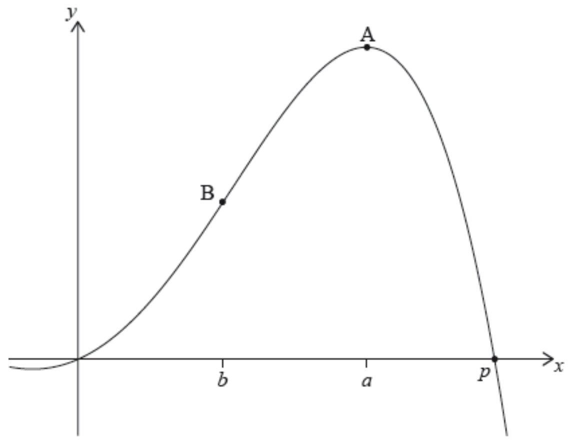
Intro derivatives *[44 marks]*

Let $f(x) = -0.5x^4 + 3x^2 + 2x$. The following diagram shows part of the graph of f.



There are x-intercepts at $x = 0$ and at $x = p$. There is a maximum at A where $x = a$, and a point of inflexion at B where $x = b$.

1a. Find the value of p. *[2 marks]*

Markscheme

evidence of valid approach ***(M1)***

eg $f(x) = 0,\ y = 0$

2.73205

$p = 2.73$ ***A1*** ***N2***

[2 marks]

1b. Write down the coordinates of A. *[2 marks]*

Markscheme

1.87938, 8.11721

$(1.88,\ 8.12)$ ***A2*** ***N2***

[2 marks]

1c. Write down the rate of change of f at A. *[1 mark]*

Markscheme

rate of change is 0 (do not accept decimals) ***A1*** ***N1***

[1 marks]

1d. Find the coordinates of B. *[4 marks]*

Markscheme

METHOD 1 (using GDC)

valid approach ***M1***

eg $f'' = 0$, max/min on f', $x = -1$

sketch of either f' or f'', with max/min or root (respectively) ***(A1)***

$x = 1$ ***A1*** ***N1***

Substituting **their** x value into f ***(M1)***

eg $f(1)$

$y = 4.5$ ***A1*** ***N1***

METHOD 2 (analytical)

$f'' = -6x^2 + 6$ ***A1***

setting $f'' = 0$ ***(M1)***

$x = 1$ ***A1*** ***N1***

substituting **their** x value into f ***(M1)***

eg $f(1)$

$y = 4.5$ ***A1*** ***N1***

[4 marks]

1e. Find the the rate of change of f at B. *[3 marks]*

Markscheme

recognizing rate of change is f' ***(M1)***

eg y', $f'(1)$

rate of change is 6 ***A1*** ***N2***

[3 marks]

1f. Let R be the region enclosed by the graph of f, the x-axis, the line $x = b$ and the line $x = a$. The region R is rotated 360° about the x-axis. Find the volume of the solid formed. *[3 marks]*

Markscheme

attempt to substitute either limits or the function into formula ***(M1)***

involving f^2 (accept absence of π and/or $\mathrm{d}x$)

eg $\pi \int (-0.5x^4 + 3x^2 + 2x)^2 \mathrm{d}x, \int_1^{1.88} f^2$

128.890

$\text{volume} = 129$ ***A2*** ***N3***

[3 marks]

Let $f(x) = \frac{1}{x-1} + 2$, for $x > 1$.

2a. Write down the equation of the horizontal asymptote of the graph of f. *[2 marks]*

Markscheme

$y = 2$ (correct equation only) ***A2*** ***N2***

[2 marks]

2b. Find $f'(x)$. *[2 marks]*

Markscheme

valid approach ***(M1)***

eg $(x-1)^{-1}+2,\ f'(x)=\frac{0(x-1)-1}{(x-1)^2}$

$-(x-1)^{-2},\ f'(x)=\frac{-1}{(x-1)^2}$ ***A1*** ***N2***

[2 marks]

Let $g(x)=ae^{-x}+b$, for $x \geqslant 1$. The graphs of f and g have the same horizontal asymptote.

2c. Write down the value of b. *[2 marks]*

Markscheme

correct equation for the asymptote of g

eg $y=b$ ***(A1)***

$b=2$ ***A1*** ***N2***

[2 marks]

2d. Given that $g'(1)=-e$, find the value of a. *[4 marks]*

Markscheme

correct derivative of g (seen anywhere) ***(A2)***

eg $g'(x)=-a\mathrm{e}^{-x}$

correct equation ***(A1)***

eg $-\mathrm{e}=-a\mathrm{e}^{-1}$

7.38905

$a=\mathrm{e}^2\ (\text{exact}),\ 7.39$ ***A1*** ***N2***

[4 marks]

2e. There is a value of x, for $1<x<4$, for which the graphs of f and g have the same gradient. Find this gradient. *[4 marks]*

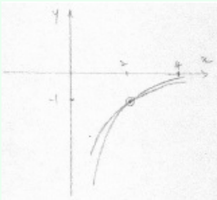
Markscheme

attempt to equate **their** derivatives ***(M1)***

eg $f'(x) = g'(x),\ \frac{-1}{(x-1)^2} = -a\mathrm{e}^{-x}$

valid attempt to solve **their** equation ***(M1)***

eg correct value outside the domain of f such as 0.522 or 4.51,



correct solution (may be seen in sketch) ***(A1)***

eg $x = 2,\ (2,\ -1)$

gradient is -1 ***A1*** ***N3***

[4 marks]

Let
$f(x) = x^3 - 4x + 1$.

3a. Expand
$(x+h)^3$. *[2 marks]*

Markscheme

attempt to expand ***(M1)***

$(x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$ ***A1*** ***N2***

[2 marks]

3b. Use the formula
$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ to show that the derivative of
$f(x)$ is
$3x^2 - 4$. *[4 marks]*

Markscheme

evidence of substituting
$x + h$ ***(M1)***

correct substitution ***A1***

e.g.
$f'(x) = \lim_{h \to 0} \frac{(x+h)^3 - 4(x+h) + 1 - (x^3 - 4x + 1)}{h}$

simplifying ***A1***

e.g.
$\frac{(x^3 + 3x^2h + 3xh^2 + h^3 - 4x - 4h + 1 - x^3 + 4x - 1)}{h}$

factoring out *h* ***A1***

e.g.
$\frac{h(3x^2 + 3xh + h^2 - 4)}{h}$

$f'(x) = 3x^2 - 4$ ***AG*** ***N0***

[4 marks]

3c. The tangent to the curve of f at the point $\mathrm{P}(1, -2)$ is parallel to the tangent at a point Q. Find the coordinates of Q. *[4 marks]*

Markscheme

$f'(1) = -1$ ***(A1)***

setting up an appropriate equation ***M1***

e.g.
$3x^2 - 4 = -1$

at Q,
$x = -1, y = 4$ (Q is
$(-1, 4)$) ***A1*** ***A1***

[4 marks]

3d. The graph of *f* is decreasing for $p < x < q$. Find the value of *p* and of *q*. *[3 marks]*

Markscheme

recognizing that *f* is decreasing when
$f'(x) < 0$ ***R1***

correct values for *p* and *q* (but do not accept
$p = 1.15, q = -1.15$) ***A1A1*** ***N1N1***

e.g.
$p = -1.15, q = 1.15$;
$\pm\frac{2}{\sqrt{3}}$; an interval such as
$-1.15 \le x \le 1.15$

[3 marks]

3e. Write down the range of values for the gradient of f . *[2 marks]*

Markscheme

$f'(x) \geq -4$,
$y \geq -4$,
$[-4, \infty[$ ***A2 N2***

[2 marks]