Gravity, Mechanics, Circular motion

[76 marks]

A planet has radius R. At a distance h above the surface of the planet the gravitational field strength is g and the gravitational potential is V.

1a. State what is meant by gravitational field strength. *[1 mark]*

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Markscheme

the «gravitational» force per unit mass exerted on a point/small/test mass

[1 mark]

1b. Show that $V = -g(R + h)$. *[2 marks]*

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Markscheme

at height h potential is $V = -\frac{GM}{(R+h)}$

field is $g = \frac{GM}{(R+h)^2}$

«dividing gives answer»

Do not allow an answer that starts with $g = -\frac{\Delta V}{\Delta r}$ and then cancels the deltas and substitutes $R + h$

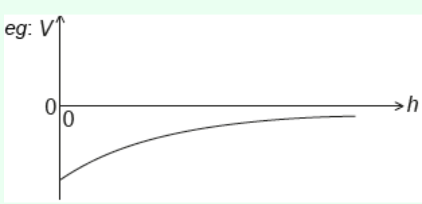
[2 marks]

1c. Draw a graph, on the axes, to show the variation of the gravitational potential V of the planet with height h above the surface of the planet. *[2 marks]*

Markscheme

correct shape and sign

non-zero negative vertical intercept



[2 marks]

1d. A planet has a radius of 3.1×10^6 m. At a point P a distance 2.4×10^7 m above the surface of the planet the gravitational field strength is 2.2 N kg^{-1}. Calculate the gravitational potential at point P, include an appropriate unit for your answer. *[1 mark]*

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Markscheme

V = «–2.2 × ($3.1 \times 10^6 + 2.4 \times 10^7$) =» «–» 6.0×10^7 J kg^{-1}

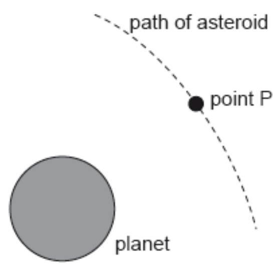
Unit is essential

Allow eg MJ kg^{-1} if power of 10 is correct

Allow other correct SI units eg m^2s^{-2}, N m kg^{-1}

[1 mark]

1e. The diagram shows the path of an asteroid as it moves past the planet. *[3 marks]*



When the asteroid was far away from the planet it had negligible speed. Estimate the speed of the asteroid at point P as defined in (b).

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Markscheme

total energy at P = 0 / KE gained = GPE lost

«

$\frac{1}{2}mv^2 + mV = 0 \Rightarrow$» $v = \sqrt{-2V}$

v = «

$\sqrt{2 \times 6.0 \times 10^7}$ =» 1.1×10^4 «ms^{-1}»

*Award **[3]** for a bald correct answer*

Ignore negative sign errors in the workings

Allow ECF from 6(b)

[3 marks]

1f. The mass of the asteroid is 6.2×10^{12} kg. Calculate the gravitational force experienced by the **planet** when the asteroid is at point P. *[2 marks]*

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Markscheme

ALTERNATIVE 1

force on asteroid is «$6.2 \times 10^{12} \times 2.2$ =» 1.4×10^{13} «N»

«by Newton’s third law» this is also the force on the planet

ALTERNATIVE 2

mass of planet = 2.4×10^{25} «kg» «from $V = -\frac{GM}{(R+h)}$»

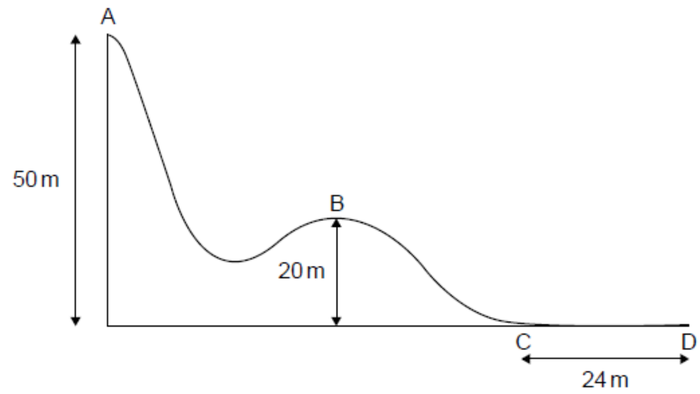
force on planet «

$\frac{GMm}{(R+h)^2}$» = 1.4×10^{13} «N»

MP2 must be explicit

[2 marks]

The diagram below shows part of a downhill ski course which starts at point A, 50 m above level ground. Point B is 20 m above level ground.



A skier of mass 65 kg starts from rest at point A and during the ski course some of the gravitational potential energy transferred to kinetic energy.

2a. From A to B, 24 % of the gravitational potential energy transferred to kinetic energy. Show that the velocity at B is 12 m s^{-1}. *[2 marks]*

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Markscheme

$\frac{1}{2}v^2 = 0.24\,\text{gh}$

$v = 11.9$ «m s^{-1}»

Award GPE lost = 65 × 9.81 × 30 = «19130 J»

Must see the 11.9 value for MP2, not simply 12.

Allow g = 9.8 ms^{-2}.

2b. Some of the gravitational potential energy transferred into internal energy of the skis, slightly increasing their temperature. Distinguish between internal energy and temperature. *[2 marks]*

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Markscheme

internal energy is the total KE «and PE» of the molecules/particles/atoms in an object

temperature is a measure of the average KE of the molecules/particles/atoms

*Award **[1 max]** if there is no mention of molecules/particles/atoms.*

2c. The dot on the following diagram represents the skier as she passes point B. Draw and label the vertical forces acting on the skier. *[2 marks]*

Markscheme

arrow vertically downwards from dot labelled weight/W/mg/gravitational force/F_g/$F_{gravitational}$ ***AND*** arrow vertically upwards from dot labelled reaction force/R/normal contact force/N/F_N

W > R

Do not allow gravity.
Do not award MP1 if additional 'centripetal' force arrow is added.
Arrows must connect to dot.
Ignore any horizontal arrow labelled friction.
Judge by eye for MP2. Arrows do not have to be correctly labelled or connect to dot for MP2.

2d. The hill at point B has a circular shape with a radius of 20 m. Determine whether the skier will lose contact with the ground at point B. *[3 marks]*

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Markscheme

ALTERNATIVE 1

recognition that centripetal force is required / $\frac{mv^2}{r}$ seen

= 468 «N»

W/640 N (weight) is larger than the centripetal force required, so the skier does not lose contact with the ground

ALTERNATIVE 2

recognition that centripetal acceleration is required / $\frac{v^2}{r}$ seen

a = 7.2 «ms^{-2}»

g is larger than the centripetal acceleration required, so the skier does not lose contact with the ground

ALTERNATIVE 3

recognition that to lose contact with the ground centripetal force ≥ weight

calculation that v ≥ 14 «ms^{-1}»

comment that 12 «ms^{-1}» is less than 14 «ms^{-1}» so the skier does not lose contact with the ground

ALTERNATIVE 4

recognition that centripetal force is required / $\frac{mv^2}{r}$ seen

calculation that reaction force = 172 «N»

reaction force > 0 so the skier does not lose contact with the ground

Do not award a mark for the bald statement that the skier does not lose contact with the ground.

2e. The skier reaches point C with a speed of 8.2 m s $^{-1}$. She stops after a distance of 24 m at point D. *[3 marks]*

Determine the coefficient of dynamic friction between the base of the skis and the snow. Assume that the frictional force is constant and that air resistance can be neglected.

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Markscheme

ALTERNATIVE 1

$0 = 8.2^2 + 2 \times a \times 24$ therefore $a =$ «–»1.40 «m s $^{-2}$»

friction force = $ma = 65 \times 1.4 = 91$ «N»

coefficient of friction = $\frac{91}{65 \times 9.81} = 0.14$

ALTERNATIVE 2

$KE = \frac{1}{2}mv^2 = 0.5 \times 65 \times 8.2^2 = 2185$ «J»

friction force = KE/distance = 2185/24 = 91 «N»

coefficient of friction = $\frac{91}{65 \times 9.81} = 0.14$

Allow ECF from MP1.

At the side of the course flexible safety nets are used. Another skier of mass 76 kg falls normally into the safety net with speed 9.6 m s $^{-1}$.

2f. Calculate the impulse required from the net to stop the skier and state an appropriate unit for your answer. *[2 marks]*

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Markscheme

«76 × 9.6»= 730
Ns ***OR*** kg ms^{-1}

2g. Explain, with reference to change in momentum, why a flexible safety net is less likely to harm the skier than a rigid barrier. *[2 marks]*

Markscheme

safety net extends stopping time

$F = \frac{\Delta p}{\Delta t}$ therefore F is smaller «with safety net»

OR

force is proportional to rate of change of momentum therefore F is smaller «with safety net»

Accept reverse argument.

3a. (i) Define *gravitational field strength.* *[2 marks]*

(ii) State the SI unit for gravitational field strength.

Markscheme

(i) «gravitational» force per unit mass on a «small **or** test» mass

(ii) N kg^{-1}

Award mark if N kg^{-1} is seen, treating any further work as neutral.
Do not accept bald m s^{-2}

3b. A planet orbits the Sun in a circular orbit with orbital period *T* and orbital radius *R*. The mass of the Sun is *M*. *[4 marks]*

(i) Show that $T = \sqrt{\frac{4\pi^2 R^3}{GM}}$.

(ii) The Earth's orbit around the Sun is almost circular with radius 1.5×10^{11} m. Estimate the mass of the Sun.

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Markscheme

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clear evidence that *v* in $v^2 = \frac{4\pi^2 R^2}{T^2}$ is equated to orbital speed $\sqrt{\frac{GM}{R}}$

OR

clear evidence that centripetal force is equated to gravitational force

OR

clear evidence that *a* in $a = \frac{v^2}{R}$ etc is equated to *g* in $g = \frac{GM}{R^2}$ with consistent use of symbols

Minimum is a statement that

$\sqrt{\frac{GM}{R}}$ *is the orbital speed which is then used in*

$v = \frac{2\pi R}{T}$

Minimum is $F_c = F_g$ *ignore any signs.*

Minimum is g = a.

substitutes and re-arranges to obtain result

Allow any legitimate method not identified here.

Do not allow spurious methods involving equations of shm etc

$$\ll T = \sqrt{\frac{4\pi^2 R}{\left(\frac{GM}{R^2}\right)}} = \sqrt{\frac{4\pi^2 R^3}{GM}} \gg$$

ii

«*T* = 365 × 24 × 60 × 60 = 3.15 × 10 ^{7} s»

$$M = \ll \frac{4\pi^2 R^3}{GT^2} = \gg = \frac{4 \times 3.14^2 \times \left(1.5 \times 10^{11}\right)^3}{6.67 \times 10^{-11} \times \left(3.15 \times 10^7\right)^2}$$

2×10^{30}«kg»

Allow use of 3.16 x 10^7 s for year length (quoted elsewhere in paper).

Condone error in power of ten in MP1.

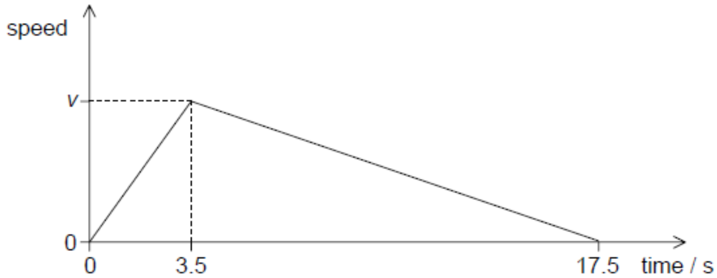
Award ***[1 max]*** *if incorrect time used (24 h is sometimes seen, leading to 2.66 x 10 ^{35} kg).*

Units are not required, but if not given assume kg and mark POT accordingly if power wrong.

Award ***[2]*** *for a bald correct answer.*

No sf penalty here.

Curling is a game played on a horizontal ice surface. A player pushes a large smooth stone across the ice for several seconds and then releases it. The stone moves until friction brings it to rest. The graph shows the variation of speed of the stone with time.



The total distance travelled by the stone in 17.5 s is 29.8 m.

4a. Determine the coefficient of dynamic friction between the stone and the ice during the last 14.0 s of the stone's motion. *[3 marks]*

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Markscheme

ALTERNATIVE 1

«deceleration» $= \frac{3.41}{14.0}$ «$= 0.243\,\mathrm{m\,s^{-2}}$»

F = 0.243 × *m*

$\mu = \frac{0.243 \times m}{m \times 9.81} = 0.025$

ALTERNATIVE 2

distance travelled after release = 23.85 «m»
KE lost = 5.81 *m* «J»

$\mu_\mathrm{d} = \frac{\text{KE lost}}{mg \times \text{distance}} = \frac{5.81m}{23.85mg} = 0.025$

*Award **[3]** for a bald correct answer.*

Ignore sign in acceleration.

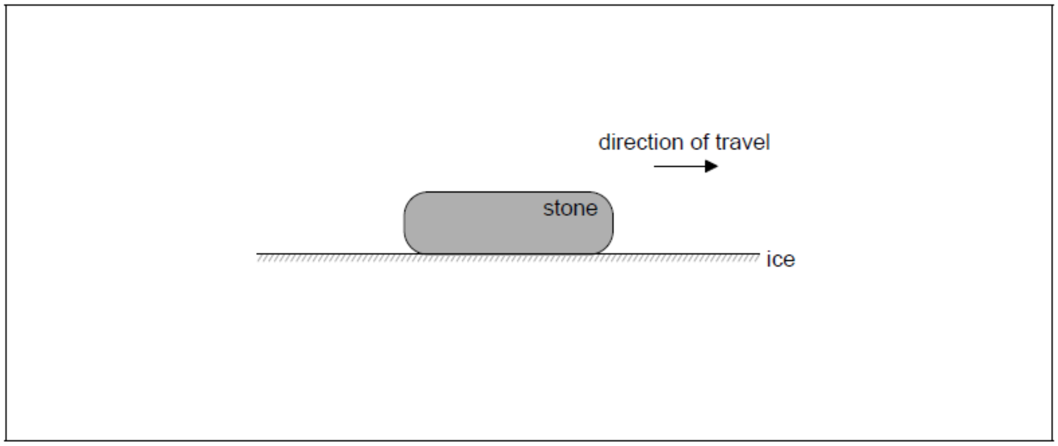
Allow ECF from (a) (note that $\mu = 0.0073$ x candidate answer to (a)).

Ignore any units in answer.

Condone omission of m in solution.

Allow g = 10 N kg^{-1} (gives 0.024).

4b. The diagram shows the stone during its motion **after** release. *[3 marks]*



Label the diagram to show the forces acting on the stone. Your answer should include the name, the direction **and** point of application of each force.

Markscheme

normal force, upwards, ignore point of application

Force must be labeled for its mark to be awarded. Blob at poa not required.
Allow OWTTE for normal force. Allow N, R, reaction.
The vertical forces must lie within the middle third of the stone

weight/weight force/force of gravity, downwards, ignore point of application

Allow mg, W but not "gravity".

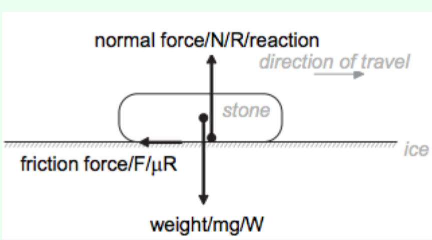
Penalise gross deviations from vertical/horizontal once only

friction/resistive force, to left, at bottom of stone, point of application must be **on** the interface between ice and stone

Allow F, μR. Only allow arrows/lines that lie on the interface. Take the tail of the arrow as the definitive point of application and expect line to be drawn horizontal.

*Award **[2 max]** if any force arrow does not touch the stone*

Do not award MP3 if a "driving force" is shown acting to the right. This need not be labelled to disqualify the mark. Treat arrows labelled "air resistance" as neutral.



N.B: *Diagram in MS is drawn with the vertical forces not direction of travel collinear for clarity*

This question is in two parts. **Part 1** is about momentum. **Part 2** is about electric point charges.

Part 1 Momentum

5a. State the law of conservation of linear momentum. *[2 marks]*

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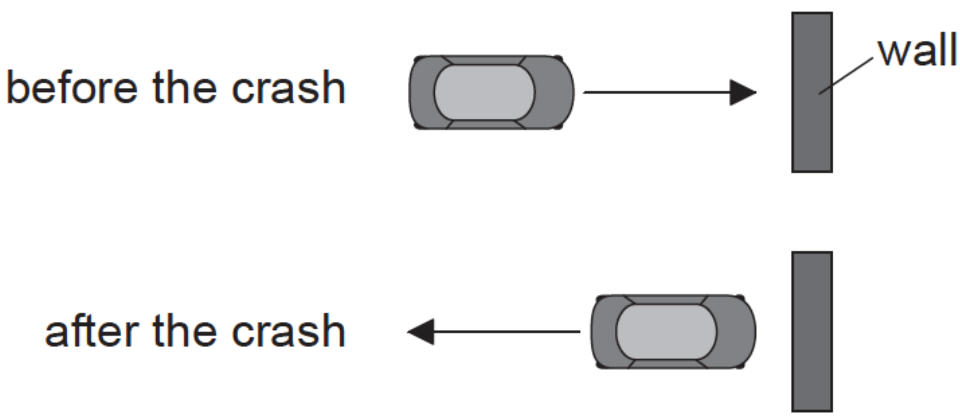
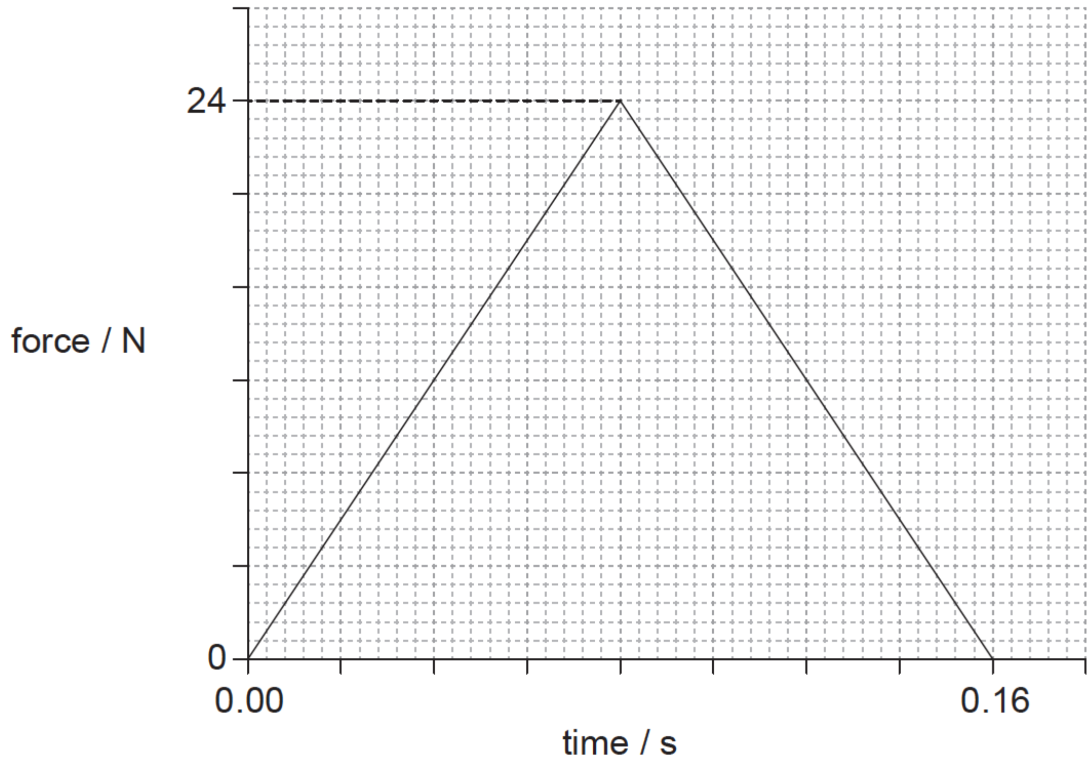
Markscheme

total momentum does not change/is constant; } *(do not allow "momentum is conserved")*
provided external force is zero / no external forces / isolated system;

5b. A toy car crashes into a wall and rebounds at right angles to the wall, as shown in the plan view. *[9 marks]*



The graph shows the variation with time of the force acting on the car due to the wall during the collision.

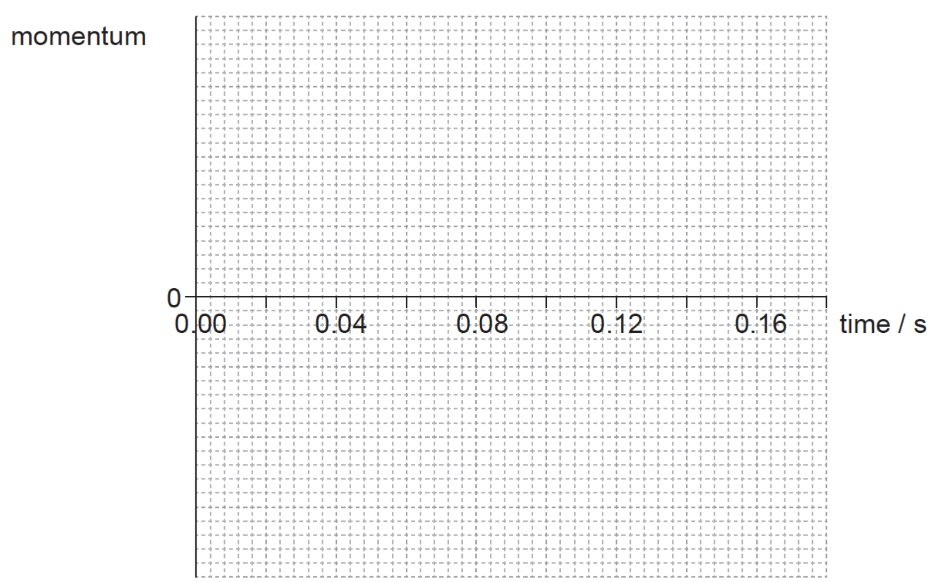


The kinetic energy of the car is unchanged after the collision. The mass of the car is 0.80 kg.

(i) Determine the initial momentum of the car.

(ii) Estimate the average acceleration of the car before it rebounds.

(iii) On the axes, draw a graph to show how the momentum of the car varies during the impact. You are not required to give values on the y-axis.



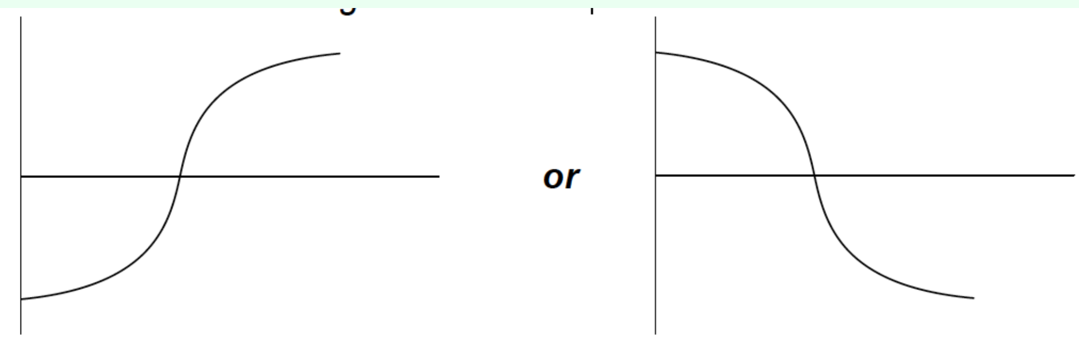
Markscheme

(i) clear attempt to calculate area under graph;
initial momentum is half change in momentum;
$\left(\frac{1}{2} \times \frac{1}{2} \times 24 \times 0.16\right) = 0.96 \left(\text{kgms}^{-1}\right)$
*Award **[2 max]** for calculation of total change (1.92kg ms^{-1})*

(ii) initial speed $= \left(\frac{0.96}{0.8} =\right) 1.2\text{ms}^{-1}$;
$a = \frac{1.2-(-1.2)}{0.16}$ ***or*** $a = \frac{-1.2-1.2}{0.16}$;
–15(ms^{-2}); *(must see negative sign or a comment that this is a deceleration)*

or
average force =12 N;
uses F=0.8×a ;
–15(ms^{-2}); *(must see negative sign or a comment that this is a deceleration)*
*Award **[3]** for a bald correct answer.*
Other solution methods involving different kinematic equations are possible.

(iii) goes through t=0.08s **and** from negative momentum to positive / positive momentum to negative;
constant sign of gradient throughout;
curve as shown;
Award marks for diagram as shown.



5c. Two identical toy cars, A and B are dropped from the same height onto a solid floor without rebounding. Car A is unprotected whilst car B is in a box with protective packaging around the toy. Explain why car B is less likely to be damaged when dropped. *[4 marks]*

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Markscheme

impulse is the same/similar in both cases / momentum change is same;
impulse is force × time / force is rate of change of momentum;
time to come to rest is longer for car B;
force experienced by car B is less (so less likely to be damaged);

Part 2 Electric point charges

5d. Define *electric field strength* at a point in an electric field. *[2 marks]*

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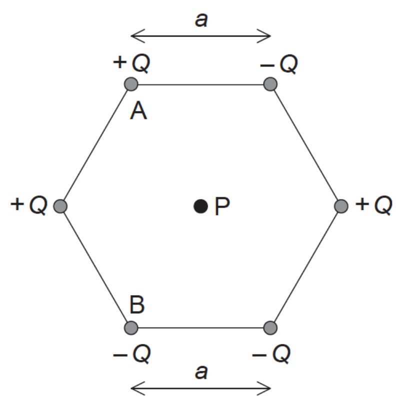
Markscheme

electric force per unit charge;
acting on a small/point positive (test) charge;

5e. Six point charges of equal magnitude Q are held at the corners of a hexagon with the signs of the charges as shown. Each side of the hexagon has a length *a*. *[8 marks]*



P is at the centre of the hexagon.

(i) Show, using Coulomb's law, that the magnitude of the electric field strength at point P due to **one** of the point charges is

$$\frac{kQ}{a^2}$$

(ii) On the diagram, draw arrows to represent the direction of the field at P due to point charge A (label this direction A) and point charge B (label this direction B).

(iii) The magnitude of Q is 3.2 μC and length a is 0.15 m. Determine the magnitude and the direction of the electric field strength at point P due to all six charges.

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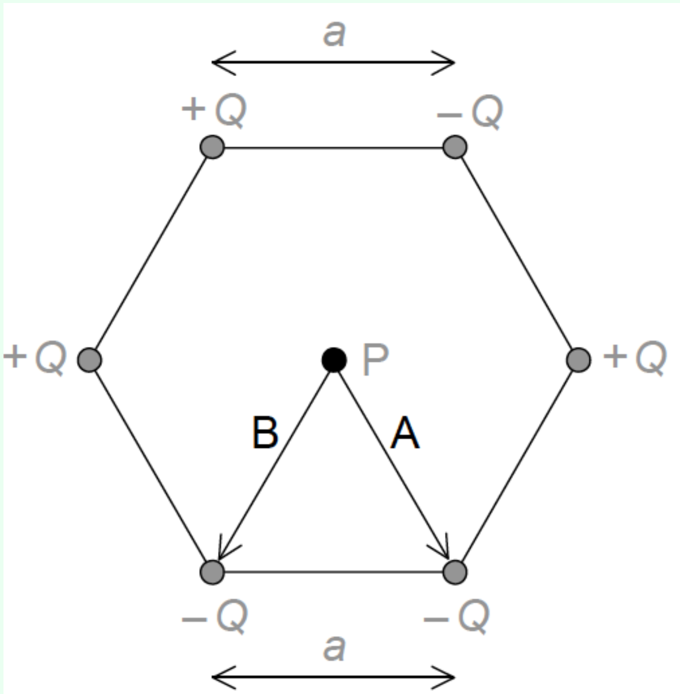
Markscheme

(i) states Coulomb's law as $\frac{kQq}{r^2}$ ***or*** $\frac{F}{q} = \frac{kQ}{r^2}$
states explicitly q=1;
states r=a;

(ii) 

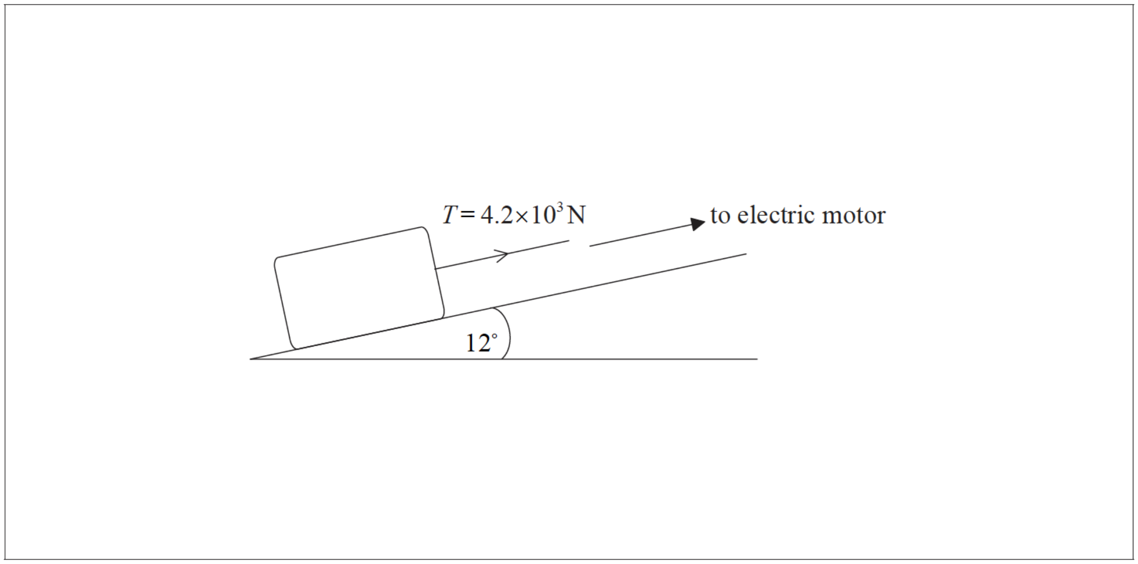
arrow labelled A pointing to lower right charge;
arrow labelled B point to lower left charge;
Arrows can be anywhere on diagram.

(iii) overall force is due to +Q top left and -Q bottom right / top right and bottom left and centre charges all cancel; } *(can be seen on diagram)*
force is therefore $\frac{2kQ}{a^2}$;

2.6×106 (N C^{-1}) ;
towards bottom right charge; *(allow clear arrow on diagram showing direction)*

This question is about forces.

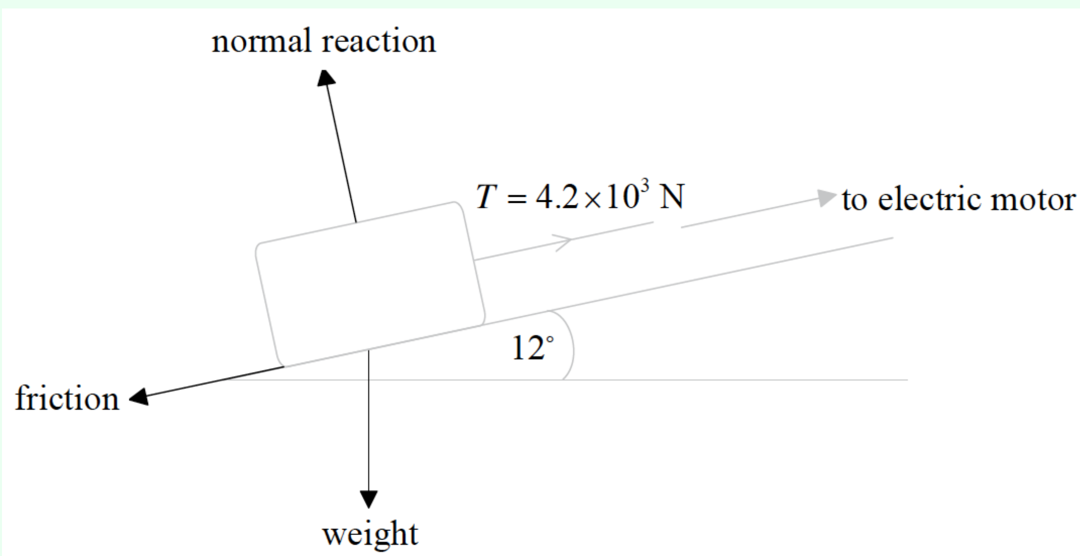
A stone block is pulled at constant speed up an incline by a cable attached to an electric motor.



The incline makes an angle of 12° with the horizontal. The weight of the block is 1.5×10^{4} N and the tension T in the cable is 4.2×10^{3} N.

6a. On the diagram draw and label arrows that represent the forces acting on the block. *[2 marks]*

Markscheme



(normal) reaction/*N*/*R* and weight/force of gravity/gravity force/gravitational force/ *mg*/*w*/*W* with correct directions;

friction/frictional force/F/F_f with arrow pointing down ramp along surface of ramp;

Do not allow "gravity" as label. Do not allow "drag" as label for friction.

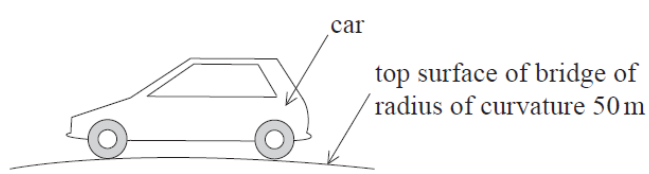
6b. Calculate the magnitude of the friction force acting on the block. *[3 marks]*

Markscheme

recognize that friction= T-$W\sin\theta$;
$W\sin\theta$=3.1×10^3N;
friction=1.1×10^3N;

This question is about circular motion.

The diagram shows a car moving at a constant speed over a curved bridge. At the position shown, the top surface of the bridge has a radius of curvature of 50 m.



7a. Explain why the car is accelerating even though it is moving with a constant speed. *[2 marks]*

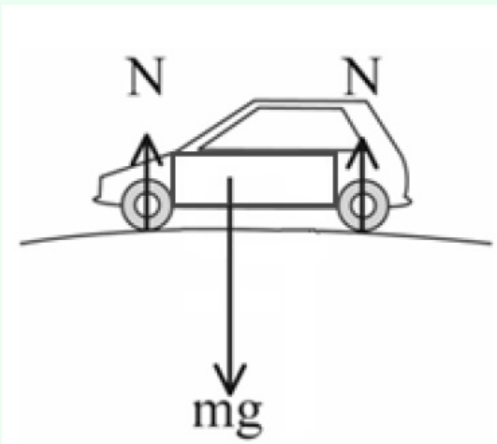
Markscheme

direction changing;

velocity changing so accelerating;

7b. On the diagram, draw and label the vertical forces acting on the car in the position shown. *[2 marks]*

Markscheme



weight/gravitational force/mg/w/F_w/F_g and reaction/normal reaction/perpendicular contact force/N/R/F_N/F_R both labelled; *(do not allow "gravity" for "weight".)*

weight between wheels (in box) from centre of mass and reactions at both wheels / single reaction acting along same line of action as the weight;

Judge by eye. Look for reasonably vertical lines with weight force longer than (sum of) reaction(s). Extra forces (eg centripetal force) loses the second mark.

7c. Calculate the maximum speed at which the car will stay in contact with the bridge. *[3 marks]*

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Markscheme

$g = \frac{v^2}{r}$;
$v = \sqrt{50 \times 9.8}$;

22(ms^{-1});

*Allow **[3]** for a bald correct answer.*