ExpLogTrigReview *[213 marks]*

Let $b = \log_2 a$, where $a > 0$. Write down each of the following expressions in terms of b.

1a. $\log_2 a^3$ *[2 marks]*

Markscheme

correct approach ***(A1)***

eg $3\log_2 a$

$\log_2 a^3 = 3b$ ***A1 N2***

[2 marks]

1b. $\log_2 8a$ *[2 marks]*

Markscheme

correct working ***(A1)***

eg $\log_2 8 + \log_2 a$, $\log_2 8 = 3$

$\log_2 8a = 3 + b$ ***A1 N2***

[2 marks]

1c. $\log_8 a$ *[2 marks]*

Markscheme

correct working ***(A1)***

eg $\frac{\log_2 a}{\log_2 8}$, $\frac{1}{3}\log_2 a$, $b\log_8 2$

$\log_8 a = \frac{b}{3}$ ***A1 N2***

[2 marks]

2. Given that $\sin x = \frac{1}{3}$, where $0 < x < \frac{\pi}{2}$, find the value of $\cos 4x$. *[6 marks]*

Markscheme

METHOD 1

correct substitution into formula for $\cos\left(2x\right)$ or $\sin\left(2x\right)$ ***(A1)***

eg $1-2\left(\frac{1}{3}\right)^2,\ 2\left(\frac{\sqrt{8}}{3}\right)^2-1,\ 2\left(\frac{1}{3}\right)\left(\frac{\sqrt{8}}{3}\right),\ \left(\frac{\sqrt{8}}{3}\right)^2-\left(\frac{1}{3}\right)^2$

$\cos\left(2x\right)=\frac{7}{9}$ or $\sin\left(2x\right)=\frac{2\sqrt{8}}{9}$ $\left(=\frac{\sqrt{32}}{9}=\frac{4\sqrt{2}}{9}\right)$ (may be seen in substitution) ***A2***

recognizing $4x$ is double angle of $2x$ (seen anywhere) ***(M1)***

eg $\cos\left(2\left(2x\right)\right),\ 2\cos^2\left(2\theta\right)-1,\ 1-2\sin^2\left(2\theta\right),\ \cos^2\left(2\theta\right)-\sin^2\left(2\theta\right)$

correct substitution of **their** value of $\cos\left(2x\right)$ and/or $\sin\left(2x\right)$ into formula for $\cos\left(4x\right)$ ***(A1)***

eg $2\left(\frac{7}{9}\right)^2-1,\ \frac{98}{81}-1,\ 1-2\left(\frac{2\sqrt{8}}{9}\right)^2,\ 1-\frac{64}{81},\ \left(\frac{7}{9}\right)^2-\left(\frac{2\sqrt{8}}{9}\right)^2,\ \frac{49}{81}-\frac{32}{81}$

$\cos\left(4x\right)=\frac{17}{81}$ ***A1 N2***

METHOD 2

recognizing $4x$ is double angle of $2x$ (seen anywhere) ***(M1)***

eg $\cos\left(2\left(2x\right)\right)$

double angle identity for $2x$ ***(M1)***

eg $2\cos^2\left(2\theta\right)-1,\ 1-2\sin^2\left(2x\right),\ \cos^2\left(2\theta\right)-\sin^2\left(2\theta\right)$

correct expression for $\cos\left(4x\right)$ in terms of $\sin x$ and/or $\cos x$ ***(A1)***

eg $2\left(1-2\sin^2\theta\right)^2-1,\ 1-2(2\sin x\cos x)^2,\ \left(1-2\sin^2\theta\right)^2-(2\sin\theta\cos\theta)^2$

correct substitution for $\sin x$ and/or $\cos x$ ***A1***

eg $2\left(1-2\left(\frac{1}{3}\right)^2\right)^2-1,\ 2\left(1-4\left(\frac{1}{3}\right)^2+4\left(\frac{1}{3}\right)^4\right)-1,\ 1-2\left(2\times\frac{1}{3}\times\frac{\sqrt{8}}{3}\right)^2$

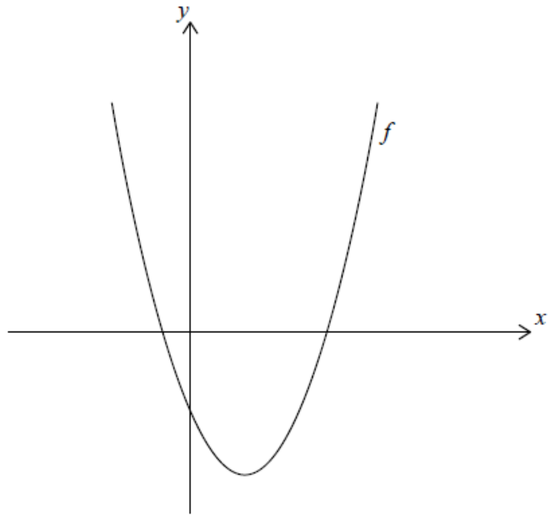
correct working ***(A1)***

eg $2\left(\frac{49}{81}\right)-1,\ 1-2\left(\frac{32}{81}\right),\ \frac{49}{81}-\frac{32}{81}$

$\cos\left(4x\right)=\frac{17}{81}$ ***A1 N2***

[6 marks]

Let $f(x) = x^2 - 4x - 5$. The following diagram shows part of the graph of f.



3a. Find the x-intercepts of the graph of f. *[5 marks]*

Markscheme

valid approach ***(M1)***

eg $f(x) = 0,\ \ x^2 - 4x - 5 = 0$

valid attempt to solve quadratic equation ***(M1)***

eg factorizing, formula, completing the square

evidence of correct working ***(A1)***

eg $(x-5)(x+1),\ \ x = \frac{4 \pm \sqrt{16 - 4(-5)}}{2}$

$x = -1,\ x = 5$ (accept (–1, 0), (5, 0)) ***A1A1 N3***

[5 marks]

3b. Find the equation of the axis of symmetry of the graph of f. *[2 marks]*

Markscheme

correct working ***(A1)***

eg $\frac{-(-4)}{2(1)},\ \frac{-1+5}{2}$

$x = 2$ (must be an equation with $x =$) ***A1 N2***

[2 marks]

The function can be written in the form $f(x) = (x - h)^2 + k$.

3c. Write down the value of h. *[1 mark]*

Markscheme

$h = 2$ ***A1 N1***

[1 mark]

3d. Find the value of k. *[3 marks]*

Markscheme

METHOD 1

valid approach ***(M1)***

eg $f(2)$

correct substitution ***(A1)***

eg $(2)^2 - 4(2) - 5$

$k = -9$ ***A1 N2***

METHOD 2

valid attempt to complete the square ***(M1)***

eg $x^2 - 4x + 4$

correct working ***(A1)***

eg $(x^2 - 4x + 4) - 4 - 5$, $(x - 2)^2 - 9$

$k = -9$ ***A1 N2***

[3 marks]

3e. The graph of a second function, g, is obtained by a reflection of the graph of f in the y-axis, followed by a translation of $\begin{pmatrix} -3 \\ 6 \end{pmatrix}$. *[5 marks]*

Find the coordinates of the vertex of the graph of g.

Markscheme

METHOD 1 (working with vertex)

vertex of f is at (2, –9)   ***(A1)***

correct horizontal reflection   ***(A1)***

eg $x = -2$, (–2, –9)

valid approach for translation of **their** x **or** y value   ***(M1)***

eg $x - 3$, $y + 6$, $\begin{pmatrix} -2 \\ -9 \end{pmatrix} + \begin{pmatrix} -3 \\ 6 \end{pmatrix}$, one correct coordinate for vertex

vertex of g is (–5, –3) (accept $x = -5$, $y = -3$)   ***A1A1 N1N1***

METHOD 2 (working with function)

correct approach for horizontal reflection   ***(A1)***

eg $f(-x)$

correct horizontal reflection   ***(A1)***

eg $(-x)^2 - 4(-x) - 5$, $x^2 + 4x - 5$, $(-x - 2)^2 - 9$

valid approach for translation of **their** x **or** y value   ***(M1)***

eg $(x + 3)^2 + 4(x + 3) - 5 + 6$, $x^2 + 10x + 22$, $(x + 5)^2 - 3$, one correct coordinate for vertex

vertex of g is (–5, –3) (accept $x = -5$, $y = -3$)   ***A1A1 N1N1***

[5 marks]

The first two terms of an infinite geometric sequence are $u_1 = 18$ and $u_2 = 12\sin^2\theta$, where $0 < \theta < 2\pi$, and $\theta \neq \pi$.

4a. Find an expression for r in terms of θ.   *[2 marks]*

Markscheme

valid approach   ***(M1)***

eg $\frac{u_2}{u_1}, \frac{u_1}{u_2}$

$r = \frac{12\sin^2\theta}{18}\left(= \frac{2\sin^2\theta}{3}\right)$ ***A1 N2***

[2 marks]

4b. Find the possible values of r.   *[3 marks]*

Markscheme

recognizing that $\sin\theta$ is bounded ***(M1)***

eg $0 \le \sin^2\theta \le 1$, $-1 \le \sin\theta \le 1$, $-1 < \sin\theta < 1$

$0 < r \le \frac{2}{3}$ ***A2 N3***

Note: If working shown, award ***M1A1*** for correct values with incorrect inequality sign(s).
If no working shown, award ***N1*** for correct values with incorrect inequality sign(s).

[3 marks]

4c. Show that the sum of the infinite sequence is $\frac{54}{2+\cos(2\theta)}$. *[4 marks]*

Markscheme

correct substitution into formula for infinite sum ***A1***

eg $\frac{18}{1-\frac{2\sin^2\theta}{3}}$

evidence of choosing an appropriate rule for $\cos 2\theta$ (seen anywhere) ***(M1)***

eg $\cos 2\theta = 1 - 2\sin^2\theta$

correct substitution of identity/working (seen anywhere) ***(A1)***

eg $\frac{18}{1-\frac{2}{3}\left(\frac{1-\cos 2\theta}{2}\right)}$, $\frac{54}{3-2\left(\frac{1-\cos 2\theta}{2}\right)}$, $\frac{18}{\frac{3-2\sin^2\theta}{3}}$

correct working that clearly leads to the given answer ***A1***

eg $\frac{18\times 3}{2+(1-2\sin^2\theta)}$, $\frac{54}{3-(1-\cos 2\theta)}$

$\frac{54}{2+\cos(2\theta)}$ ***AG N0***

[4 marks]

4d. Find the values of θ which give the greatest value of the sum. *[6 marks]*

Markscheme

METHOD 1 (using differentiation)

recognizing $\frac{\mathrm{d}S_\infty}{\mathrm{d}\theta} = 0$ (seen anywhere) ***(M1)***

finding any correct expression for $\frac{\mathrm{d}S_\infty}{\mathrm{d}\theta}$ ***(A1)***

eg $\frac{0-54\times(-2\sin 2\theta)}{(2+\cos 2\theta)^2},\ -54(2+\cos 2\theta)^{-2}\ (-2\sin 2\theta)$

correct working ***(A1)***

eg $\sin 2\theta = 0$

any correct value for $\sin^{-1}(0)$ (seen anywhere) ***(A1)***

eg $0, \pi, \ldots$, sketch of sine curve with x-intercept(s) marked both correct values for 2θ (ignore additional values) ***(A1)***

$2\theta = \pi, 3\pi$ (accept values in degrees)

both correct answers $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ ***A1 N4***

Note: Award ***A0*** if either or both correct answers are given in degrees.
Award ***A0*** if additional values are given.

METHOD 2 (using denominator)

recognizing when S_∞ is greatest ***(M1)***

eg $2 + \cos 2\theta$ is a minimum, $1-r$ is smallest
correct working ***(A1)***

eg minimum value of $2 + \cos 2\theta$ is 1, minimum $r = \frac{2}{3}$

correct working ***(A1)***

eg $\cos 2\theta = -1,\ \frac{2}{3}\sin^2\theta = \frac{2}{3},\ \sin^2\theta = 1$

EITHER (using $\cos 2\theta$)

any correct value for $\cos^{-1}(-1)$ (seen anywhere) ***(A1)***

eg $\pi, 3\pi, \ldots$ (accept values in degrees), sketch of cosine curve with x-intercept(s) marked

both correct values for 2θ (ignore additional values) ***(A1)***

$2\theta = \pi, 3\pi$ (accept values in degrees)

OR (using $\sin\theta$)

$\sin\theta = \pm 1$ (A1)

$\sin^{-1}(1) = \frac{\pi}{2}$ (accept values in degrees) (seen anywhere) ***A1***

THEN

both correct answers $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ ***A1 N4***

Note: Award ***A0*** if either or both correct answers are given in degrees.
Award ***A0*** if additional values are given.

[6 marks]

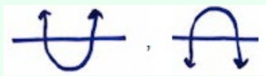
Let $f(x) = ax^2 - 4x - c$. A horizontal line, L, intersects the graph of f at $x = -1$ and $x = 3$.

5a. The equation of the axis of symmetry is $x = p$. Find p. *[2 marks]*

Markscheme

METHOD 1 (using symmetry to find p)

valid approach ***(M1)***

eg $\frac{-1+3}{2}$, 

$p = 1$ ***A1 N2***

Note: Award no marks if they work backwards by substituting $a = 2$ into $-\frac{b}{2a}$ to find p.

Do not accept $p = \frac{2}{a}$.

METHOD 2 (calculating a first)
(i) & (ii) valid approach to calculate a ***M1***

eg $a + 4 - c = a(3^2) - 4(3) - c$, $f(-1) = f(3)$

correct working ***A1***

eg $8a = 16$

$a = 2$ ***AG N0***

valid approach to find p ***(M1)***

eg $-\frac{b}{2a}$, $\frac{4}{2(2)}$

$p = 1$ ***A1 N2***

[2 marks]

5b. Hence, show that $a = 2$. *[2 marks]*

Markscheme

METHOD 1

valid approach ***M1***

eg $-\frac{b}{2a}$, $\frac{4}{2a}$ (might be seen in (i)), $f'(1) = 0$

correct equation ***A1***

eg $\frac{4}{2a} = 1$, $2a(1) - 4 = 0$

$a = 2$ ***AG N0***

METHOD 2 (calculating a first)
(i) & (ii) valid approach to calculate a ***M1***

eg $a + 4 - c = a(3^2) - 4(3) - c$, $f(-1) = f(3)$

correct working ***A1***

eg $8a = 16$

$a = 2$ ***AG N0***

[2 marks]

5c. The equation of L is $y = 5$. Find the value of c. *[3 marks]*

Markscheme

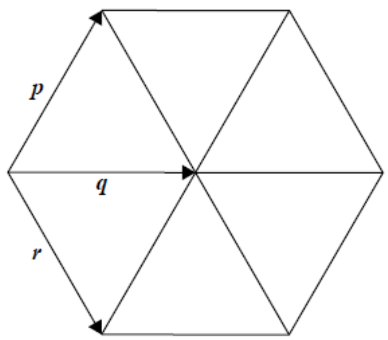
valid approach ***(M1)***
eg $f(-1) = 5$, $f(3) = 5$

correct working ***(A1)***
eg $2 + 4 - c = 5$, $18 - 12 - c = 5$

$c = 1$ ***A1 N2***

[3 marks]

6. Six equilateral triangles, each with side length 3 cm, are arranged to form a hexagon. *[6 marks]*
This is shown in the following diagram.



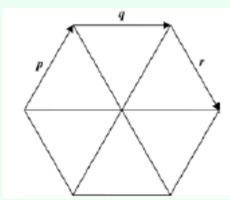
The vectors $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$ are shown on the diagram.

Find $\boldsymbol{p}\cdot(\boldsymbol{p}+\boldsymbol{q}+\boldsymbol{r})$.

Markscheme

METHOD 1 (using $|\boldsymbol{p}|\,|2\boldsymbol{q}|\cos\theta$)

finding $\boldsymbol{p}+\boldsymbol{q}+\boldsymbol{r}$ ***(A1)***

eg $2\boldsymbol{q}$, 

$|\boldsymbol{p}+\boldsymbol{q}+\boldsymbol{r}| = 2 \times 3\ (=6)$ (seen anywhere) ***A1***

correct angle between $\boldsymbol{p}$ and $\boldsymbol{q}$ (seen anywhere) ***(A1)***

$\frac{\pi}{3}$ (accept $60°$)

substitution of **their** values ***(M1)***

eg $3 \times 6 \times \cos\left(\frac{\pi}{3}\right)$

correct value for $\cos\left(\frac{\pi}{3}\right)$ (seen anywhere) ***(A1)***

eg $\frac{1}{2},\ 3 \times 6 \times \frac{1}{2}$

$\boldsymbol{p}\cdot(\boldsymbol{p}+\boldsymbol{q}+\boldsymbol{r}) = 9$ ***A1 N3***

METHOD 2 (scalar product using distributive law)

correct expression for scalar distribution ***(A1)***

eg $\boldsymbol{p}\cdot\boldsymbol{p}+\boldsymbol{p}\cdot\boldsymbol{q}+\boldsymbol{p}\cdot\boldsymbol{r}$

three correct angles between the vector pairs (seen anywhere) ***(A2)***

eg $0°$ between $\boldsymbol{p}$ and $\boldsymbol{p}$, $\frac{\pi}{3}$ between $\boldsymbol{p}$ and $\boldsymbol{q}$, $\frac{2\pi}{3}$ between $\boldsymbol{p}$ and $\boldsymbol{r}$

Note: Award ***A1*** for only two correct angles.

substitution of **their** values ***(M1)***

eg $3.3.\cos 0 + 3.3.\cos\frac{\pi}{3} + 3.3.\cos 120$

one correct value for $\cos 0$, $\cos\left(\frac{\pi}{3}\right)$ or $\cos\left(\frac{2\pi}{3}\right)$ (seen anywhere) ***A1***

eg $\frac{1}{2},\ 3 \times 6 \times \frac{1}{2}$

p*•(*p* + *q* + *r*) = 9** ***A1 N3

METHOD 3 (scalar product using relative position vectors)

valid attempt to find one component of ***p*** or ***r*** ***(M1)***

eg sin 60 = $\frac{x}{3}$, cos 60 = $\frac{x}{3}$, one correct value $\frac{3}{2}$, $\frac{3\sqrt{3}}{2}$, $\frac{-3\sqrt{3}}{2}$

one correct vector (two or three dimensions) (seen anywhere) ***A1***

eg $p = \begin{pmatrix} \frac{3}{2} \\ \frac{3\sqrt{3}}{2} \end{pmatrix}$, $q = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$, $r = \begin{pmatrix} \frac{3}{2} \\ -\frac{3\sqrt{3}}{2} \\ 0 \end{pmatrix}$

three correct vectors ***p*** + ***q*** + ***r*** = 2***q*** ***(A1)***

p + ***q*** + ***r*** = $\begin{pmatrix} 6 \\ 0 \end{pmatrix}$ or $\begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix}$ (seen anywhere, including scalar product) ***(A1)***

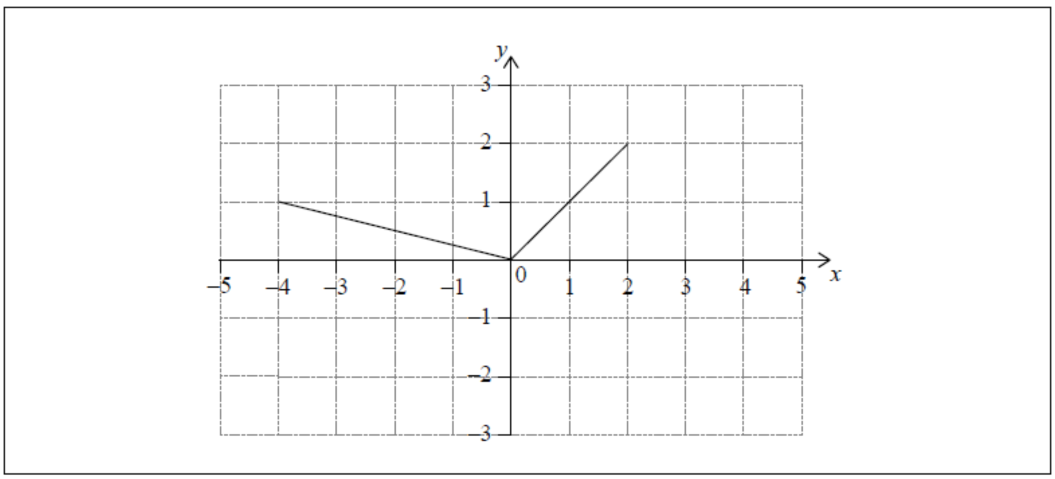
correct working ***(A1)***

eg $\left(\frac{3}{2} \times 6\right) + \left(\frac{3\sqrt{3}}{2} \times 0\right)$, $9 + 0 + 0$

p*•(*p* + *q* + *r*) = 9** ***A1 N3

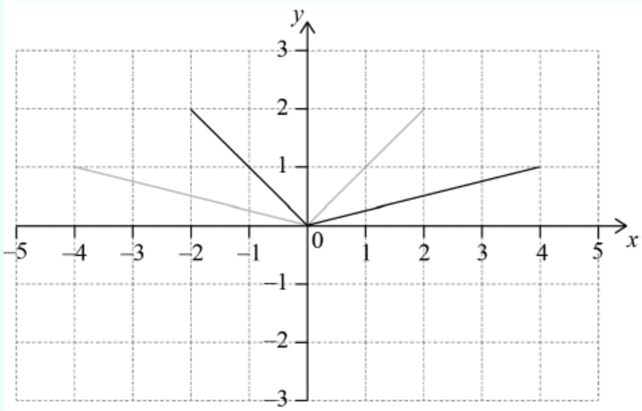
[6 marks]

The following diagram shows the graph of a function f, for $-4 \le x \le 2$.



7a. On the same axes, sketch the graph of $f(-x)$. *[2 marks]*

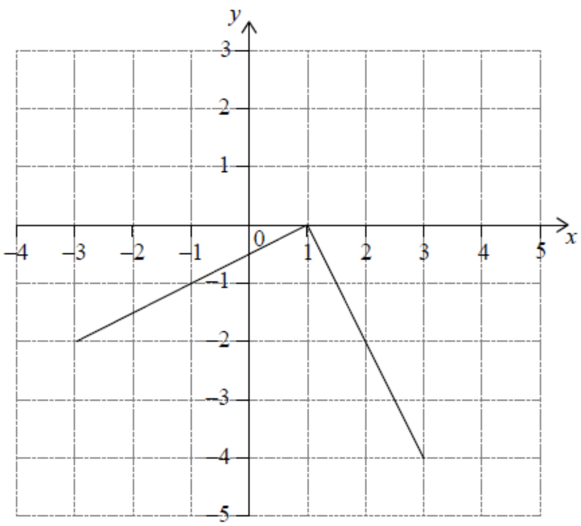
Markscheme



A2 N2

[2 marks]

7b. Another function, g, can be written in the form $g(x) = a \times f(x + b)$. The following diagram shows the graph of g. *[4 marks]*



Write down the value of a and of b.

Markscheme

recognizing horizontal shift/translation of 1 unit ***(M1)***

eg $b = 1$, moved 1 right

recognizing vertical stretch/dilation with scale factor 2 ***(M1)***

eg $a = 2$, $y \times (-2)$

$a = -2$, $b = -1$ ***A1A1 N2N2***

[4 marks]

8. Let $f(x) = px^2 + qx - 4p$, where $p \neq 0$. Find Find the number of roots for the equation $f(x) = 0$. *[7 marks]*

Justify your answer.

Markscheme

METHOD 1

evidence of discriminant ***(M1)***
eg $b^2 - 4ac, \ \Delta$

correct substitution into discriminant ***(A1)***
eg $q^2 - 4p(-4p)$

correct discriminant ***A1***
eg $q^2 + 16p^2$

$16p^2 > 0 \ \ (\text{accept } p^2 > 0)$ ***A1***

$q^2 \geqslant 0 \ \ (\text{do not accept } q^2 > 0)$ ***A1***

$q^2 + 16p^2 > 0$ ***A1***

f has 2 roots ***A1 N0***

METHOD 2

y-intercept = −4*p* (seen anywhere) ***A1***

if *p* is positive, then the *y*-intercept will be negative ***A1***

an upward-opening parabola with a negative *y*-intercept ***R1***
eg sketch that must indicate *p* > 0.

if *p* is negative, then the y-intercept will be positive ***A1***

a downward-opening parabola with a positive y-intercept ***R1***
eg sketch that must indicate *p* > 0.

f has 2 roots ***A2 N0***

[7 marks]

An arithmetic sequence has $u_1 = \log_c(p)$ and $u_2 = \log_c(pq)$, where $c > 1$ and $p, \ q > 0$.

9a. Show that $d = \log_c(q)$. *[2 marks]*

Markscheme

valid approach involving addition or subtraction ***M1***
eg $u_2 = \log_c p + d,\ u_1 - u_2$

correct application of log law ***A1***
eg $\log_c (pq) = \log_c p + \log_c q,\ \log_c \left(\frac{pq}{p}\right)$

$d = \log_c q$ ***AG N0***

[2 marks]

9b. *[6 marks]*

Let $p = c^2$ and $q = c^3$. Find the value of $\sum_{n=1}^{20} u_n$.

Markscheme

METHOD 1 (finding u_1 and *d*)

recognizing $\sum = S_{20}$ (seen anywhere) ***(A1)***

attempt to find u_1 or *d* using $\log_c c^k = k$ ***(M1)***
eg $\log_c c$, $3\log_c c$, correct value of u_1 or *d*

u_1 = 2, *d* = 3 (seen anywhere) ***(A1)(A1)***

correct working ***(A1)***
eg $S_{20} = \frac{20}{2}(2 \times 2 + 19 \times 3)$, $S_{20} = \frac{20}{2}(2 + 59)$, $10(61)$

$\sum_{n=1}^{20} u_n$ = 610 ***A1 N2***

METHOD 2 (expressing *S* in terms of *c*)

recognizing $\sum = S_{20}$ (seen anywhere) ***(A1)***

correct expression for *S* in terms of *c* ***(A1)***
eg $10\left(2\log_c c^2 + 19\log_c c^3\right)$

$\log_c c^2 = 2,\ \log_c c^3 = 3$ (seen anywhere) ***(A1)(A1)***

correct working ***(A1)***

eg $S_{20} = \frac{20}{2}(2 \times 2 + 19 \times 3)$, $S_{20} = \frac{20}{2}(2 + 59)$, $10(61)$

$\sum_{n=1}^{20} u_n$ = 610 ***A1 N2***

METHOD 3 (expressing *S* in terms of *c*)

recognizing $\sum = S_{20}$ (seen anywhere) ***(A1)***

correct expression for *S* in terms of *c* ***(A1)***
eg $10\left(2\log_c c^2 + 19\log_c c^3\right)$

correct application of log law ***(A1)***
eg
$2\log_c c^2 = \log_c c^4,\ 19\log_c c^3 = \log_c c^{57},\ 10\left(\log_c \left(c^2\right)^2 + \log_c \left(c^3\right)^{19}\right),\ 10\left(\log_c c^4 + \log_c c^{57}\right),\ 10$

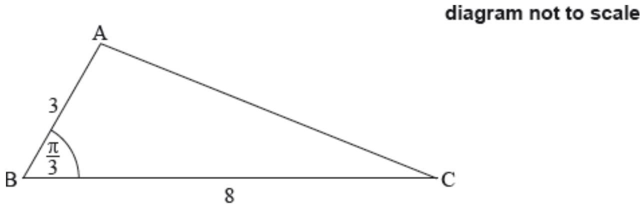
correct application of definition of log ***(A1)***
eg $\log_c c^{61} = 61,\ \log_c c^4 = 4,\ \log_c c^{57} = 57$

correct working ***(A1)***
eg $S_{20} = \frac{20}{2}(4 + 57)$, $10(61)$

$\sum_{n=1}^{20} u_n$ = 610 ***A1 N2***

[6 marks]

The following diagram shows triangle ABC, with $\mathrm{AB} = 3\ \mathrm{cm}$, $\mathrm{BC} = 8\ \mathrm{cm}$, and $\mathrm{A\hat{B}C} = \frac{\pi}{3}$.



10a. Show that $\mathrm{AC} = 7\ \mathrm{cm}$. *[4 marks]*

Markscheme

evidence of choosing the cosine rule ***(M1)***

eg $c^2 = a^2 + b^2 - ab\cos C$

correct substitution into RHS of cosine rule ***(A1)***

eg $3^2 + 8^2 - 2 \times 3 \times 8 \times \cos\frac{\pi}{3}$

evidence of correct value for $\cos\frac{\pi}{3}$ (may be seen anywhere, including in cosine rule) ***A1***

eg $\cos\frac{\pi}{3} = \frac{1}{2},\ \mathrm{AC}^2 = 9 + 64 - \left(48 \times \frac{1}{2}\right),\ 9 + 64 - 24$

correct working clearly leading to answer ***A1***

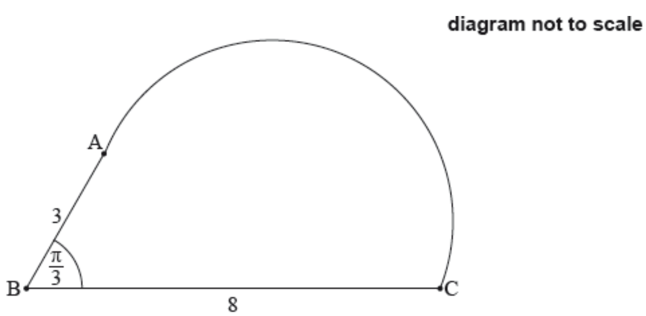
eg $\mathrm{AC}^2 = 49,\ b = \sqrt{49}$

$\mathrm{AC} = 7\ (\mathrm{cm})$ ***AG*** ***N0***

Note: Award no marks if the only working seen is $\mathrm{AC}^2 = 49$ or $\mathrm{AC} = \sqrt{49}$ (or similar).

[4 marks]

10b. The shape in the following diagram is formed by adding a semicircle with diameter [AC] to the triangle. *[3 marks]*



Find the exact perimeter of this shape.

Markscheme

correct substitution for semicircle ***(A1)***

eg $\text{semicircle} = \frac{1}{2}(2\pi \times 3.5),\ \frac{1}{2} \times \pi \times 7,\ 3.5\pi$

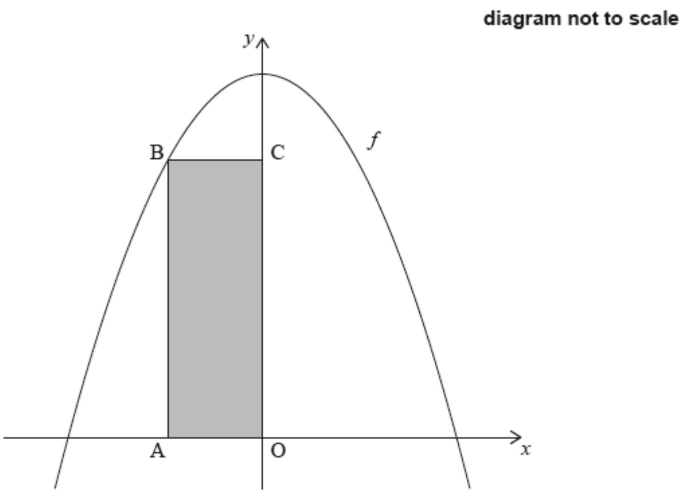
valid approach (seen anywhere) ***(M1)***

eg $\text{perimeter} = \text{AB} + \text{BC} + \text{semicircle}, 3 + 8 + \left(\frac{1}{2} \times 2 \times \pi \times \frac{7}{2}\right), 8 + 3 + 3.5\pi$

$11 + \frac{7}{2}\pi\ (= 3.5\pi + 11)\ (\text{cm})$ ***A1 N2***

[3 marks]

11. Let $f(x) = 15 - x^2$, for $x \in \mathbb{R}$. The following diagram shows part of the graph of f and the rectangle OABC, where A is on the negative x-axis, B is on the graph of f, and C is on the y-axis. *[7 marks]*



Find the x-coordinate of A that gives the maximum area of OABC.

Markscheme

attempt to find the area of OABC ***(M1)***

eg $\mathrm{OA} \times \mathrm{OC}, x \times f(x),\ f(x) \times (-x)$

correct expression for area in one variable ***(A1)***

eg $\text{area} = x(15 - x^2),\ 15x - x^3,\ x^3 - 15x$

valid approach to find maximum **area** (seen anywhere) ***(M1)***

eg $A'(x) = 0$

correct derivative ***A1***

eg $15 - 3x^2,\ (15 - x^2) + x(-2x) = 0,\ -15 + 3x^2$

correct working ***(A1)***

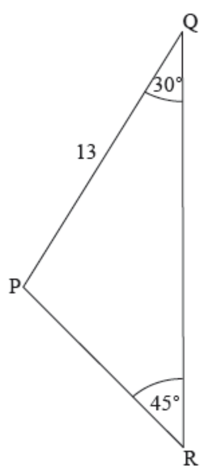
eg $15 = 3x^2,\ x^2 = 5,\ x = \sqrt{5}$

$x = -\sqrt{5}\ \left(\text{accept A}\left(-\sqrt{5},\ 0\right)\right)$ ***A2 N3***

[7 marks]

12. The following diagram shows triangle PQR. *[6 marks]*

diagram not to scale



$\mathrm{P\hat{Q}R} = 30°$, $\mathrm{Q\hat{R}P} = 45°$ and $\mathrm{PQ} = 13\ \mathrm{cm}$.

Find PR.

Markscheme

METHOD 1

evidence of choosing the sine rule ***(M1)***

eg $\frac{a}{\sin A} = \frac{b}{\sin B}$

correct substitution ***A1***

eg $\frac{x}{\sin 30} = \frac{13}{\sin 45}, \frac{13 \sin 30}{\sin 45}$

$\sin 30 = \frac{1}{2}, \sin 45 = \frac{1}{\sqrt{2}}$ ***(A1)(A1)***

correct working ***A1***

eg $\frac{1}{2} \times \frac{13}{\frac{1}{\sqrt{2}}}, \frac{1}{2} \times 13 \times \frac{2}{\sqrt{2}}, 13 \times \frac{1}{2} \times \sqrt{2}$

correct answer ***A1 N3***

eg $\text{PR} = \frac{13\sqrt{2}}{2}, \frac{13}{\sqrt{2}}\ (\text{cm})$

METHOD 2 (using height of Δ PQR)

valid approach to find height of ΔPQR ***(M1)***

eg $\sin 30 = \frac{x}{13}, \cos 60 = \frac{x}{13}$

$\sin 30 = \frac{1}{2}$ or $\cos 60 = \frac{1}{2}$ ***(A1)***

$\text{height} = 6.5$ ***A1***

correct working ***A1***

eg $\sin 45 = \frac{6.5}{\text{PR}}, \sqrt{6.5^2 + 6.5^2}$

correct working ***(A1)***

eg $\sin 45 = \frac{1}{\sqrt{2}}, \cos 45 = \frac{1}{\sqrt{2}}, \sqrt{\frac{169 \times 2}{4}}$

correct answer ***A1 N3***

eg $\text{PR} = \frac{13\sqrt{2}}{2}, \frac{13}{\sqrt{2}}\ (\text{cm})$

[6 marks]

The first three terms of a geometric sequence are $\ln x^{16}, \ln x^8, \ln x^4$, for $x > 0$.

13a. Find the common ratio. *[3 marks]*

Markscheme

correct use $\log x^n = n \log x$ ***A1***

eg $16 \ln x$

valid approach to find r ***(M1)***

eg $\frac{u_{n+1}}{u_n}$, $\frac{\ln x^8}{\ln x^{16}}$, $\frac{4 \ln x}{8 \ln x}$, $\ln x^4 = \ln x^{16} \times r^2$

$r = \frac{1}{2}$ ***A1*** ***N2***

[3 marks]

13b. Solve $\sum_{k=1}^{\infty} 2^{5-k} \ln x = 64$. *[5 marks]*

Markscheme

recognizing a sum (finite or infinite) ***(M1)***

eg $2^4 \ln x + 2^3 \ln x$, $\frac{a}{1-r}$, S_∞, $16 \ln x + \ldots$

valid approach (seen anywhere) ***(M1)***

eg recognizing GP is the same as part (a), using **their** r value from part (a), $r = \frac{1}{2}$

correct substitution into infinite sum (only if $|r|$ is a constant and less than 1) ***A1***

eg $\frac{2^4 \ln x}{1-\frac{1}{2}}$, $\frac{\ln x^{16}}{\frac{1}{2}}$, $32 \ln x$

correct working ***(A1)***

eg $\ln x = 2$

$x = \mathrm{e}^2$ ***A1*** ***N3***

[5 marks]

14. Solve $\log_2(2 \sin x) + \log_2(\cos x) = -1$, for $2\pi < x < \frac{5\pi}{2}$. *[7 marks]*

Markscheme

correct application of $\log a + \log b = \log ab$ ***(A1)***

eg $\log_2(2\sin x\cos x),\ \log 2 + \log(\sin x) + \log(\cos x)$

correct equation without logs ***A1***

eg $2\sin x\cos x = 2^{-1},\ \sin x\cos x = \frac{1}{4},\ \sin 2x = \frac{1}{2}$

recognizing double-angle identity (seen anywhere) ***A1***

eg $\log(\sin 2x),\ 2\sin x\cos x = \sin 2x,\ \sin 2x = \frac{1}{2}$

evaluating $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}\ (30°)$ ***(A1)***

correct working ***A1***

eg $x = \frac{\pi}{12} + 2\pi,\ 2x = \frac{25\pi}{6},\ \frac{29\pi}{6},\ 750°,\ 870°,\ x = \frac{\pi}{12}$ **and** $x = \frac{5\pi}{12}$, one correct final answer

$x = \frac{25\pi}{12},\ \frac{29\pi}{12}$ (do not accept additional values) ***A2*** ***N0***

[7 marks]

Let $\sin\theta = \frac{\sqrt{5}}{3}$, where θ is acute.

15a. Find $\cos\theta$. *[3 marks]*

Markscheme

evidence of valid approach ***(M1)***

eg right triangle, $\cos^2\theta = 1 - \sin^2\theta$

correct working ***(A1)***

eg missing side is 2, $\sqrt{1 - \left(\frac{\sqrt{5}}{3}\right)^2}$

$\cos\theta = \frac{2}{3}$ ***A1*** ***N2***

[3 marks]

15b. Find $\cos 2\theta$. *[2 marks]*

Markscheme

correct substitution into formula for $\cos 2\theta$ ***(A1)***

eg $2 \times \left(\frac{2}{3}\right)^2 - 1,\ 1 - 2\left(\frac{\sqrt{5}}{3}\right)^2,\ \left(\frac{2}{3}\right)^2 - \left(\frac{\sqrt{5}}{3}\right)^2$

$\cos 2\theta = -\frac{1}{9}$ ***A1 N2***

[2 marks]

The first two terms of an infinite geometric sequence, in order, are

$2\log_2 x,\ \log_2 x$, where $x > 0$.

16a. Find r. *[2 marks]*

Markscheme

evidence of dividing terms (in any order) ***(M1)***

eg $\frac{\mu_2}{\mu_1},\ \frac{2\log_2 x}{\log_2 x}$

$r = \frac{1}{2}$ ***A1 N2***

[2 marks]

16b. Show that the sum of the infinite sequence is $4\log_2 x$. *[2 marks]*

Markscheme

correct substitution ***(A1)***

eg $\frac{2\log_2 x}{1-\frac{1}{2}}$

correct working ***A1***

eg $\frac{2\log_2 x}{\frac{1}{2}}$

$S_\infty = 4\log_2 x$ ***AG N0***

[2 marks]

The first three terms of an arithmetic sequence, in order, are

$\log_2 x,\ \log_2\left(\frac{x}{2}\right),\ \log_2\left(\frac{x}{4}\right)$, where $x > 0$.

16c. Find d, giving your answer as an integer. *[4 marks]*

Markscheme

evidence of subtracting two terms (in any order) ***(M1)***

eg $u_3 - u_2,\ \log_2 x - \log_2 \frac{x}{2}$

correct application of the properties of logs ***(A1)***

eg $\log_2\left(\frac{\frac{x}{2}}{x}\right),\ \log_2\left(\frac{x}{2} \times \frac{1}{x}\right),\ (\log_2 x - \log_2 2) - \log_2 x$

correct working ***(A1)***

eg $\log_2 \frac{1}{2},\ -\log_2 2$

$d = -1$ ***A1 N3***

[4 marks]

Let S_{12} be the sum of the first 12 terms of the arithmetic sequence.

16d. Show that $S_{12} = 12\log_2 x - 66$. *[2 marks]*

Markscheme

correct substitution into the formula for the sum of an arithmetic sequence ***(A1)***

eg $\frac{12}{2}(2\log_2 x + (12-1)(-1))$

correct working ***A1***

eg $6(2\log_2 x - 11),\ \frac{12}{2}(2\log_2 x - 11)$

$12\log_2 x - 66$ ***AG N0***

[2 marks]

16e. Given that S_{12} is equal to half the sum of the infinite geometric sequence, find x, giving your answer in the form 2^p, where $p \in \mathbb{Q}$. *[3 marks]*

Markscheme

correct equation ***(A1)***

eg $12\log_2 x - 66 = 2\log_2 x$

correct working ***(A1)***

eg $10\log_2 x = 66,\ \log_2 x = 6.6,\ 2^{66} = x^{10},\ \log_2\left(\frac{x^{12}}{x^2}\right) = 66$

$x = 2^{6.6}$ (accept $p = \frac{66}{10}$) ***A1 N2***

[3 marks]

Let $f(x) = 3\sin\left(\frac{\pi}{2}x\right)$, for $0 \leqslant x \leqslant 4$.

17a. (i) Write down the amplitude of f. *[3 marks]*

(ii) Find the period of f.

Markscheme

(i) 3 ***A1 N1***

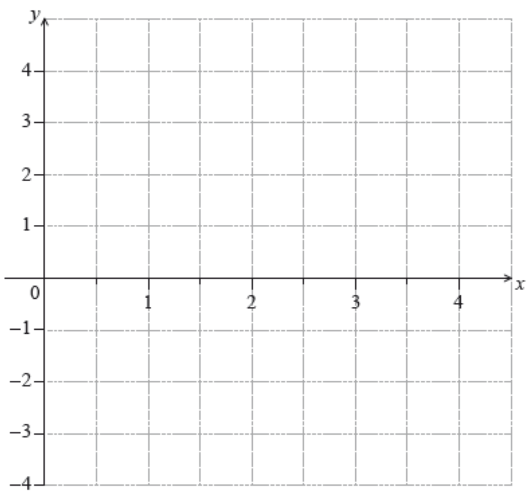
(ii) valid attempt to find the period ***(M1)***

eg $\frac{2\pi}{b}, \frac{2\pi}{\frac{\pi}{2}}$

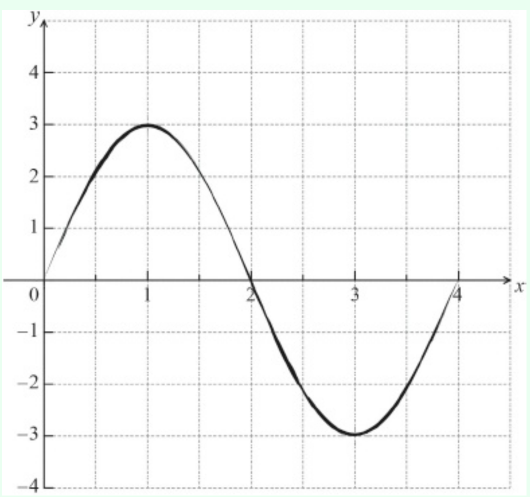
period $= 4$ ***A1 N2***

[3 marks]

17b. On the following grid sketch the graph of f. *[4 marks]*



Markscheme



A1A1A1A1 N4

[4 marks]

Consider $f(x) = x^2 + qx + r$. The graph of f has a minimum value when $x = -1.5$.

The distance between the two zeros of f is 9.

18a. Show that the two zeros are 3 and -6. *[2 marks]*

Markscheme

recognition that the x-coordinate of the vertex is -1.5 (seen anywhere) ***(M1)***

eg axis of symmetry is -1.5, sketch, $f'(-1.5) = 0$

correct working to find the zeroes ***A1***

eg -1.5 ± 4.5

$x = -6$ and $x = 3$ ***AG*** ***N0***

[2 marks]

18b. Find the value of q and of r. *[4 marks]*

Markscheme

METHOD 1 (using factors)

attempt to write factors ***(M1)***

eg $(x-6)(x+3)$

correct factors ***A1***

eg $(x-3)(x+6)$

$q=3,\ r=-18$ ***A1A1*** ***N3***

METHOD 2 (using derivative or vertex)

valid approach to find q ***(M1)***

eg $f'(-1.5)=0,\ -\frac{q}{2a}=-1.5$

$q=3$ ***A1***

correct substitution ***A1***

eg $3^2+3(3)+r=0,\ (-6)^2+3(-6)+r=0$

$r=-18$ ***A1***

$q=3,\ r=-18$ ***N3***

METHOD 3 (solving simultaneously)

valid approach setting up system of two equations ***(M1)***

eg $9+3q+r=0,\ 36-6q+r=0$

one correct value

eg $q=3,\ r=-18$ ***A1***

correct substitution ***A1***

eg $3^2+3(3)+r=0,\ (-6)^2+3(-6)+r=0,\ 3^2+3q-18=0,\ 36-6q-18=0$

second correct value ***A1***

eg $q=3,\ r=-18$

$q=3,\ r=-18$ ***N3***

[4 marks]

Let $f'(x)=\frac{6-2x}{6x-x^2}$, for $0<x<6$.

The graph of f has a maximum point at P.

19a. Find the x-coordinate of P. *[3 marks]*

Markscheme

recognizing $f'(x) = 0$ ***(M1)***

correct working ***(A1)***

eg $6 - 2x = 0$

$x = 3$ ***A1*** ***N2***

[3 marks]

The y-coordinate of P is $\ln 27$.

19b. Find $f(x)$, expressing your answer as a single logarithm. *[8 marks]*

Markscheme

evidence of integration ***(M1)***

eg $\int f',\ \int \frac{6-2x}{6x-x^2} \mathrm{d}x$

using substitution ***(A1)***

eg $\int \frac{1}{u} \mathrm{d}u$ where $u = 6x - x^2$

correct integral ***A1***

eg $\ln(u) + c,\ \ln(6x - x^2)$

substituting $(3,\ \ln 27)$ into **their** integrated expression (must have c) ***(M1)***

eg $\ln(6 \times 3 - 3^2) + c = \ln 27,\ \ln(18 - 9) + \ln k = \ln 27$

correct working ***(A1)***

eg $c = \ln 27 - \ln 9$

EITHER

$c = \ln 3$ ***(A1)***

attempt to substitute **their** value of c into $f(x)$ ***(M1)***

eg $f(x) = \ln(6x - x^2) + \ln 3$ ***A1*** ***N4***

OR

attempt to substitute **their** value of c into $f(x)$ ***(M1)***

eg $f(x) = \ln(6x - x^2) + \ln 27 - \ln 9$

correct use of a log law ***(A1)***

eg $f(x) = \ln(6x - x^2) + \ln\left(\frac{27}{9}\right),\ f(x) = \ln\left(27(6x - x^2)\right) - \ln 9$

$f(x) = \ln\left(3(6x - x^2)\right)$ ***A1*** ***N4***

[8 marks]

19c. The graph of f is transformed by a vertical stretch with scale factor $\frac{1}{\ln 3}$. The image of P under this transformation has coordinates $(a,\ b)$.

Find the value of a and of b, where $a,\ b \in \mathbb{N}$.

Markscheme

$a = 3$ ***A1*** ***N1***

correct working ***A1***

eg $\frac{\ln 27}{\ln 3}$

correct use of log law ***(A1)***

eg $\frac{3 \ln 3}{\ln 3},\ \log_3 27$

$b = 3$ ***A1*** ***N2***

[4 marks]

Let $x = \ln 3$ and $y = \ln 5$. Write the following expressions in terms of x and y.

20a. $\ln\left(\frac{5}{3}\right)$. *[2 marks]*

Markscheme

correct approach ***(A1)***

eg $\ln 5 - \ln 3$

$\ln\left(\frac{5}{3}\right) = y - x$ ***A1*** ***N2***

[2 marks]

20b. $\ln 45$. *[4 marks]*

Markscheme

recognizing factors of 45 (may be seen in log expansion) ***(M1)***

eg $\ln(9 \times 5),\ 3 \times 3 \times 5,\ \log 3^2 \times \log 5$

correct application of $\log(ab) = \log a + \log b$ ***(A1)***

eg $\ln 9 + \ln 5,\ \ln 3 + \ln 3 + \ln 5,\ \ln 3^2 + \ln 5$

correct working ***(A1)***

eg $2 \ln 3 + \ln 5,\ x + x + y$

$\ln 45 = 2x + y$ ***A1*** ***N3***

[4 marks]

Let $f(x) = 6x\sqrt{1-x^2}$, for $-1 \leqslant x \leqslant 1$, and $g(x) = \cos(x)$, for $0 \leqslant x \leqslant \pi$.

Let $h(x) = (f \circ g)(x)$.

21a. Write $h(x)$ in the form $a\sin(bx)$, where $a,\ b \in \mathbb{Z}$. *[5 marks]*

Markscheme

attempt to form composite in any order ***(M1)***

eg $f(g(x))$, $\cos\left(6x\sqrt{1-x^2}\right)$

correct working ***(A1)***

eg $6\cos x\sqrt{1-\cos^2 x}$

correct application of Pythagorean identity (do not accept $\sin^2 x + \cos^2 x = 1$) ***(A1)***

eg $\sin^2 x = 1 - \cos^2 x,\ 6\cos x \sin x,\ 6\cos x\left|\sin x\right|$

valid approach (do not accept $2\sin x\cos x = \sin 2x$) ***(M1)***

eg $3(2\cos x\sin x)$

$h(x) = 3\sin 2x$ ***A1*** ***N3***

[5 marks]

21b. Hence find the range of h. *[2 marks]*

Markscheme

valid approach ***(M1)***

eg amplitude $= 3$, sketch with max and min y-values labelled, $-3 < y < 3$

correct range ***A1*** ***N2***

eg $-3 \leqslant y \leqslant 3,\ [-3,\ 3]$ from -3 to 3

Note: Do not award ***A1*** for $-3 < y < 3$ or for “between -3 and 3”.

[2 marks]

22. An arithmetic sequence has the first term $\ln a$ and a common difference $\ln 3$. *[6 marks]*

The 13th term in the sequence is $8\ln 9$. Find the value of a.

Markscheme

Note: There are many approaches to this question, and the steps may be done in any order. There are 3 relationships they may need to apply at some stage, for the 3rd, 4th and 5th marks. These are

equating bases *eg* recognising 9 is 3^2

log rules: $\ln b + \ln c = \ln(bc),\ \ln b - \ln c = \ln\left(\frac{b}{c}\right),$

exponent rule: $\ln b^n = n \ln b$.

The exception to the ***FT*** rule applies here, so that if they demonstrate correct application of the 3 relationships, they may be awarded the ***A*** marks, even if they have made a previous error. However all applications of a relationship need to be correct. Once an error has been made, do not award ***A1FT*** for their final answer, even if it follows from their working.

Please check working and award marks in line with the markscheme.

correct substitution into u_{13} formula ***(A1)***

eg $\ln a + (13 - 1)\ln 3$

set up equation for u_{13} in any form (seen anywhere) ***(M1)***

eg $\ln a + 12 \ln 3 = 8 \ln 9$

correct application of relationships ***(A1)(A1)(A1)***

$a = 81$ ***A1*** ***N3***

[6 marks]

Examples of application of relationships

Example 1

correct application of exponent rule for logs ***(A1)***

eg $\ln a + \ln 3^{12} = \ln 9^8$

correct application of addition rule for logs ***(A1)***

eg $\ln(a3^{12}) = \ln 9^8$

substituting for 9 or 3 in ln expression in equation ***(A1)***

eg $\ln(a3^{12}) = \ln 3^{16},\ \ln(a9^6) = \ln 9^8$

Example 2

recognising $9 = 3^2$ ***(A1)***

eg $\ln a + 12 \ln 3 = 8 \ln 3^2,\ \ln a + 12 \ln 9^{\frac{1}{2}} = 8 \ln 9$

one correct application of exponent rule for logs relating $\ln 9$ to $\ln 3$ ***(A1)***

eg $\ln a + 12 \ln 3 = 16 \ln 3,\ \ln a + 6 \ln 9 = 8 \ln 9$

another correct application of exponent rule for logs ***(A1)***

eg $\ln a = \ln 3^4,\ \ln a = \ln 9^2$

23a. Given that $2^m = 8$ and $2^n = 16$, write down the value of m and of n. *[2 marks]*

Markscheme

$m = 3,\ n = 4$ ***A1A1*** ***N2***

[2 marks]

23b. Hence or otherwise solve $8^{2x+1} = 16^{2x-3}$. *[4 marks]*

Markscheme

attempt to apply $(2^a)^b = 2^{ab}$ ***(M1)***

eg $6x + 3,\ 4(2x - 3)$

equating **their** powers of 2 (seen anywhere) ***M1***

eg $3(2x + 1) = 8x - 12$

correct working ***A1***

eg $8x - 12 = 6x + 3,\ 2x = 15$

$x = \frac{15}{2}\quad(7.5)$ ***A1*** ***N2***

[4 marks]

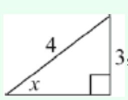
Total [6 marks]

Given that $\sin x = \frac{3}{4}$, where x is an obtuse angle,

24a. find the value of $\cos x$; *[4 marks]*

Markscheme

valid approach ***(M1)***

eg 

$3,\ \sin^2 x + \cos^2 x = 1$

correct working ***(A1)***

eg $4^2 - 3^2,\ \cos^2 x = 1 - \left(\frac{3}{4}\right)^2$

correct calculation ***(A1)***

eg $\frac{\sqrt{7}}{4},\ \cos^2 x = \frac{7}{16}$

$\cos x = -\frac{\sqrt{7}}{4}$ ***A1*** ***N3***

[4 marks]

24b. find the value of $\cos 2x$. *[3 marks]*

Markscheme

correct substitution (accept missing minus with cos) ***(A1)***

eg $1-2\left(\frac{3}{4}\right)^2,\ 2\left(-\frac{\sqrt{7}}{4}\right)^2-1,\ \left(\frac{\sqrt{7}}{4}\right)^2-\left(\frac{3}{4}\right)^2$

correct working ***A1***

eg $2\left(\frac{7}{16}\right)-1,\ 1-\frac{18}{16},\ \frac{7}{16}-\frac{9}{16}$

$\cos 2x=-\frac{2}{16}\quad\left(=-\frac{1}{8}\right)$ ***A1*** ***N2***

[3 marks]

Total [7 marks]

Let $f(x)=x^2+x-6$.

25a. Write down the y-intercept of the graph of f. *[1 mark]*

Markscheme

y-intercept is $-6,\ (0,\ -6),\ y=-6$ ***A1***

[1 mark]

25b. Solve $f(x)=0$. *[3 marks]*

Markscheme

valid attempt to solve ***(M1)***

eg $(x-2)(x+3)=0,\ x=\frac{-1\pm\sqrt{1+24}}{2}$, one correct answer

$x=2,\ x=-3$ ***A1A1*** ***N3***

[3 marks]

26a. Write the expression $3\ln 2-\ln 4$ in the form $\ln k$, where $k\in\mathbb{Z}$. *[3 marks]*

Markscheme

correct application of $\ln a^b = b \ln a$ (seen anywhere) ***(A1)***

eg $\ln 4 = 2\ln 2,\ 3\ln 2 = \ln 2^3,\ 3\log 2 = \log 8$

correct working ***(A1)***

eg $3\ln 2 - 2\ln 2,\ \ln 8 - \ln 4$

$\ln 2 \quad (\text{accept } k = 2)$ ***A1 N2***

[3 marks]

26b. Hence or otherwise, solve $3\ln 2 - \ln 4 = -\ln x$. *[3 marks]*

Markscheme

METHOD 1

attempt to substitute **their** answer into the equation ***(M1)***

eg $\ln 2 = -\ln x$

correct application of a log rule ***(A1)***

eg $\ln \frac{1}{x},\ \ln \frac{1}{2} = \ln x,\ \ln 2 + \ln x = \ln 2x \quad (= 0)$

$x = \frac{1}{2}$ ***A1 N2***

METHOD 2

attempt to rearrange equation, with $3\ln 2$ written as $\ln 2^3$ or $\ln 8$ ***(M1)***

eg $\ln x = \ln 4 - \ln 2^3,\ \ln 8 + \ln x = \ln 4,\ \ln 2^3 = \ln 4 - \ln x$

correct working applying $\ln a \pm \ln b$ ***(A1)***

eg $\frac{4}{8},\ 8x = 4,\ \ln 2^3 = \ln \frac{4}{x}$

$x = \frac{1}{2}$ ***A1 N2***

[3 marks]

Total [6 marks]

Let
$f(x) = a(x-h)^2 + k$. The vertex of the graph of
f is at
$(2, 3)$ and the graph passes through
$(1, 7)$.

27a. Write down the value of h and of k. *[2 marks]*

Markscheme

$h = 2,\ k = 3$ ***A1A1*** ***N2***

[2 marks]

27b. Find the value of a. *[3 marks]*

Markscheme

attempt to substitute
$(1, 7)$ in any order into **their**
$f(x)$ ***(M1)***

eg $7 = a(1-2)^2 + 3, 7 = a(1-3)^2 + 2, 1 = a(7-2)^2 + 3$

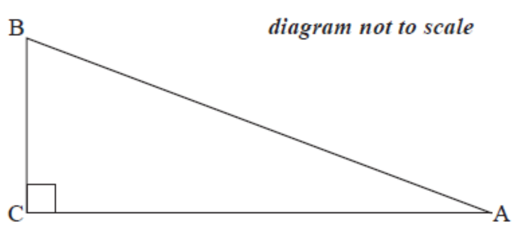
correct equation ***(A1)***

eg $7 = a + 3$

a = 4 ***A1*** ***N2***

[3 marks]

The following diagram shows a right-angled triangle,
ABC, where
$\sin \mathrm{A} = \frac{5}{13}$.



28a. Show that $\cos A = \frac{12}{13}$. *[2 marks]*

Markscheme

METHOD 1

approach involving Pythagoras' theorem ***(M1)***

eg $5^2 + x^2 = 13^2$, labelling correct sides on triangle

finding third side is 12 (may be seen on diagram) ***A1***

$\cos A = \frac{12}{13}$ ***AG*** ***N0***

METHOD 2

approach involving $\sin^2\theta + \cos^2\theta = 1$ ***(M1)***

eg $\left(\frac{5}{13}\right)^2 + \cos^2\theta = 1,\ x^2 + \frac{25}{169} = 1$

correct working ***A1***

eg $\cos^2\theta = \frac{144}{169}$

$\cos A = \frac{12}{13}$ ***AG*** ***N0***

[2 marks]

28b. Find $\cos 2A$. *[3 marks]*

Markscheme

correct substitution into $\cos 2\theta$ ***(A1)***

eg $1 - 2\left(\frac{5}{13}\right)^2,\ 2\left(\frac{12}{13}\right)^2 - 1,\ \left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2$

correct working ***(A1)***

eg $1 - \frac{50}{169},\ \frac{288}{169} - 1,\ \frac{144}{169} - \frac{25}{169}$

$\cos 2A = \frac{119}{169}$ ***A1*** ***N2***

[3 marks]

Find the value of each of the following, giving your answer as an integer.

29a. $\log_6 36$ *[2 marks]*

Markscheme

correct approach ***(A1)***

eg $6^x = 36,\ 6^2$

2 ***A1*** ***N2***

[2 marks]

29b. $\log_6 4 + \log_6 9$ *[2 marks]*

Markscheme

correct simplification ***(A1)***

eg $\log_6 36,\ \log(4 \times 9)$

2 ***A1 N2***

[2 marks]

29c. $\log_6 2 - \log_6 12$ *[3 marks]*

Markscheme

correct simplification ***(A1)***

eg $\log_6 \frac{2}{12},\ \log(2 \div 12)$

correct working ***(A1)***

eg $\log_6 \frac{1}{6},\ 6^{-1} = \frac{1}{6}, 6^x = \frac{1}{6}$

-1 ***A1 N2***

[3 marks]