

Integration Summative [49 marks]

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **Mathematics SL** formula booklet is required for this paper.
- The maximum mark for this examination paper is [49 marks].

A function $f(x)$ has derivative $f'(x) = 3x^2 + 18x$. The graph of f has an x -intercept at $x = -1$.

1a. Find $f(x)$.

[6 marks]

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Markscheme

evidence of integration (M1)

eg $\int f'(x)$

correct integration (accept absence of C) (A1)(A1)

eg $x^3 + \frac{18}{2}x^2 + C$, $x^3 + 9x^2$

attempt to substitute $x = -1$ into **their** $f = 0$ (must have C) M1

eg $(-1)^3 + 9(-1)^2 + C = 0$, $-1 + 9 + C = 0$

Note: Award M0 if they substitute into original or differentiated function.

correct working (A1)

eg $8 + C = 0$, $C = -8$

$f(x) = x^3 + 9x^2 - 8$ A1 N5

[6 marks]

1b. The graph of f has a point of inflexion at $x = p$. Find p .

[4 marks]

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Markscheme

METHOD 1 (using 2nd derivative)

recognizing that $f'' = 0$ (seen anywhere) **M1**

correct expression for f'' **(A1)**

eg $6x + 18, 6p + 18$

correct working **(A1)**

$$6p + 18 = 0$$

$$p = -3 \quad \mathbf{A1\ N3}$$

METHOD 1 (using 1st derivative)

recognizing the vertex of f' is needed **(M2)**

eg $-\frac{b}{2a}$ (must be clear this is for f')

correct substitution **(A1)**

$$\text{eg } \frac{-18}{2 \times 3}$$

$$p = -3 \quad \mathbf{A1\ N3}$$

[4 marks]

Markscheme

valid approach to set up integration by substitution/inspection (M1)

eg $u = x^2 - 1$, $du = 2x$, $\int 2xe^{x^2-1}dx$

correct expression (A1)

eg $\frac{1}{2} \int 2xe^{x^2-1}dx$, $\frac{1}{2} \int e^u du$

$\frac{1}{2}e^{x^2-1} + c$ A2 N4

Notes: Award A1 if missing "+c".

[4 marks]

2b. Find $f(x)$, given that $f'(x) = xe^{x^2-1}$ and $f(-1) = 3$.

[3 marks]

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Markscheme

substituting $x = -1$ into **their** answer from (a) (M1)

eg $\frac{1}{2}e^0$, $\frac{1}{2}e^{1-1} = 3$

correct working (A1)

eg $\frac{1}{2} + c = 3$, $c = 2.5$

$f(x) = \frac{1}{2}e^{x^2-1} + 2.5$ A1 N2

[3 marks]

Let

$$f(x) = \frac{(\ln x)^2}{2}, \text{ for}$$

$$x > 0.$$

3a. Show that $f'(x) = \frac{\ln x}{x}$.

[2 marks]

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Markscheme

METHOD 1

correct use of chain rule **A1A1**

$$\text{eg } \frac{2\ln x}{2} \times \frac{1}{x}, \frac{2\ln x}{2x}$$

Note: Award **A1** for $\frac{2\ln x}{2x}$, **A1** for $\times \frac{1}{x}$.

$$f'(x) = \frac{\ln x}{x} \quad \mathbf{AG} \quad \mathbf{NO}$$

[2 marks]

METHOD 2

correct substitution into quotient rule, with derivatives seen **A1**

$$\text{eg } \frac{2 \times 2\ln x \times \frac{1}{x} - 0 \times (\ln x)^2}{4}$$

correct working **A1**

$$\text{eg } \frac{4\ln x \times \frac{1}{x}}{4}$$

$$f'(x) = \frac{\ln x}{x} \quad \mathbf{AG} \quad \mathbf{NO}$$

[2 marks]

3b. There is a minimum on the graph of f . Find the x -coordinate of this minimum.

[3 marks]

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Markscheme

setting derivative = 0 (M1)

eg $f'(x) = 0, \frac{\ln x}{x} = 0$

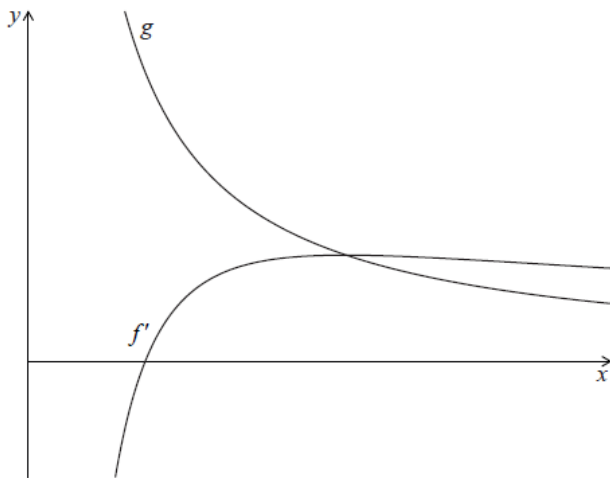
correct working (A1)

eg $\ln x = 0, x = e^0$

$x = 1$ A1 N2

[3 marks]

Let $g(x) = \frac{1}{x}$. The following diagram shows parts of the graphs of f' and g .



The graph of f' has an x-intercept at $x = p$.

3c. Write down the value of p .

[2 marks]

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Markscheme

intercept when $f'(x) = 0$ (M1)

$p = 1$ A1 N2

[2 marks]

3d. The graph of g intersects the graph of f' when $x = q$.

[3 marks]

Find the value of q .

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Markscheme

equating functions (M1)

eg $f' = g, \frac{\ln x}{x} = \frac{1}{x}$

correct working (A1)

eg $\ln x = 1$

$q = e$ (accept $x = e$) A1 N2

[3 marks]

3e. The graph of g intersects the graph of f' when $x = q$.

[5 marks]

Let R be the region enclosed by the graph of f' , the graph of g and the line $x = p$.

Show that the area of R is $\frac{1}{2}$.

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Markscheme

evidence of integrating and subtracting functions (in any order, seen anywhere) **(M1)**

eg $\int_q^e \left(\frac{1}{x} - \frac{\ln x}{x} \right) dx, \int f' - g$

correct integration $\ln x - \frac{(\ln x)^2}{2}$ **A2**

substituting limits into **their** integrated function and subtracting (in any order) **(M1)**

eg $(\ln e - \ln 1) - \left(\frac{(\ln e)^2}{2} - \frac{(\ln 1)^2}{2} \right)$

Note: Do not award **M1** if the integrated function has only one term.

correct working **A1**

eg $(1 - 0) - \left(\frac{1}{2} - 0 \right), 1 - \frac{1}{2}$

area = $\frac{1}{2}$ **AG NO**

Notes: Candidates may work with two separate integrals, and only combine them at the end. Award marks in line with the markscheme.

[5 marks]

Let

$$f(x) = (x - 1)(x - 4).$$

4a. Find the x -intercepts of the graph of f .

[3 marks]

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Markscheme

valid approach (M1)

eg $f(x) = 0$, sketch of parabola showing two x -intercepts

$x = 1, x = 4$ (accept $(1, 0), (4, 0)$) A1A1 N3

[3 marks]

4b. The region enclosed by the graph of f and the x -axis is rotated 360° about the x -axis [3 marks]

Find the volume of the solid formed.

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Markscheme

attempt to substitute either limits or the function into formula involving f^2 (M1)

eg $\int_1^4 (f(x))^2 dx, \pi \int ((x-1)(x-4))^2$

volume = 8.1π (exact), 25.4 A2 N3

[3 marks]

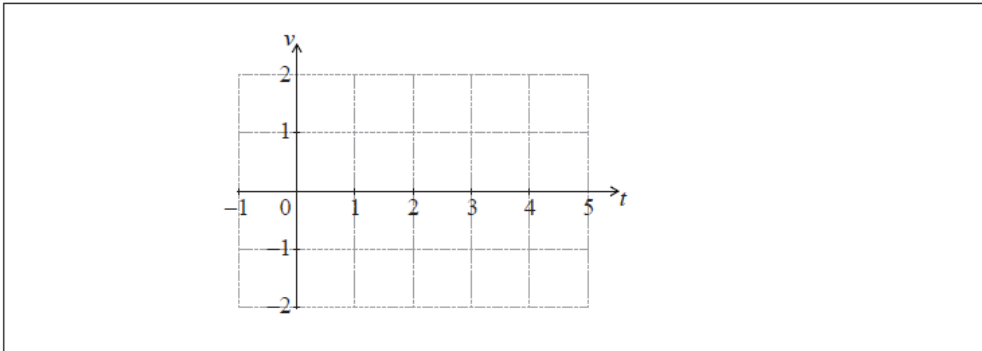
The velocity of a particle in ms^{-1} is given by

$$v = e^{\sin t} - 1, \text{ for}$$

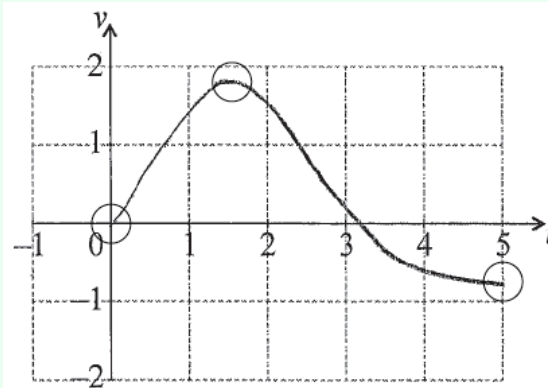
$$0 \leq t \leq 5.$$

5a. On the grid below, sketch the graph of v .

[3 marks]



Markscheme



A1A1A1 N3

Note: Award **A1** for approximately correct shape crossing x -axis with $3 < x < 3.5$.

Only if this **A1** is awarded, award the following:

A1 for maximum in circle, **A1** for endpoints in circle.

[3 marks]

5b. Find the total distance travelled by the particle in the first five seconds.

[1 mark]

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Markscheme

$$t = \pi \text{ (exact), } 3.14 \quad \mathbf{A1} \quad \mathbf{N1}$$

[1 mark]

5c. Write down the positive t -intercept.

[4 marks]

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Markscheme

recognizing distance is area under velocity curve **(M1)**

eg $s = \int v$, shading on diagram, attempt to integrate

valid approach to find the total area **(M1)**

eg area A + area B, $\int v dt - \int v dt$, $\int_0^{3.14} v dt + \int_{3.14}^5 v dt$, $\int |v|$

correct working with integration and limits (accept dx or missing dt) **(A1)**

eg $\int_0^{3.14} v dt + \int_5^{3.14} v dt$, $3.067\dots + 0.878\dots$, $\int_0^5 |e^{\sin t} - 1|$

distance = 3.95 (m) **A1 N3**

[4 marks]