 10 V-I readings are obtained.
    a. One should aim to obtain values of I over as wide a range as possible, depending on the maximum current the supply can hold, to increase the precision of the results.
8. Do not leave the current running for too long, especially at high currents, to ensure nothing overheats.
9. Repeat steps 1-8 for the two other power supplies.

Data & Analysis

After performing the experiment, the following results were obtained for Supply A:

Supply A (13.8V)										
Current / ± 0.01 A	0.57	0.85	1.10	1.35	1.60	1.85	2.10	2.35	2.60	2.85
Voltage / ± 0.2 V	13.4	13.1	12.8	12.5	12.3	12.2	12.0	11.6	11.3	11.0

Table 2 The raw data from Supply A.

The uncertainty for the current is derived from the accuracy of the ammeter. The device displayed the value of the current, which almost did not fluctuate at all, up to two decimals places, making ± 0.01 A the appropriate uncertainty for the current. However, even though the voltmeter displayed the value of the potential difference up to three decimals, it would not be appropriate to use ± 0.001 V as its uncertainty. This is because the voltage measured was highly unstable and fluctuated within a range of approximately 0.4 V. For this reason, it is much more appropriate to use ± 0.2 V as the uncertainty for the voltage.

Using this data, a graph relating the terminal voltage and the current can be produced:

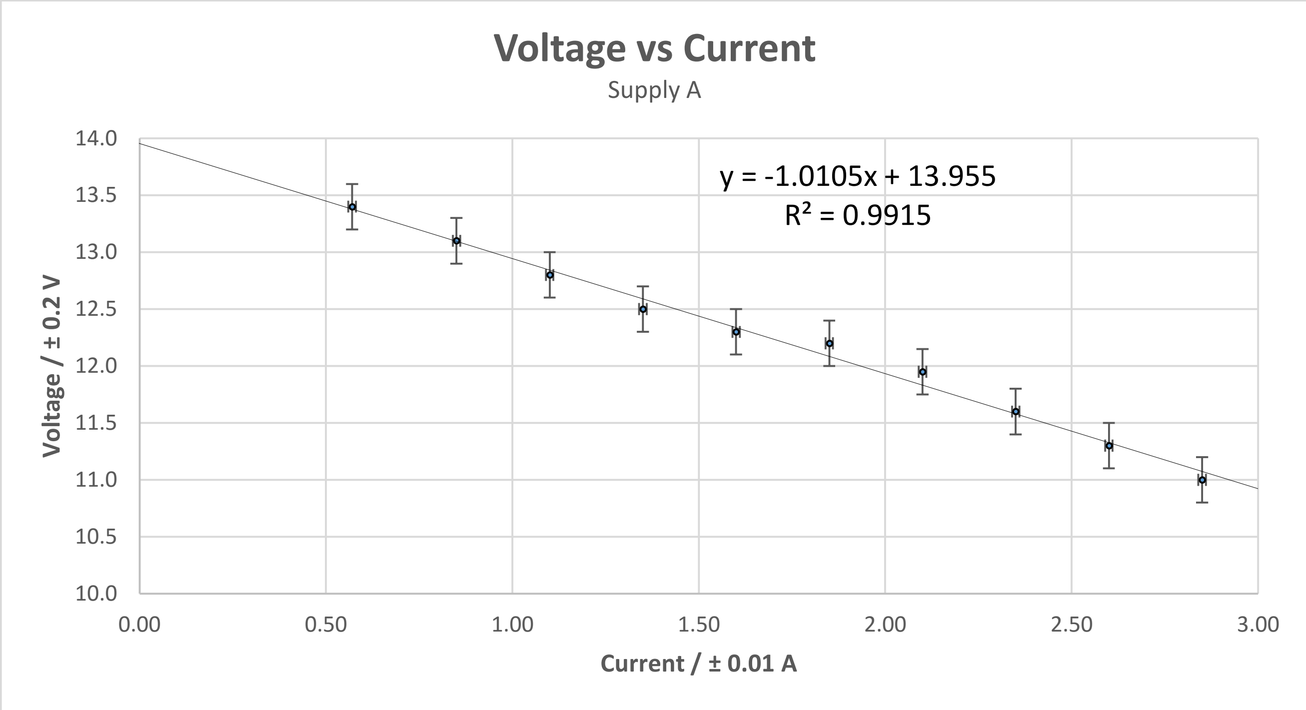


Figure 2 The relationship between terminal voltage and current for Supply A.

From this graph, one could conclude that the internal resistance of the small power supply equals 1.0105 Ohms, since the graph represents $V = \varepsilon - R_{int}I$. However, the uncertainties of the data points should also be taken into account. This can be done by constructing a maximum and minimum gradient using the error bars, which would represent the upper and lower extreme of the value of the internal resistance.

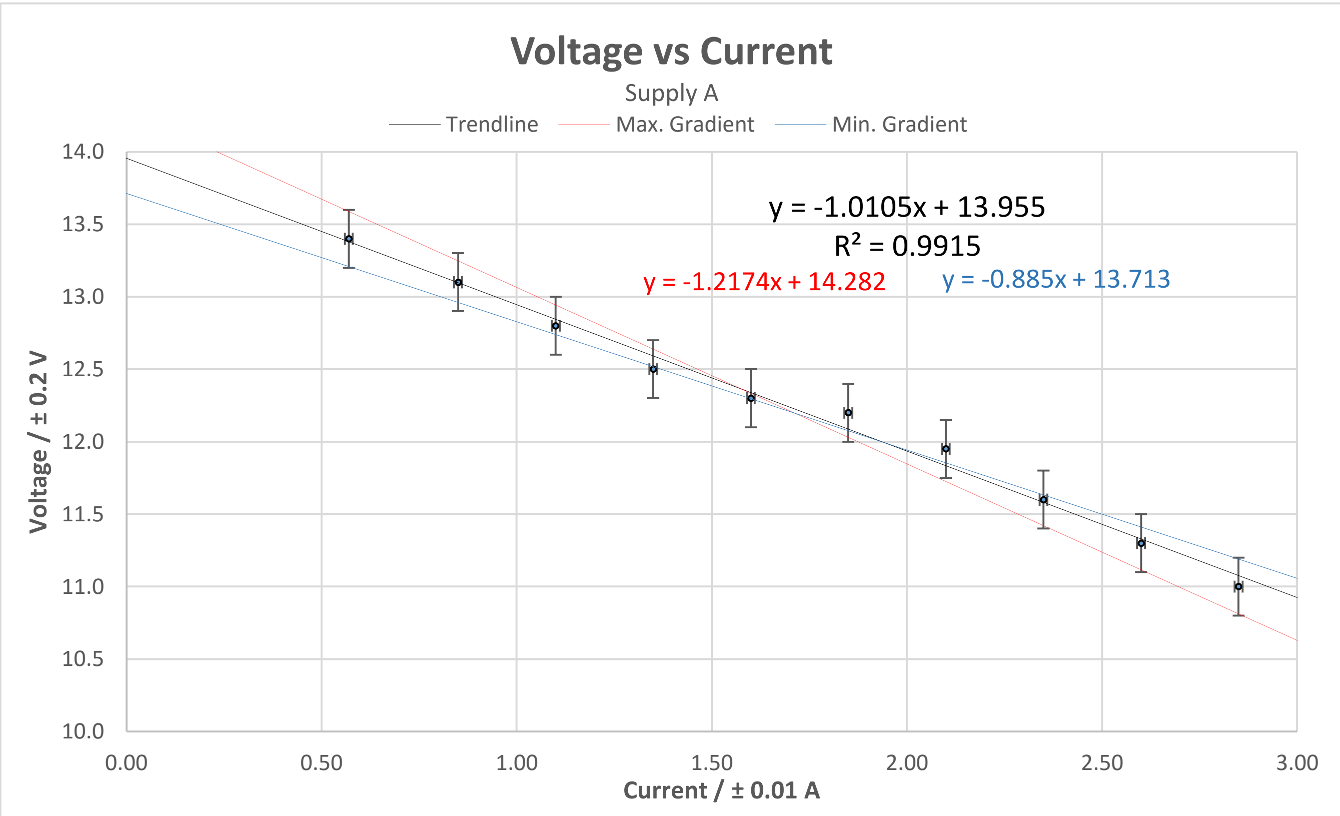


Figure 3 The same graph as above, now with maximum and minimum gradients added to it.

Using the minimum and maximum gradient, one could say that the internal resistance of the power supply A falls between 0.885 and 1.2174. For this reason, one could also state the internal resistance as 1.0 Ω ± 0.2 or 1.0 Ω ± 20%. This allows us to effectively demonstrate the value of the internal resistance and its degree of uncertainty.

The same process can then be applied to the two other power supplies:

Supply B (20V)										
Current / ± 0.01 A	0.90	1.30	1.70	2.10	2.50	2.90	3.30	3.70	4.10	4.50
Voltage / ± 0.2 V	18.8	18.6	18.5	18.3	18.2	18.0	17.8	17.6	17.4	17.2

Table 3 The raw data from Supply B.

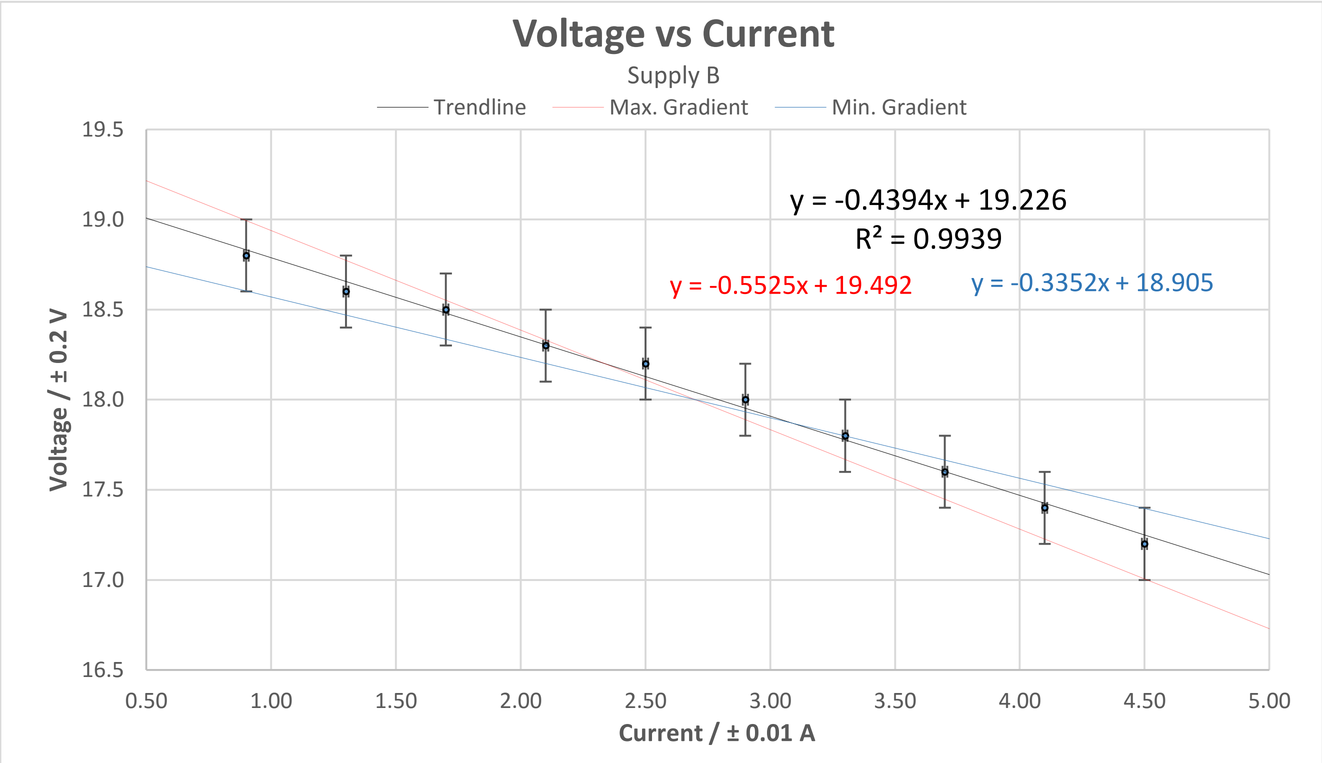


Figure 4 The graph relating terminal voltage and current for Supply B, including maximum and minimum gradients.

Similarly, one can find the internal resistance of this power supply to be 0.4394. Using the maximum and minimum gradient, one can determine its value with the appropriate uncertainty: 0.4 Ω ± 0.1 or 0.4 Ω ± 25%.

For supply C:

Supply C (30V)										
Current / ± 0.01 A	1.51	1.65	1.80	1.95	2.10	2.25	2.40	2.55	2.70	2.85
Voltage / ± 0.2 V	27.2	26.9	26.6	26.3	26.1	25.9	25.7	25.6	25.4	25.2

Table 4 The raw data from Supply C.

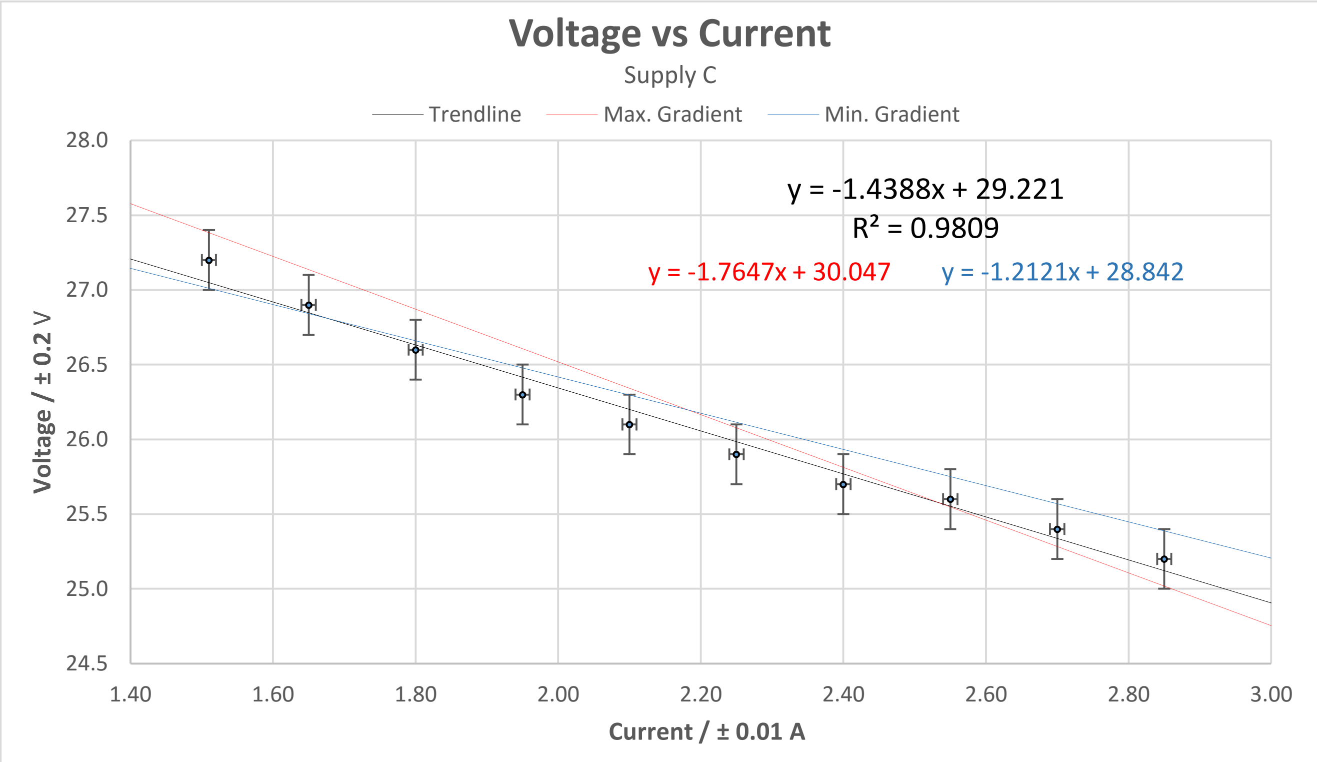


Figure 5 The graph relating terminal voltage and current for Supply C, including maximum and minimum gradients.

For the supply C, the internal resistance appears to be 1.4388. When taking into account the values of the maximum and minimum gradient, one finds the internal resistance to be 1.5 Ω ± 0.3 or 1.5 Ω ± 20%.

Conclusion

In conclusion, the internal resistances of the power supplies are larger than expected. It is important to note that Supply C had previously been plugged into the wrong outlet (220V instead of 110V), which caused one of its safety resistances to burn out. Even though the supply still functions properly, the burned out component could have significantly increased its internal resistance, which provides an explanation for its high internal resistance relative to the other supplies. Apart from that, the differences could also be due to the physical size of the supplies and the materials they are made of.

From the internal resistances, it is possible to calculate another interesting feature of the power supplies: the theoretical maximum current produced, which demonstrates the effect of the internal resistance. It is derived using the equation that was also used earlier on in the investigation: $I = \frac{\varepsilon}{R_{ext}+R_{int}}$. This maximum current would occur when the external resistance is zero. Therefore, it is possible to find the theoretical maximum current produced by the power supply: $I_{max} = \frac{\varepsilon}{R_{int}}$. However, in real life the power supplies would not be able to withstand such high currents, which is why this value is purely theoretical. The results of the investigation are summarized in the data table below:

	Internal Resistance (Ω)	Maximum Current (A)	Threshold External Resistance (Ω)
Supply A (13.8V)	1.0 ± 20%	13.5 ± 20%	19.0
Supply B (20V)	0.4 ± 25%	45.5 ± 25%	7.6
Supply C (30V)	1.5 ± 20%	20.1 ± 20%	28.5

Table 5 The results of the investigation.

I personally find it particularly fascinating that supply B can provide a higher theoretical maximum current than supply C, even though its emf is lower by 10V, due to the difference in internal resistance. With the results shown above, it is now possible to evaluate whether or not the internal resistance of a power supply is indeed negligible. Using the established definition of negligibility (less than 5% of total resistance), a threshold external resistance is calculated. For circuits with an external resistance less than this threshold resistance, the internal resistance of the power supply cannot be neglected. Since typical high school circuits have relatively small external resistances (10-20Ω), the internal resistance of Supply A & C can usually not be neglected, while that of Supply B can be considered negligible even at such low external resistances. Nonetheless, as the external resistance becomes larger and exceeds all threshold resistances, the internal resistances of all power supplies become more and more insignificant and eventually, negligible. In short, the internal resistance of power supplies cannot simply be neglected in typical high school circuitry problems and should therefore always be taken into account.

Evaluation

Since the power supplies did not publish their internal resistances, there is unfortunately no way to compare the experimental values to expected values, which makes it rather difficult to evaluate the errors of the experiment. However, there are certain systematic errors that have in fact affected the results of the experiment and caused the rather high percentage uncertainties.

A major cause of the lack of precision of the results is the limited value of the current the power supplies could withstand. The power supplies could withstand either three or five amps at their maximum, which severely limits the range of data points that can be measured. A greater amount and range of data points would increase the precision of the trendline, which would ultimately lead to a greater precision in the value of the internal resistance of the power supplies. This could possibly be resolved by using other power supplies that are able to withstand higher values of current, increasing the possible range of data points and therefore the overall precision of the experiment. Another way of potentially reducing this error would be adding more variable resistors or substituting them with ones that can go up to higher resistances. This would increase the maximum external resistance, which ultimately also increases the range of the possible values for the current by lowering the minimum value of the current.

Another source of error that occurred during the experiment is the heating effect of currents. At high values of current, the resistors absorb high amounts of kinetic energy from the electrons, which causes them to heat up. The problem with this is that as the temperature of the resistors increases, their resistance increases as well. Due to their changing resistance, the resistors no longer obey Ohm's Law, making the equation the investigation is based on invalid, which could majorly affect the results. The changing resistance occurs because at higher temperatures the metal cations are vibrating more and are hence obstructing the free flow of electrons by colliding more with them. Now, a changing external resistance is not a problem in this case, since we have been doing that throughout the entire experiment and this will only alter the value of the current. The source of error lies in the fact that the power supplies themselves also started to become warmer. This would mean that their internal resistances were not constant, but actually increased at higher values of current. However, this cannot have affected the results by much, since this would have resulted in a steeper slope at the higher value of current, which is not clearly visible. Nonetheless, this could possibly be improved by collecting the data at separate moments in time, so that the current would not flow continuously for a long period of time. This would limit the heating of the power supply and hence reduce this source of error.

Overall, the investigation has been very successful. I personally enjoyed the process very much and was intrigued by the final results, especially since most problems assume that such internal resistance is in fact negligible. I found it particularly interesting that the internal resistances of the power supplies are so different and the effect this has on their theoretical maximum current produced. I look forward to learning more about this in the future and perhaps investigating this with more precise measuring devices. This could possibly reduce the uncertainty of the voltage measured, which would ultimately reduce the uncertainty of the value of the internal resistance.