IB Derivatives *[253 marks]*

A function $f(x)$ has derivative $f'(x) = 3x^2 + 18x$. The graph of f has an x-intercept at $x = -1$.

1a. Find $f(x)$. *[6 marks]*

Markscheme

evidence of integration ***(M1)***

eg $\int f'(x)$

correct integration (accept absence of *C*) ***(A1)(A1)***

eg $x^3 + \frac{18}{2}x^2 + C,\ x^3 + 9x^2$

attempt to substitute *x* = −1 into **their** *f* = 0 (must have *C*) ***M1***

eg $(-1)^3 + 9(-1)^2 + C = 0,\ -1 + 9 + C = 0$

Note: Award ***M0*** if they substitute into original or differentiated function.

correct working ***(A1)***

eg $8 + C = 0,\ C = -8$

$f(x) = x^3 + 9x^2 - 8$ ***A1 N5***

[6 marks]

1b. The graph of *f* has a point of inflexion at *x* = *p*. Find *p*. *[4 marks]*

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Markscheme

METHOD 1 (using 2nd derivative)

recognizing that $f'' = 0$ (seen anywhere) ***M1***

correct expression for f'' ***(A1)***

eg $6x + 18$, $6p + 18$

correct working ***(A1)***

$6p + 18 = 0$

$p = -3$ ***A1 N3***

METHOD 1 (using 1st derivative)

recognizing the vertex of f' is needed ***(M2)***

eg $-\frac{b}{2a}$ (must be clear this is for f')

correct substitution ***(A1)***

eg $\frac{-18}{2\times 3}$

$p = -3$ ***A1 N3***

[4 marks]

1c. Find the values of x for which the graph of f is concave-down. *[3 marks]*

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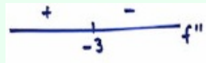
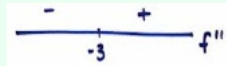
Markscheme

valid attempt to use $f''(x)$ to determine concavity *(M1)*

eg $f''(x) < 0$, $f''(-2)$, $f''(-4)$, $6x + 18 \le 0$ 

correct working *(A1)*

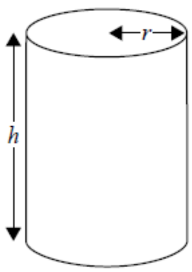
eg $6x + 18 < 0$, $f''(-2) = 6$, $f''(-4) = -6$ 

f concave down for $x < -3$ (do not accept $x \le -3$) ***A1 N2***

[3 marks]

A closed cylindrical can with radius r centimetres and height h centimetres has a volume of 20 π cm^3.

diagram not to scale



2a. Express *h* in terms of *r*. *[2 marks]*

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Markscheme

correct equation for volume *(A1)*
eg $\pi r^2 h = 20\pi$

$h = \frac{20}{r^2}$ ***A1 N2***

[2 marks]

The material for the base and top of the can costs 10 cents per cm ^{2} and the material for the curved side costs 8 cents per cm^2. The total cost of the material, in cents, is C.

2b. Show that $C = 20\pi r^2 + \frac{320\pi}{r}$. *[4 marks]*

Markscheme

attempt to find formula for cost of parts ***(M1)***
eg 10 × two circles, 8 × curved side

correct expression for cost of two circles in terms of r (seen anywhere) ***A1***
eg $2\pi r^2 \times 10$

correct expression for cost of curved side (seen anywhere) ***(A1)***
eg $2\pi r \times h \times 8$

correct expression for cost of curved side in terms of r ***A1***
eg $8 \times 2\pi r \times \frac{20}{r^2}$, $\frac{320\pi}{r^2}$

$C = 20\pi r^2 + \frac{320\pi}{r}$ ***AG N0***

[4 marks]

2c. Given that there is a minimum value for C, find this minimum value in terms of π. *[9 marks]*

Markscheme

recognize $C' = 0$ at minimum *(R1)*
eg $C' = 0,\ \frac{\mathrm{d}C}{\mathrm{d}r} = 0$

correct differentiation (may be seen in equation)

$C' = 40\pi r - \frac{320\pi}{r^2}$ ***A1A1***

correct equation ***A1***
eg $40\pi r - \frac{320\pi}{r^2} = 0,\ 40\pi r\frac{320\pi}{r^2}$

correct working ***(A1)***
eg $40r^3 = 320,\ r^3 = 8$

r = 2 (m) ***A1***

attempt to substitute **their** value of *r* into *C*
eg $20\pi \times 4 + 320 \times \frac{\pi}{2}$ ***(M1)***

correct working
eg $80\pi + 160\pi$ ***(A1)***

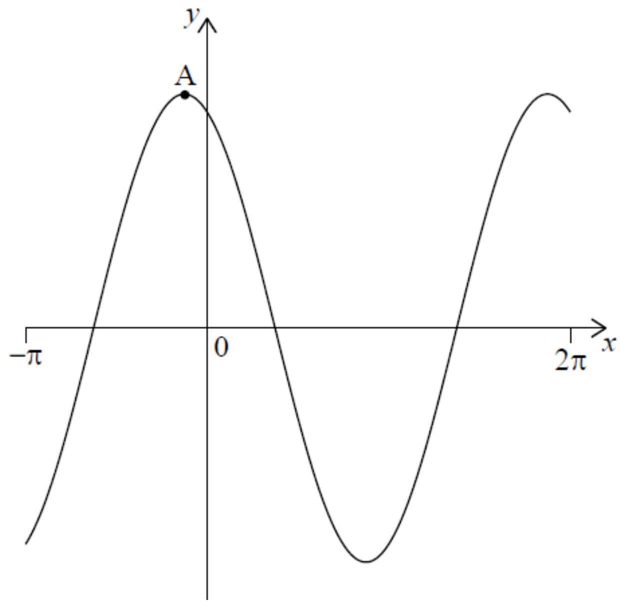
240π (cents) ***A1 N3***

Note: Do not accept 753.6, 753.98 or 754, even if 240 π is seen.

[9 marks]

Let $f(x) = 12\cos x - 5\sin x, \ -\pi \leqslant x \leqslant 2\pi$, be a periodic function with $f(x) = f(x + 2\pi)$

The following diagram shows the graph of f.



There is a maximum point at A. The minimum value of f is −13 .

3a. Find the coordinates of A. *[2 marks]*

Markscheme

−0.394791,13

A(−0.395, 13) ***A1A1 N2***

[2 marks]

3b. For the graph of f, write down the amplitude. *[1 mark]*

Markscheme

13 ***A1 N1***

[1 mark]

3c. For the graph of f, write down the period. *[1 mark]*

Markscheme

2π, 6.28 ***A1 N1***

[1 mark]

3d. Hence, write $f(x)$ in the form $p\cos(x+r)$. *[3 marks]*

Markscheme

valid approach *(M1)*

eg recognizing that amplitude is p or shift is r

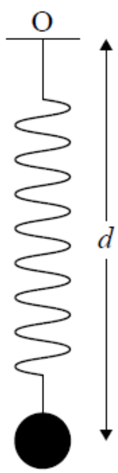
$f\left(x\right) = 13\ \cos\ \left(x + 0.395\right)$ (accept p = 13, r = 0.395) ***A1A1 N3***

Note: Accept any value of r of the form $0.395 + 2\pi k,\ k \in \mathbb{Z}$

[3 marks]

A ball on a spring is attached to a fixed point O. The ball is then pulled down and released, so that it moves back and forth vertically.

diagram not to scale



The distance, d centimetres, of the centre of the ball from O at time t seconds, is given by

$d\left(t\right) = f\left(t\right) + 17,\ 0 \leqslant t \leqslant 5.$

3e. Find the maximum speed of the ball. *[3 marks]*

Markscheme

recognizing need for $d'(t)$ ***(M1)***

eg $-12\sin(t) - 5\cos(t)$

correct approach (accept any variable for t) ***(A1)***

eg $-13\sin(t + 0.395)$, sketch of d', $(1.18, -13)$, $t = 4.32$

maximum speed = 13 (cms^{-1}) ***A1 N2***

[3 marks]

3f. Find the first time when the ball's speed is changing at a rate of 2 cm s^{-2}. *[5 marks]*

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Markscheme

recognizing that acceleration is needed *(M1)*

eg a(t), d''(t)

correct equation (accept any variable for t) *(A1)*

eg $a\left(t\right)=-2,\ \left|\frac{\mathrm{d}}{\mathrm{d}t}\left(d'\left(t\right)\right)\right|=2,\ -12\ \cos\ \left(t\right)+5\ \sin\ \left(t\right)=-2$

valid attempt to solve **their** equation *(M1)*

eg sketch, 1.33

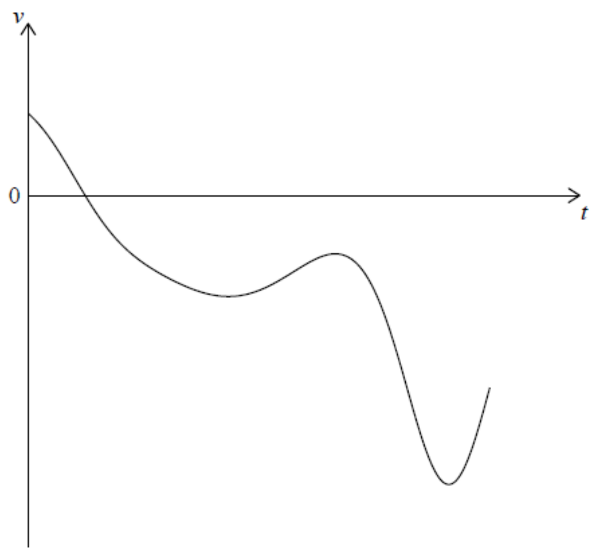
1.02154

1.02 ***A2 N3***

[5 marks]

A particle P moves along a straight line. The velocity v m s^{-1} of P after t seconds is given by $v(t) = 7\cos t - 5t^{\cos t}$, for $0 \le t \le 7$.

The following diagram shows the graph of v.



4a. Find the initial velocity of P. *[2 marks]*

Markscheme

initial velocity when $t = 0$ ***(M1)***

eg $v(0)$

$v = 17$ (m s^{-1}) ***A1 N2***

[2 marks]

4b. Find the maximum speed of P. *[3 marks]*

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Markscheme

recognizing maximum speed when $|v|$ is greatest ***(M1)***

eg minimum, maximum, $v' = 0$

one correct coordinate for minimum ***(A1)***

eg 6.37896, –24.6571

24.7 (ms^{-1}) ***A1 N2***

[3 marks]

4c. Write down the number of times that the acceleration of P is 0 m s $^{-2}$. *[3 marks]*

Markscheme

recognizing a = *v*′ ***(M1)***

eg $a = \frac{\mathrm{d}v}{\mathrm{d}t}$, correct derivative of first term

identifying when a = 0 ***(M1)***

eg turning points of *v*, *t*-intercepts of *v*′

3 ***A1 N3***

[3 marks]

4d. Find the acceleration of P when it changes direction. *[4 marks]*

Markscheme

recognizing P changes direction when $v = 0$ ***(M1)***

$t = 0.863851$ ***(A1)***

-9.24689

$a = -9.25$ (ms^{-2}) ***A2 N3***

[4 marks]

4e. Find the total distance travelled by P. *[3 marks]*

Markscheme

correct substitution of limits or function into formula ***(A1)***

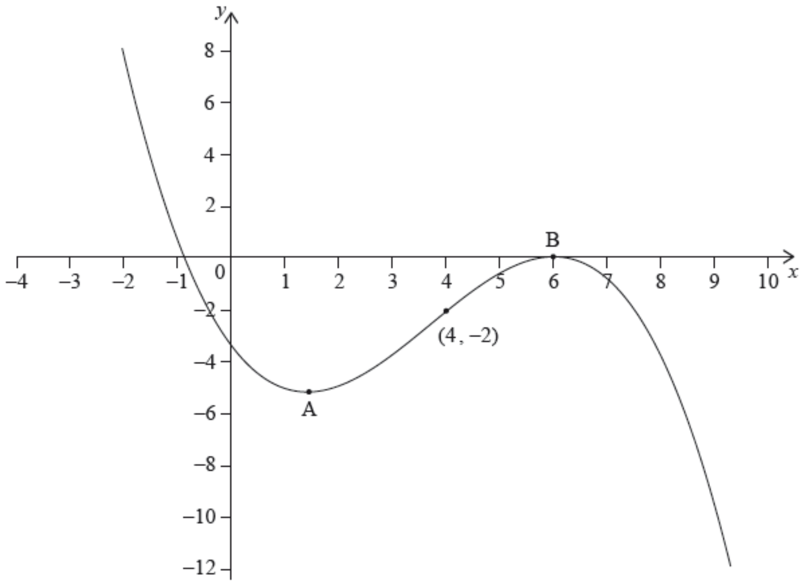
eg $\int_0^7 |v|, \int_0^{0.8638} v\mathrm{d}t - \int_{0.8638}^7 v\mathrm{d}t, \int |7\cos x - 5x^{\cos x}| dx, 3.32 = 60.6$

63.8874

63.9 (metres) ***A2 N3***

[3 marks]

The following diagram shows the graph of f', the derivative of f.



The graph of f' has a local minimum at A, a local maximum at B and passes through $(4,\ -2)$.

The point $\mathrm{P}(4,\ 3)$ lies on the graph of the function, f.

5a. Write down the gradient of the curve of f at P. *[1 mark]*

Markscheme

-2 ***A1*** ***N1***

[1 mark]

5b. Find the equation of the normal to the curve of f at P. *[3 marks]*

Markscheme

gradient of normal $= \frac{1}{2}$ ***(A1)***

attempt to substitute their normal gradient and coordinates of P (in any order) ***(M1)***

eg $y - 4 = \frac{1}{2}(x - 3),\ 3 = \frac{1}{2}(4) + b,\ b = 1$

$y - 3 = \frac{1}{2}(x - 4),\ y = \frac{1}{2}x + 1,\ x - 2y + 2 = 0$ ***A1*** ***N3***

[3 marks]

5c. Determine the concavity of the graph of f when $4 < x < 5$ **and** justify your answer. *[2 marks]*

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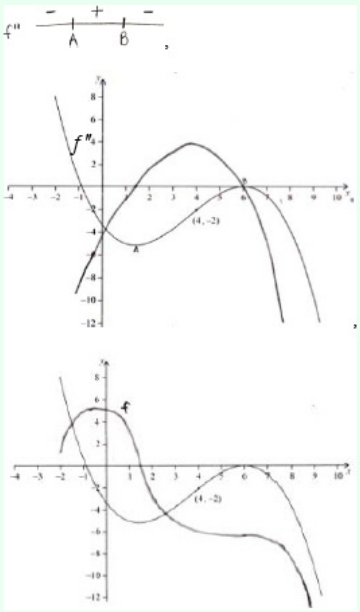
Markscheme

correct answer **and** valid reasoning ***A2*** ***N2***

answer: *eg* graph of f is concave up, concavity is positive (between $4 < x < 5$)

reason: *eg* slope of f' is positive, f' is increasing, $f'' > 0$,

sign chart (must clearly be for f'' and show A and B)



Note: The reason given must refer to a specific function/graph. Referring to “the graph” or “it” is not sufficient.

[2 marks]

Let $f(x) = \cos x$.

6a. (i) Find the first four derivatives of $f(x)$. *[4 marks]*

(ii) Find $f^{(19)}(x)$.

Markscheme

(i)

$f'(x) = -\sin x,\ f''(x) = -\cos x,\ f^{(3)}(x) = \sin x,\ f^{(4)}(x) = \cos x$ ***A2*** ***N2***

(ii) valid approach ***(M1)***

eg recognizing that 19 is one less than a multiple of 4, $f^{(19)}(x) = f^{(3)}(x)$

$f^{(19)}(x) = \sin x$ ***A1*** ***N2***

[4 marks]

Let $g(x) = x^k$, where $k \in \mathbb{Z}^+$.

6b. (i) Find the first three derivatives of $g(x)$. *[5 marks]*

(ii) Given that $g^{(19)}(x) = \frac{k!}{(k-p)!}(x^{k-19})$, find p.

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Markscheme

(i)

$g'(x) = kx^{k-1}$

$g''(x) = k(k-1)x^{k-2},\ g^{(3)}(x) = k(k-1)(k-2)x^{k-3}$ ***A1A1 N2***

(ii) **METHOD 1**

correct working that leads to the correct answer, involving the correct expression for the 19th derivative ***A2***

eg $k(k-1)(k-2)\ldots(k-18) \times \frac{(k-19)!}{(k-19)!},\ {}_kP_{19}$

$p = 19$ (accept $\frac{k!}{(k-19)!}x^{k-19}$) ***A1 N1***

METHOD 2

correct working involving recognizing patterns in coefficients of first three derivatives (may be seen in part (b)(i)) leading to a general rule for 19th coefficient ***A2***

eg $g'' = 2!\begin{pmatrix} k \\ 2 \end{pmatrix},\ k(k-1)(k-2) = \frac{k!}{(k-3)!},\ g^{(3)}(x) = {}_kP_3(x^{k-3})$

$g^{(19)}(x) = 19!\begin{pmatrix} k \\ 19 \end{pmatrix},\ 19! \times \frac{k!}{(k-19)! \times 19!},\ {}_kP_{19}$

$p = 19$ (accept $\frac{k!}{(k-19)!}x^{k-19}$) ***A1 N1***

[5 marks]

Let $k = 21$ and $h(x) = \left(f^{(19)}(x) \times g^{(19)}(x)\right)$.

6c. (i) Find $h'(x)$. *[7 marks]*

(ii) Hence, show that $h'(\pi) = \frac{-21!}{2}\pi^2$.

Markscheme

(i) valid approach using product rule ***(M1)***

eg $uv' + vu',\ f^{(19)}g^{(20)} + f^{(20)}g^{(19)}$

correct 20th derivatives (must be seen in product rule) ***(A1)(A1)***

eg $g^{(20)}(x) = \frac{21!}{(21-20)!}x,\ f^{(20)}(x) = \cos x$

$h'(x) = \sin x(21!x) + \cos x\left(\frac{21!}{2}x^2\right)$ $\left(\text{accept } \sin x\left(\frac{21!}{1!}x\right) + \cos x\left(\frac{21!}{2!}x^2\right)\right)$ ***A1 N3***

(ii) substituting $x = \pi$ (seen anywhere) ***(A1)***

eg $f^{(19)}(\pi)g^{(20)}(\pi) + f^{(20)}(\pi)g^{(19)}(\pi),\ \sin\pi\frac{21!}{1!}\pi + \cos\pi\frac{21!}{2!}\pi^2$

evidence of one correct value for $\sin\pi$ or $\cos\pi$ (seen anywhere) ***(A1)***

eg $\sin\pi = 0,\ \cos\pi = -1$

evidence of correct values substituted into $h'(\pi)$ ***A1***

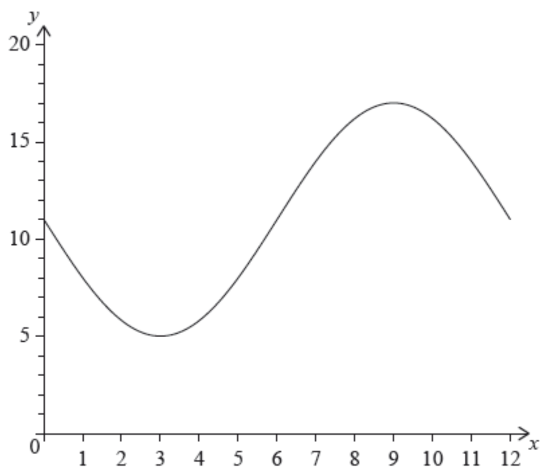
eg $21!(\pi)\left(0 - \frac{\pi}{2!}\right),\ 21!(\pi)\left(-\frac{\pi}{2}\right),\ 0 + (-1)\frac{21!}{2}\pi^2$

Note: If candidates write only the first line followed by the answer, award ***A1A0A0***.

$\frac{-21!}{2}\pi^2$ ***AG N0***

[7 marks]

The following diagram shows the graph of $f(x) = a\sin bx + c$, for $0 \leqslant x \leqslant 12$.



The graph of f has a minimum point at $(3,\ 5)$ and a maximum point at $(9,\ 17)$.

7a. (i) Find the value of c. *[6 marks]*

(ii) Show that $b = \frac{\pi}{6}$.

(iii) Find the value of a.

Markscheme

(i) valid approach ***(M1)***

eg $\frac{5+17}{2}$

$c = 11$ ***A1*** ***N2***

(ii) valid approach ***(M1)***

eg period is 12, per $= \frac{2\pi}{b}$, $9 - 3$

$b = \frac{2\pi}{12}$ ***A1***

$b = \frac{\pi}{6}$ ***AG*** ***N0***

(iii) **METHOD 1**

valid approach ***(M1)***

eg $5 = a\sin\left(\frac{\pi}{6} \times 3\right) + 11$, substitution of points

$a = -6$ ***A1*** ***N2***

METHOD 2

valid approach ***(M1)***

eg $\frac{17-5}{2}$, amplitude is 6

$a = -6$ ***A1*** ***N2***

[6 marks]

The graph of g is obtained from the graph of f by a translation of $\begin{pmatrix} k \\ 0 \end{pmatrix}$. The maximum point on the graph of g has coordinates $(11.5,\ 17)$.

7b. (i) Write down the value of k. *[3 marks]*

(ii) Find $g(x)$.

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Markscheme

(i)

$k = 2.5$ ***A1*** ***N1***

(ii)

$g(x) = -6\sin\left(\frac{\pi}{6}(x - 2.5)\right) + 11$ ***A2*** ***N2***

[3 marks]

The graph of g changes from concave-up to concave-down when $x = w$.

7c. (i) Find w. *[6 marks]*

(ii) Hence or otherwise, find the maximum positive rate of change of g.

Markscheme

(i) **METHOD 1** Using g

recognizing that a point of inflexion is required ***M1***

eg sketch, recognizing change in concavity

evidence of valid approach ***(M1)***

eg $g''(x) = 0$, sketch, coordinates of max/min on g'

$w = 8.5$ (exact) ***A1*** ***N2***

METHOD 2 Using f

recognizing that a point of inflexion is required ***M1***

eg sketch, recognizing change in concavity

evidence of valid approach involving translation ***(M1)***

eg $x = w - k$, sketch, $6 + 2.5$

$w = 8.5$ (exact) ***A1*** ***N2***

(ii) valid approach involving the derivative of g or f (seen anywhere) ***(M1)***

eg $g'(w),\ -\pi\cos\left(\frac{\pi}{6}x\right)$, max on derivative, sketch of derivative

attempt to find max value on derivative ***M1***

eg $-\pi\cos\left(\frac{\pi}{6}(8.5-2.5)\right),\ f'(6)$, dot on max of sketch

3.14159

max rate of change $= \pi$ (exact), 3.14 ***A1*** ***N2***

[6 marks]

Let $f(x) = \sqrt{4x+5}$, for $x \geqslant -1.25$.

8a. Find $f'(1)$. *[4 marks]*

Markscheme

choosing chain rule ***(M1)***

eg $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \times \frac{\mathrm{d}u}{\mathrm{d}x},\ u = 4x+5,\ u' = 4$

correct derivative of f ***A2***

eg $\frac{1}{2}(4x+5)^{-\frac{1}{2}} \times 4,\ f'(x) = \frac{2}{\sqrt{4x+5}}$

$f'(1) = \frac{2}{3}$ ***A1*** ***N2***

[4 marks]

Consider another function g. Let R be a point on the graph of g. The x-coordinate of R is 1. The equation of the tangent to the graph at R is $y = 3x + 6$.

8b. Write down $g'(1)$. *[2 marks]*

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Markscheme

recognize that $g'(x)$ is the gradient of the tangent ***(M1)***

eg $g'(x) = m$

$g'(1) = 3$ ***A1*** ***N2***

[2 marks]

8c. Find $g(1)$. *[2 marks]*

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Markscheme

recognize that R is on the tangent ***(M1)***

eg $g(1) = 3 \times 1 + 6$, sketch

$g(1) = 9$ ***A1*** ***N2***

[2 marks]

8d. Let $h(x) = f(x) \times g(x)$. Find the equation of the tangent to the graph of h at the point where $x = 1$. *[7 marks]*

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Markscheme

$f(1) = \sqrt{4+5}\ (= 3)$ (seen anywhere)   ***A1***

$h(1) = 3 \times 9\ (= 27)$ (seen anywhere)   ***A1***

choosing product rule to find $h'(x)$ ***(M1)***

eg $uv' + u'v$

correct substitution to find $h'(1)$ ***(A1)***

eg $f(1) \times g'(1) + f'(1) \times g(1)$

$h'(1) = 3 \times 3 + \frac{2}{3} \times 9\ (= 15)$ ***A1***

EITHER

attempt to substitute coordinates (in any order) into the equation of a straight line   ***(M1)***

eg $y - 27 = h'(1)(x-1),\ y - 1 = 15(x - 27)$

$y - 27 = 15(x-1)$ ***A1***   ***N2***

OR

attempt to substitute coordinates (in any order) to find the y-intercept   ***(M1)***

eg $27 = 15 \times 1 + b,\ 1 = 15 \times 27 + b$

$y = 15x + 12$ ***A1***   ***N2***

[7 marks]

Let $f'(x) = \frac{6-2x}{6x-x^2}$, for $0 < x < 6$.

The graph of f has a maximum point at P.

9a. Find the x-coordinate of P. *[3 marks]*

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Markscheme

recognizing $f'(x) = 0$ ***(M1)***

correct working ***(A1)***

eg $6 - 2x = 0$

$x = 3$ ***A1*** ***N2***

[3 marks]

The y-coordinate of P is $\ln 27$.

9b. Find $f(x)$, expressing your answer as a single logarithm. *[8 marks]*

Markscheme

evidence of integration ***(M1)***

eg $\int f', \int \frac{6-2x}{6x-x^2}\mathrm{d}x$

using substitution ***(A1)***

eg $\int \frac{1}{u}\mathrm{d}u$ where $u = 6x - x^2$

correct integral ***A1***

eg $\ln(u) + c, \ \ln(6x - x^2)$

substituting $(3, \ \ln 27)$ into **their** integrated expression (must have c) ***(M1)***

eg $\ln(6 \times 3 - 3^2) + c = \ln 27, \ \ln(18 - 9) + \ln k = \ln 27$

correct working ***(A1)***

eg $c = \ln 27 - \ln 9$

EITHER

$c = \ln 3$ ***(A1)***

attempt to substitute **their** value of c into $f(x)$ ***(M1)***

eg $f(x) = \ln(6x - x^2) + \ln 3$ ***A1 N4***

OR

attempt to substitute **their** value of c into $f(x)$ ***(M1)***

eg $f(x) = \ln(6x - x^2) + \ln 27 - \ln 9$

correct use of a log law ***(A1)***

eg $f(x) = \ln(6x - x^2) + \ln\left(\frac{27}{9}\right), \ f(x) = \ln\left(27(6x - x^2)\right) - \ln 9$

$f(x) = \ln\left(3(6x - x^2)\right)$ ***A1 N4***

[8 marks]

9c. The graph of f is transformed by a vertical stretch with scale factor $\frac{1}{\ln 3}$. The image of P under this transformation has coordinates $(a, \ b)$.

Find the value of a and of b, where $a, \ b \in \mathbb{N}$.

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Markscheme

$a = 3$ ***A1*** ***N1***

correct working ***A1***

eg $\frac{\ln 27}{\ln 3}$

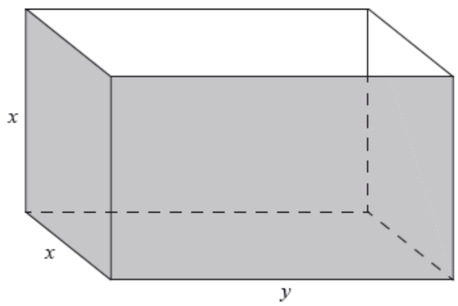
correct use of log law ***(A1)***

eg $\frac{3\ln 3}{\ln 3}$, $\log_3 27$

$b = 3$ ***A1*** ***N2***

[4 marks]

Fred makes an open metal container in the shape of a cuboid, as shown in the following diagram.



The container has height x m, width x m and length y m. The volume is $36\ \mathrm{m}^3$.

Let $A(x)$ be the outside surface area of the container.

10a. Show that $A(x) = \frac{108}{x} + 2x^2$. *[4 marks]*

Markscheme

correct substitution into the formula for volume ***A1***

eg $36 = y \times x \times x$

valid approach to eliminate y (may be seen in formula/substitution) ***M1***

eg $y = \frac{36}{x^2}$, $xy = \frac{36}{x}$

correct expression for surface area ***A1***

eg $xy + xy + xy + x^2 + x^2$, $\text{area} = 3xy + 2x^2$

correct expression in terms of x only ***A1***

eg $3x\left(\frac{36}{x^2}\right) + 2x^2$, $x^2 + x^2 + \frac{36}{x} + \frac{36}{x} + \frac{36}{x}$, $2x^2 + 3\left(\frac{36}{x}\right)$

$A(x) = \frac{108}{x} + 2x^2$ ***AG*** ***N0***

[4 marks]

10b. Find $A'(x)$. *[2 marks]*

....

....

....

Markscheme

$A'(x) = -\frac{108}{x^2} + 4x$, $4x - 108x^{-2}$ ***A1A1*** ***N2***

Note: Award ***A1*** for each term.

[2 marks]

10c. Given that the outside surface area is a minimum, find the height of the container. *[5 marks]*

Markscheme

recognizing that minimum is when $A'(x) = 0$ ***(M1)***

correct equation ***(A1)***

eg $-\frac{108}{x^2} + 4x = 0,\ 4x = \frac{108}{x^2}$

correct simplification ***(A1)***

eg $-108 + 4x^3 = 0,\ 4x^3 = 108$

correct working ***(A1)***

eg $x^3 = 27$

$\text{height} = 3\ (\text{m})\ (\text{accept } x = 3)$ ***A1*** ***N2***

[5 marks]

10d. Fred paints the outside of the container. A tin of paint covers a surface area of $10\ \mathrm{m}^2$ *[5 marks]* and costs $20. Find the total cost of the tins needed to paint the container.

Markscheme

attempt to find area using **their** height ***(M1)***

eg $\frac{108}{3}+2(3)^2,\ 9+9+12+12+12$

minimum surface area $=54\ \mathrm{m}^2$ (may be seen in part (c)) ***A1***

attempt to find the number of tins ***(M1)***

eg $\frac{54}{10},\ 5.4$

6 (tins) ***(A1)***

$120 ***A1*** ***N3***

[5 marks]

Let $f(x)=\frac{1}{x-1}+2$, for $x>1$.

11a. Write down the equation of the horizontal asymptote of the graph of f. *[2 marks]*

Markscheme

$y=2$ (correct equation only) ***A2*** ***N2***

[2 marks]

11b. Find $f'(x)$. *[2 marks]*

Markscheme

valid approach ***(M1)***

eg $(x-1)^{-1}+2,\ f'(x)=\frac{0(x-1)-1}{(x-1)^2}$

$-(x-1)^{-2},\ f'(x)=\frac{-1}{(x-1)^2}$ ***A1 N2***

[2 marks]

Let $g(x)=ae^{-x}+b$, for $x\geqslant 1$. The graphs of f and g have the same horizontal asymptote.

11c. Write down the value of b. *[2 marks]*

Markscheme

correct equation for the asymptote of g

eg $y=b$ ***(A1)***

$b=2$ ***A1 N2***

[2 marks]

11d. Given that $g'(1) = -e$, find the value of a. *[4 marks]*

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Markscheme

correct derivative of *g* (seen anywhere) ***(A2)***

eg $g'(x) = -a\mathrm{e}^{-x}$

correct equation ***(A1)***

eg $-\mathrm{e} = -a\mathrm{e}^{-1}$

7.38905

$a = \mathrm{e}^2\ (\text{exact}),\ 7.39$ ***A1*** ***N2***

[4 marks]

11e. There is a value of x, for $1 < x < 4$, for which the graphs of f and g have the same gradient. Find this gradient. *[4 marks]*

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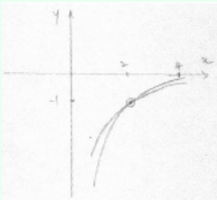
Markscheme

attempt to equate **their** derivatives ***(M1)***

eg $f'(x) = g'(x),\ \frac{-1}{(x-1)^2} = -a\mathrm{e}^{-x}$

valid attempt to solve **their** equation ***(M1)***

eg correct value outside the domain of f such as 0.522 or 4.51,



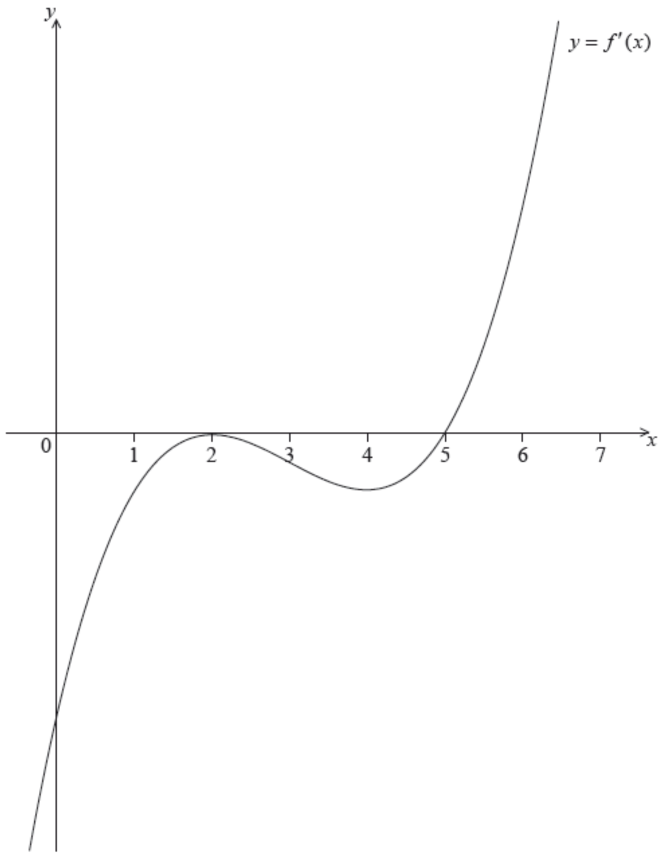
correct solution (may be seen in sketch) ***(A1)***

eg $x = 2,\ (2,\ -1)$

gradient is -1 ***A1*** ***N3***

[4 marks]

Let $y = f(x)$, for $-0.5 \leq$ x ≤ 6.5. The following diagram shows the graph of f', the derivative of f.



The graph of f' has a local maximum when $x = 2$, a local minimum when $x = 4$, and it crosses the
x-axis at the point $(5,\ 0)$.

12a. Explain why the graph of f has a local minimum when $x = 5$. *[2 marks]*

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Markscheme

METHOD 1

$f'(5) = 0$ ***(A1)***

valid reasoning including reference to the graph of f' ***R1***

eg f' changes sign from negative to positive at $x = 5$, labelled sign chart for f'

so f has a local minimum at $x = 5$ ***AG*** ***N0***

Note: It must be clear that any description is referring to the graph of f', simply giving the conditions for a minimum without relating them to f' does not gain the ***R1***.

METHOD 2

$f'(5) = 0$ ***A1***

valid reasoning referring to second derivative ***R1***

eg $f''(5) > 0$

so f has a local minimum at $x = 5$ ***AG*** ***N0***

[2 marks]

12b. Find the set of values of x for which the graph of f is concave down. *[2 marks]*

....

....

....

Markscheme

attempt to find relevant interval ***(M1)***

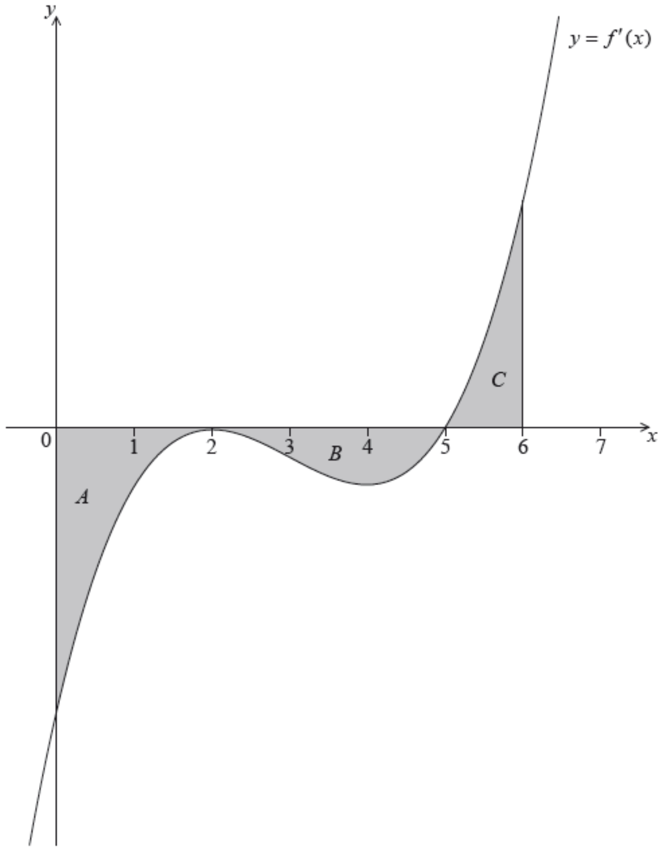
eg f' is decreasing, gradient of f' is negative, $f'' < 0$

$2 < x < 4$ (accept "between 2 and 4") ***A1*** ***N2***

Notes: If no other working shown, award ***M1A0*** for incorrect inequalities such as $2 \leq x \leq 4$, or "from 2 to 4"

[2 marks]

12c. The following diagram shows the shaded regions A, B and C. *[5 marks]*



The regions are enclosed by the graph of f', the x-axis, the y-axis, and the line $x = 6$.

The area of region A is 12, the area of region B is 6.75 and the area of region C is 6.75.

Given that $f(0) = 14$, find $f(6)$.

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Markscheme

METHOD 1 (one integral)

correct application of Fundamental Theorem of Calculus ***(A1)***

eg $\int_0^6 f'(x)\mathrm{d}x = f(6) - f(0),\ f(6) = 14 + \int_0^6 f'(x)\mathrm{d}x$

attempt to link definite integral with areas ***(M1)***

eg $\int_0^6 f'(x)\mathrm{d}x = -12 - 6.75 + 6.75,\ \int_0^6 f'(x)\mathrm{d}x = \text{Area } A + \text{Area } B + \text{ Area } C$

correct value for $\int_0^6 f'(x)\mathrm{d}x$ ***(A1)***

eg $\int_0^6 f'(x)\mathrm{d}x = -12$

correct working ***A1***

eg $f(6) - 14 = -12,\ f(6) = -12 + f(0)$

$f(6) = 2$ ***A1 N3***

METHOD 2 (more than one integral)

correct application of Fundamental Theorem of Calculus ***(A1)***

eg $\int_0^2 f'(x)\mathrm{d}x = f(2) - f(0),\ f(2) = 14 + \int_0^2 f'(x)$

attempt to link definite integrals with areas ***(M1)***

eg $\int_0^2 f'(x)\mathrm{d}x = 12,\ \int_2^5 f'(x)\mathrm{d}x = -6.75,\ \int_0^6 f'(x) = 0$

correct values for integrals ***(A1)***

eg $\int_0^2 f'(x)\mathrm{d}x = -12,\ \int_5^2 f'(x)\mathrm{d}x = 6.75,\ f(6) - f(2) = 0$

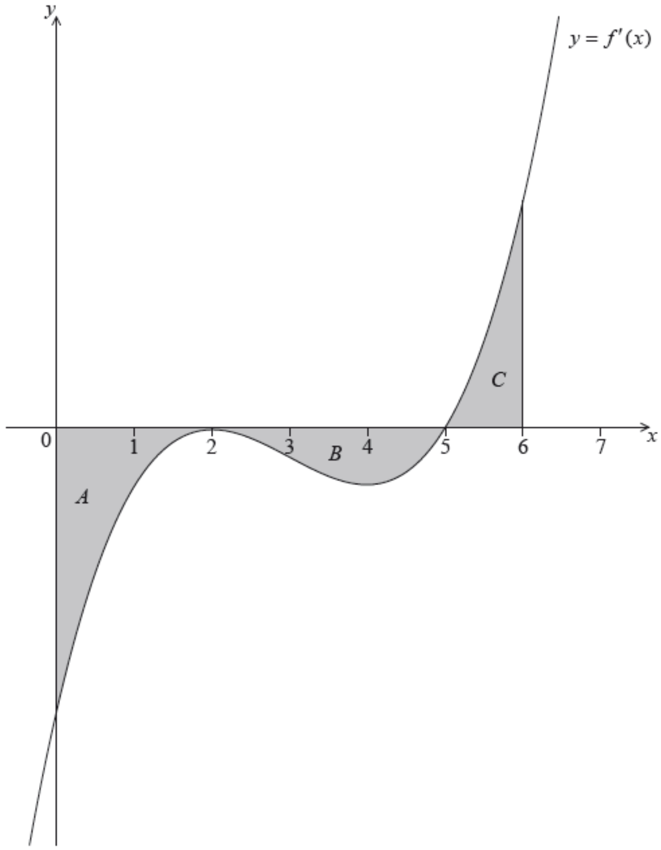
one correct intermediate value ***A1***

eg $f(2) = 2,\ f(5) = -4.75$

$f(6) = 2$ ***A1 N3***

[5 marks]

12d. The following diagram shows the shaded regions A, B and C. *[6 marks]*



The regions are enclosed by the graph of f', the *x*-axis, the *y*-axis, and the line $x = 6$.

The area of region A is 12, the area of region B is 6.75 and the area of region C is 6.75.

Let $g(x) = (f(x))^2$. Given that $f'(6) = 16$, find the equation of the tangent to the graph of g at the point where $x = 6$.

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Markscheme

correct calculation of $g(6)$ (seen anywhere) ***A1***

eg $2^2,\ g(6) = 4$

choosing chain rule or product rule ***(M1)***

eg $g'(f(x))\,f'(x),\ \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \times \frac{\mathrm{d}u}{\mathrm{d}x},\ f(x)f'(x) + f'(x)f(x)$

correct derivative ***(A1)***

eg $g'(x) = 2f(x)f'(x),\ f(x)f'(x) + f'(x)f(x)$

correct calculation of $g'(6)$ (seen anywhere) ***A1***

eg $2(2)(16),\ g'(6) = 64$

attempt to substitute **their** values of $g'(6)$ and $g(6)$ (in any order) into equation of a line ***(M1)***

eg $2^2 = (2 \times 2 \times 16)6 + b,\ y - 6 = 64(x - 4)$

correct equation in any form ***A1*** ***N2***

eg $y - 4 = 64(x - 6),\ y = 64x - 380$

[6 marks]

[Total 15 marks]

A function f has its derivative given by $f'(x) = 3x^2 - 2kx - 9$, where k is a constant.

13a. Find $f''(x)$. *[2 marks]*

....

....

....

Markscheme

$f''(x) = 6x - 2k$ ***A1A1*** ***N2***

[2 marks]

13b. The graph of f has a point of inflexion when $x = 1$. *[3 marks]*

Show that $k = 3$.

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Markscheme

substituting $x = 1$ into f'' ***(M1)***

eg $f''(1),\ 6(1) - 2k$

recognizing $f''(x) = 0$ (seen anywhere) ***M1***

correct equation ***A1***

eg $6 - 2k = 0$

$k = 3$ ***AG*** ***N0***

[3 marks]

13c. Find $f'(-2)$. *[2 marks]*

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Markscheme

correct substitution into $f'(x)$ ***(A1)***

eg $3(-2)^2 - 6(-2) - 9$

$f'(-2) = 15$ ***A1*** ***N2***

[2 marks]

13d. Find the equation of the tangent to the curve of f at $(-2,\ 1)$, giving your answer in the form $y = ax + b$. *[4 marks]*

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Markscheme

recognizing gradient value (may be seen in equation) ***M1***

eg $a = 15,\ y = 15x + b$

attempt to substitute $(-2,\ 1)$ into equation of a straight line ***M1***

eg $1 = 15(-2) + b,\ (y - 1) = m(x + 2),\ (y + 2) = 15(x - 1)$

correct working ***(A1)***

eg $31 = b,\ y = 15x + 30 + 1$

$y = 15x + 31$ ***A1*** ***N2***

[4 marks]

13e. Given that $f'(-1) = 0$, explain why the graph of f has a local maximum when $x = -1$. *[3 marks]*

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Markscheme

METHOD 1 (2^{nd} derivative)

recognizing $f'' < 0$ (seen anywhere) ***R1***

substituting $x = -1$ into f'' ***(M1)***

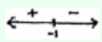
eg $f''(-1),\ 6(-1) - 6$

$f''(-1) = -12$ ***A1***

therefore the graph of f has a local maximum when $x = -1$ ***AG*** ***N0***

METHOD 2 (1^{st} derivative)

recognizing change of sign of $f'(x)$ (seen anywhere) ***R1***

eg *sign chart* 

correct value of f' for $-1 < x < 3$ ***A1***

eg $f'(0) = -9$

correct value of f' for x value to the left of -1 ***A1***

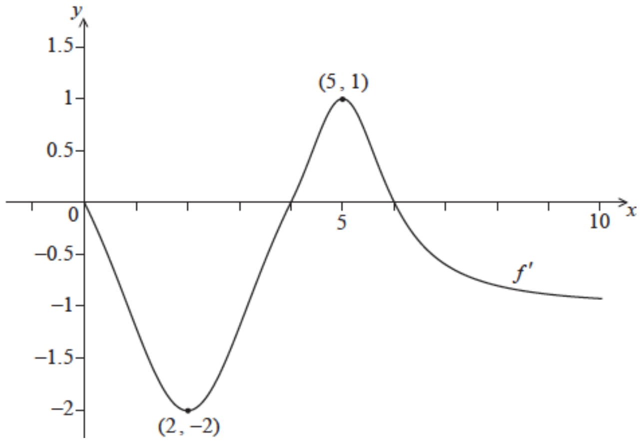
eg $f'(-2) = 15$

therefore the graph of f has a local maximum when $x = -1$ ***AG*** ***N0***

[3 marks]

Total [14 marks]

Consider a function f, for $0 \leq x \leq 10$. The following diagram shows the graph of f', the derivative of f.



The graph of f' passes through $(2, \ -2)$ and $(5, \ 1)$, and has x-intercepts at 0, 4 and 6.

14a. The graph of f has a local maximum point when $x = p$. State the value of p, and justify your answer. *[3 marks]*

Markscheme

$p = 6$ ***A1 N1***

recognizing that turning points occur when $f'(x) = 0$ ***R1 N1***

eg correct sign diagram

f' changes from positive to negative at $x = 6$ ***R1 N1***

[3 marks]

14b. Write down $f'(2)$. *[1 mark]*

Markscheme

$f'(2) = -2$ ***A1*** ***N1***

[1 mark]

14c. Let $g(x) = \ln(f(x))$ and $f(2) = 3$. *[4 marks]*

Find $g'(2)$.

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Markscheme

attempt to apply chain rule ***(M1)***

eg $\ln(x)' \times f'(x)$

correct expression for $g'(x)$ ***(A1)***

eg $g'(x) = \frac{1}{f(x)} \times f'(x)$

substituting $x = 2$ into **their** g' ***(M1)***

eg $\frac{f'(2)}{f(2)}$

-0.666667

$g'(2) = -\frac{2}{3}$ (exact), -0.667 ***A1*** ***N3***

[4 marks]

14d. Verify that $\ln 3 + \int_2^a g'(x)\mathrm{d}x = g(a)$, where $0 \leq a \leq 10$. *[4 marks]*

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Markscheme

evidence of integrating $g'(x)$ ***(M1)***

eg $g(x)|_2^a,\ g(x)|_a^2$

applying the fundamental theorem of calculus (seen anywhere) ***R1***

eg $\int_2^a g'(x) = g(a) - g(2)$

correct substitution into integral ***(A1)***

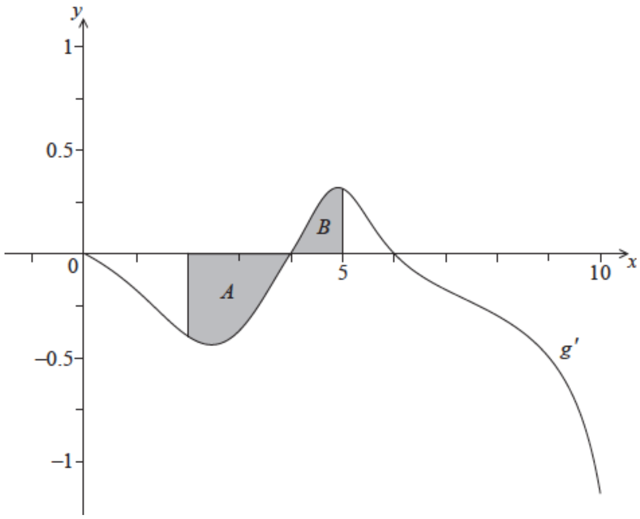
eg $\ln 3 + g(a) - g(2),\ \ln 3 + g(a) - \ln(f(2))$

$\ln 3 + g(a) - \ln 3$ ***A1***

$\ln 3 + \int_2^a g'(x) = g(a)$ ***AG*** ***N0***

[4 marks]

14e. The following diagram shows the graph of g', the derivative of g. *[4 marks]*



The shaded region A is enclosed by the curve, the x-axis and the line $x = 2$, and has area $0.66\ \text{units}^2$.

The shaded region B is enclosed by the curve, the x-axis and the line $x = 5$, and has area $0.21\ \text{units}^2$.

Find $g(5)$.

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Markscheme

METHOD 1

substituting $a = 5$ into the formula for $g(a)$ ***(M1)***

eg $\int_2^5 g'(x)\mathrm{d}x,\ g(5) = \ln 3 + \int_2^5 g'(x)\mathrm{d}x \quad (\text{do not accept only } g(5))$

attempt to substitute areas ***(M1)***

eg $\ln 3 + 0.66 - 0.21,\ \ln 3 + 0.66 + 0.21$

correct working

eg $g(5) = \ln 3 + (-0.66 + 0.21)$ ***(A1)***

0.648612

$g(5) = \ln 3 - 0.45\ (\text{exact}), 0.649$ ***A1 N3***

METHOD 2

attempt to set up an equation for one shaded region ***(M1)***

eg $\int_4^5 g'(x)\mathrm{d}x = 0.21,\ \int_2^4 g'(x)\mathrm{d}x = -0.66,\ \int_2^5 g'(x)\mathrm{d}x = -0.45$

two correct equations ***(A1)***

eg $g(5) - g(4) = 0.21,\ g(2) - g(4) = 0.66$

combining equations to eliminate $g(4)$ ***(M1)***

eg $g(5) - [\ln 3 - 0.66] = 0.21$

0.648612

$g(5) = \ln 3 - 0.45\ (\text{exact}), 0.649$ ***A1 N3***

METHOD 3

attempt to set up a definite integral ***(M1)***

eg $\int_2^5 g'(x)\mathrm{d}x = -0.66 + 0.21,\ \int_2^5 g'(x)\mathrm{d}x = -0.45$

correct working ***(A1)***

eg $g(5) - g(2) = -0.45$

correct substitution ***(A1)***

eg $g(5) - \ln 3 = -0.45$

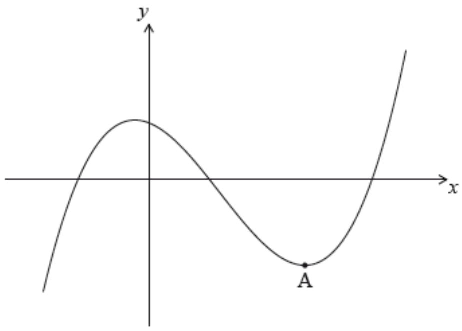
0.648612

$g(5) = \ln 3 - 0.45\ (\text{exact}), 0.649$ ***A1 N3***

[4 marks]

Total [16 marks]

The following diagram shows the graph of a function f. There is a local minimum point at A, where $x > 0$.



The derivative of f is given by $f'(x) = 3x^2 - 8x - 3$.

15a. Find the x-coordinate of A. *[5 marks]*

....

....

....

Markscheme

recognizing that the local minimum occurs when $f'(x) = 0$ ***(M1)***

valid attempt to solve $3x^2 - 8x - 3 = 0$ ***(M1)***

eg factorization, formula

correct working ***A1***

$(3x + 1)(x - 3),\ x = \frac{8 \pm \sqrt{64+36}}{6}$

$x = 3$ ***A2*** ***N3***

Note: Award ***A1*** if both values $x = \frac{-1}{3},\ x = 3$ are given.

[5 marks]

15b. The y-intercept of the graph is at $(0, 6)$. Find an expression for $f(x)$. *[6 marks]*

The graph of a function g is obtained by reflecting the graph of f in the y-axis, followed by a translation of $\begin{pmatrix} m \\ n \end{pmatrix}$.

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Markscheme

valid approach ***(M1)***

$f(x) = \int f'(x)\mathrm{d}x$

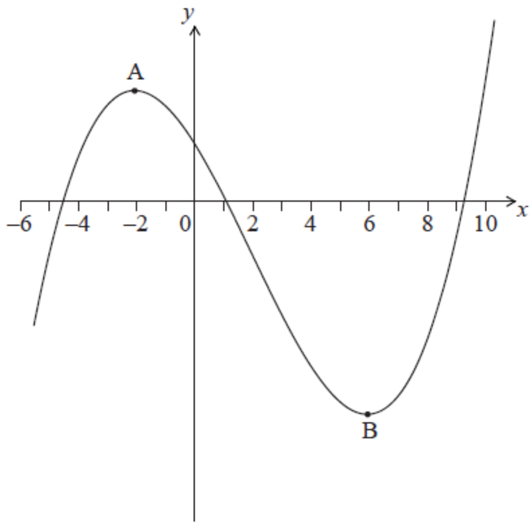
$f(x) = x^3 - 4x^2 - 3x + c$ (do not penalize for missing "$+c$") ***A1A1A1***

$c = 6$ ***(A1)***

$f(x) = x^3 - 4x^2 - 3x + 6$ ***A1*** ***N6***

[6 marks]

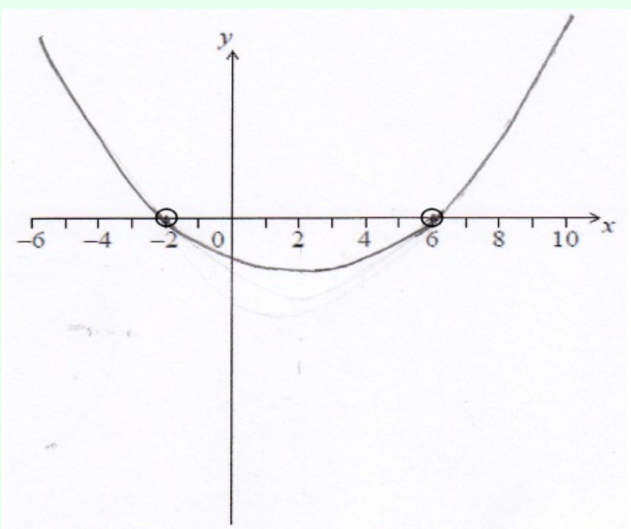
The following diagram shows part of the graph of $y = f(x)$.



The graph has a local maximum at A, where $x = -2$, and a local minimum at B, where $x = 6$.

16a. On the following axes, sketch the graph of $y = f'(x)$. *[4 marks]*

Markscheme



A1A1A1A1 ***N4***

Note: Award ***A1*** for *x*-intercept in circle at -2, ***A1*** for *x*-intercept in circle at 6.

Award ***A1*** for approximately correct shape.

Only if this ***A1*** is awarded, award ***A1*** for a negative *y*-intercept.

[4 marks]

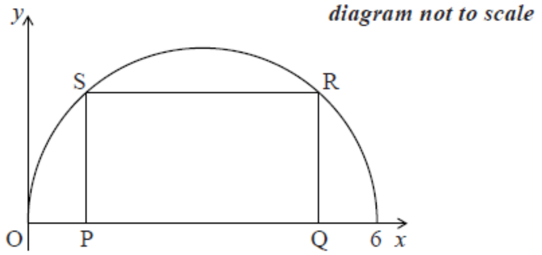
16b. Write down the following in order from least to greatest: $f(0),\ f'(6),\ f''(-2)$. *[2 marks]*

Markscheme

$f''(-2),\ f'(6),\ f(0)$ ***A2*** ***N2***

[2 marks]

Consider the graph of the semicircle given by $f(x)=\sqrt{6x-x^2}$, for $0\leqslant x\leqslant 6$. A rectangle PQRS is drawn with upper vertices R and S on the graph of f, and PQ on the x-axis, as shown in the following diagram.



17a. Let $\mathrm{OP}=x$.

(i) Find PQ, giving your answer in terms of x.

(ii) Hence, write down an expression for the area of the rectangle, giving your answer in terms of x.

Markscheme

(i) valid approach (may be seen on diagram) ***(M1)***

eg Q to 6 is x

$\mathrm{PQ}=6-2x$ ***A1*** ***N2***

(ii) $A=(6-2x)\sqrt{6x-x^2}$ ***A1*** ***N1***

[3 marks]

17b. Find the rate of change of area when $x=2$. *[2 marks]*

Markscheme

recognising $\frac{\mathrm{d}A}{\mathrm{d}x}$ at $x = 2$ needed (must be the derivative of area) ***(M1)***

$\frac{\mathrm{d}A}{\mathrm{d}x} = -\frac{7\sqrt{2}}{2}, \ -4.95$ ***A1*** ***N2***

[2 marks]

17c. The area is decreasing for $a < x < b$. Find the value of a and of b. *[2 marks]*

Markscheme

$a = 0.879\ b = 3$ ***A1A1*** ***N2***

[4 marks]

Let
$f(x) = \frac{6x}{x+1}$, for
$x > 0$.

18a. Find $f'(x)$. *[5 marks]*

Markscheme

METHOD 1

evidence of choosing quotient rule ***(M1)***

e.g. $\frac{u'v-uv'}{v^2}$

evidence of correct differentiation (must be seen in quotient rule) ***(A1)(A1)***

e.g. $\frac{\mathrm{d}}{\mathrm{d}x}(6x)=6$, $\frac{\mathrm{d}}{\mathrm{d}x}(x+1)=1$

correct substitution into quotient rule ***A1***

e.g. $\frac{(x+1)6-6x}{(x+1)^2}$, $\frac{6x+6-6x}{(x+1)^2}$

$f'(x)=\frac{6}{(x+1)^2}$ ***A1 N4***

[5 marks]

METHOD 2

evidence of choosing product rule ***(M1)***

e.g. $6x(x+1)^{-1}$, $uv'+vu'$

evidence of correct differentiation (must be seen in product rule) ***(A1)(A1)***

e.g. $\frac{\mathrm{d}}{\mathrm{d}x}(6x)=6$, $\frac{\mathrm{d}}{\mathrm{d}x}(x+1)^{-1}=-1(x+1)^{-2}\times 1$

correct working ***A1***

e.g. $6x\times -(x+1)^{-2}+(x+1)^{-1}\times 6$, $\frac{-6x+6(x+1)}{(x+1)^2}$

$f'(x)=\frac{6}{(x+1)^2}$ ***A1 N4***

[5 marks]

18b. Let $g(x)=\ln\left(\frac{6x}{x+1}\right)$, for $x>0$. *[4 marks]*

Show that $g'(x)=\frac{1}{x(x+1)}$.

Markscheme

METHOD 1

evidence of choosing chain rule ***(M1)***

e.g. formula, $\frac{1}{\left(\frac{6x}{x+1}\right)} \times \left(\frac{6x}{x+1}\right)$

correct reciprocal of $\frac{1}{\left(\frac{6x}{x+1}\right)}$ is $\frac{x+1}{6x}$ (seen anywhere) ***A1***

correct substitution into chain rule ***A1***

e.g. $\frac{1}{\left(\frac{6x}{x+1}\right)} \times \frac{6}{(x+1)^2}$, $\left(\frac{6}{(x+1)^2}\right)\left(\frac{x+1}{6x}\right)$

working that clearly leads to the answer ***A1***

e.g. $\left(\frac{6}{(x+1)}\right)\left(\frac{1}{6x}\right)$, $\left(\frac{1}{(x+1)^2}\right)\left(\frac{x+1}{x}\right)$, $\frac{6(x+1)}{6x(x+1)^2}$

$g'(x) = \frac{1}{x(x+1)}$ ***AG N0***

[4 marks]

METHOD 2

attempt to subtract logs ***(M1)***

e.g. $\ln a - \ln b$, $\ln 6x - \ln(x+1)$

correct derivatives (must be seen in correct expression) ***A1A1***

e.g. $\frac{6}{6x} - \frac{1}{x+1}$, $\frac{1}{x} - \frac{1}{x+1}$

working that clearly leads to the answer ***A1***

e.g. $\frac{x+1-x}{x(x+1)}$, $\frac{6x+6-6x}{6x(x+1)}$, $\frac{6(x+1-x)}{6x(x+1)}$

$g'(x) = \frac{1}{x(x+1)}$ ***AG N0***

[4 marks]

18c. Let $h(x) = \frac{1}{x(x+1)}$. The area enclosed by the graph of h, the x-axis and the lines $x = \frac{1}{5}$ and $x = k$ is $\ln 4$. Given that $k > \frac{1}{5}$, find the value of k . *[7 marks]*

Markscheme

valid method using integral of *h*(*x*) (accept missing/incorrect limits or missing $\mathrm{d}x$) ***(M1)***

e.g. $\text{area} = \int_{\frac{1}{5}}^{k} h(x)\mathrm{d}x$, $\int \left(\frac{1}{x(x+1)}\right)$

recognizing that integral of derivative will give original function ***(R1)***

e.g. $\int \left(\frac{1}{x(x+1)}\right)\mathrm{d}x = \ln\left(\frac{6x}{x+1}\right)$

correct substitution and subtraction ***A1***

e.g. $\ln\left(\frac{6k}{k+1}\right) - \ln\left(\frac{6\times\frac{1}{5}}{\frac{1}{5}+1}\right)$, $\ln\left(\frac{6k}{k+1}\right) - \ln(1)$

setting **their** expression equal to $\ln 4$ ***(M1)***

e.g. $\ln\left(\frac{6k}{k+1}\right) - \ln(1) = \ln 4$, $\ln\left(\frac{6k}{k+1}\right) = \ln 4$, $\int_{\frac{1}{5}}^{k} h(x)\mathrm{d}x = \ln 4$

correct equation without logs ***A1***

e.g.$\frac{6k}{k+1} = 4$, $6k = 4(k+1)$

correct working ***(A1)***

e.g. $6k = 4k + 4$, $2k = 4$

$k = 2$ ***A1*** ***N4***

[7 marks]

Let
$f(x) = \cos(\mathrm{e}^x)$, for
$-2 \le x \le 2$.

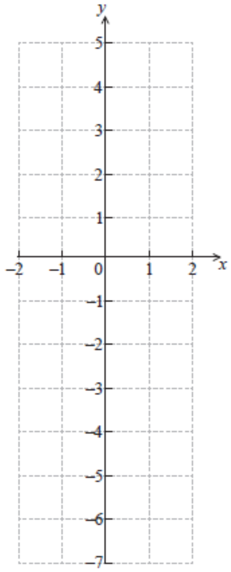
19a. Find $f'(x)$. *[2 marks]*

Markscheme

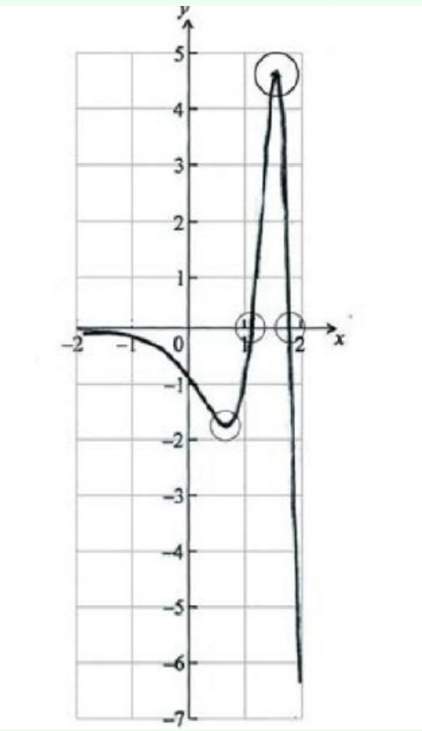
$f'(x) = -\mathrm{e}^x \sin(\mathrm{e}^x)$ ***A1A1*** ***N2***

[2 marks]

19b. On the grid below, sketch the graph of $f'(x)$. *[4 marks]*



Markscheme

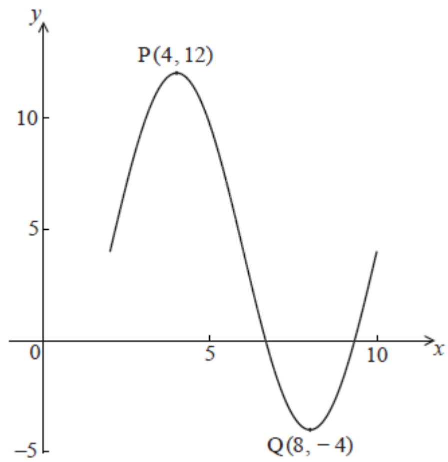


A1A1A1A1 ***N4***

Note: Award ***A1*** for shape that must have the correct domain (from -2 to $+2$) and correct range (from -6 to 4), ***A1*** for minimum in circle, ***A1*** for maximum in circle and ***A1*** for intercepts in circles.

[4 marks]

The following diagram shows the graph of
$f(x) = a\sin(b(x - c)) + d$, for
$2 \le x \le 10$.



There is a maximum point at P(4, 12) and a minimum point at Q(8, −4) .

20a. Use the graph to write down the value of *[3 marks]*

(i) a;

(ii) c;

(iii) d.

Markscheme

(i) $a = 8$ ***A1 N1***

(ii) $c = 2$ ***A1 N1***

(iii) $d = 4$ ***A1 N1***

[3 marks]

20b. Show that $b = \frac{\pi}{4}$. *[2 marks]*

Markscheme

METHOD 1

recognizing that period $= 8$ ***(A1)***

correct working ***A1***

e.g. $8 = \frac{2\pi}{b}$, $b = \frac{2\pi}{8}$

$b = \frac{\pi}{4}$ ***AG*** ***N0***

METHOD 2

attempt to substitute ***M1***

e.g. $12 = 8\sin(b(4-2)) + 4$

correct working ***A1***

e.g. $\sin 2b = 1$

$b = \frac{\pi}{4}$ ***AG*** ***N0***

[2 marks]

20c. Find $f'(x)$. *[3 marks]*

Markscheme

evidence of attempt to differentiate or choosing chain rule ***(M1)***

e.g. $\cos\frac{\pi}{4}(x-2)$, $\frac{\pi}{4} \times 8$

$f'(x) = 2\pi\cos\left(\frac{\pi}{4}(x-2)\right)$ (accept $2\pi\cos\frac{\pi}{4}(x-2)$) ***A2*** ***N3***

[3 marks]

20d. At a point R, the gradient is -2π . Find the x-coordinate of R. *[6 marks]*

Markscheme

recognizing that gradient is $f'(x)$ ***(M1)***

e.g. $f'(x) = m$

correct equation ***A1***

e.g. $-2\pi = 2\pi\cos\left(\frac{\pi}{4}(x-2)\right)$, $-1 = \cos\left(\frac{\pi}{4}(x-2)\right)$

correct working ***(A1)***

e.g. $\cos^{-1}(-1) = \frac{\pi}{4}(x-2)$

using $\cos^{-1}(-1) = \pi$ (seen anywhere) ***(A1)***

e.g. $\pi = \frac{\pi}{4}(x-2)$

simplifying ***(A1)***

e.g. $4 = (x-2)$

$x = 6$ ***A1*** ***N4***

[6 marks]