

ExpLogTrigReview [213 marks]

Let $b = \log_2 a$, where $a > 0$. Write down each of the following expressions in terms of b .

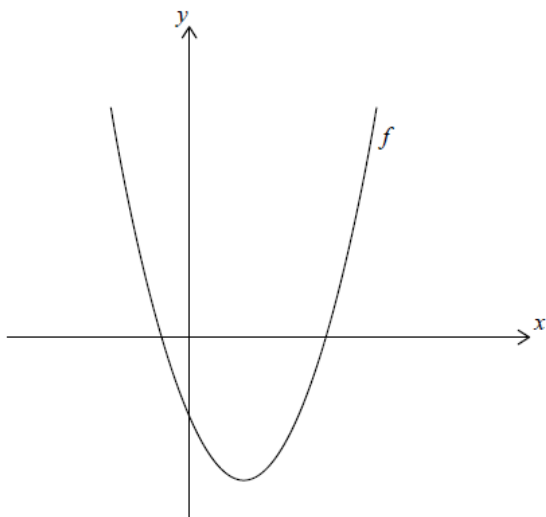
1a. $\log_2 a^3$ [2 marks]

1b. $\log_2 8a$ [2 marks]

1c. $\log_8 a$ [2 marks]

2. Given that $\sin x = \frac{1}{3}$, where $0 < x < \frac{\pi}{2}$, find the value of $\cos 4x$. [6 marks]

Let $f(x) = x^2 - 4x - 5$. The following diagram shows part of the graph of f .



3a. Find the x -intercepts of the graph of f . [5 marks]

3b. Find the equation of the axis of symmetry of the graph of f . [2 marks]

The function can be written in the form $f(x) = (x - h)^2 + k$.

3c. Write down the value of h . [1 mark]

3d. Find the value of k . [3 marks]

- 3e. The graph of a second function, g , is obtained by a reflection of the graph of f in the y -axis, followed by a translation of $\begin{pmatrix} -3 \\ 6 \end{pmatrix}$. [5 marks]

Find the coordinates of the vertex of the graph of g .

The first two terms of an infinite geometric sequence are $u_1 = 18$ and $u_2 = 12\sin^2 \theta$, where $0 < \theta < 2\pi$, and $\theta \neq \pi$.

- 4a. Find an expression for r in terms of θ . [2 marks]

- 4b. Find the possible values of r . [3 marks]

- 4c. Show that the sum of the infinite sequence is $\frac{54}{2+\cos(2\theta)}$. [4 marks]

- 4d. Find the values of θ which give the greatest value of the sum. [6 marks]

Let $f(x) = ax^2 - 4x - c$. A horizontal line, L , intersects the graph of f at $x = -1$ and $x = 3$.

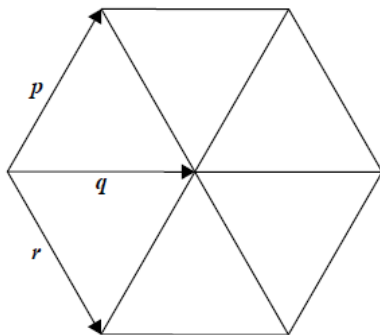
- 5a. The equation of the axis of symmetry is $x = p$. Find p . [2 marks]

- 5b. Hence, show that $a = 2$. [2 marks]

- 5c. The equation of L is $y = 5$. Find the value of c . [3 marks]

6. Six equilateral triangles, each with side length 3 cm, are arranged to form a hexagon. [6 marks]
This is shown in the following diagram.

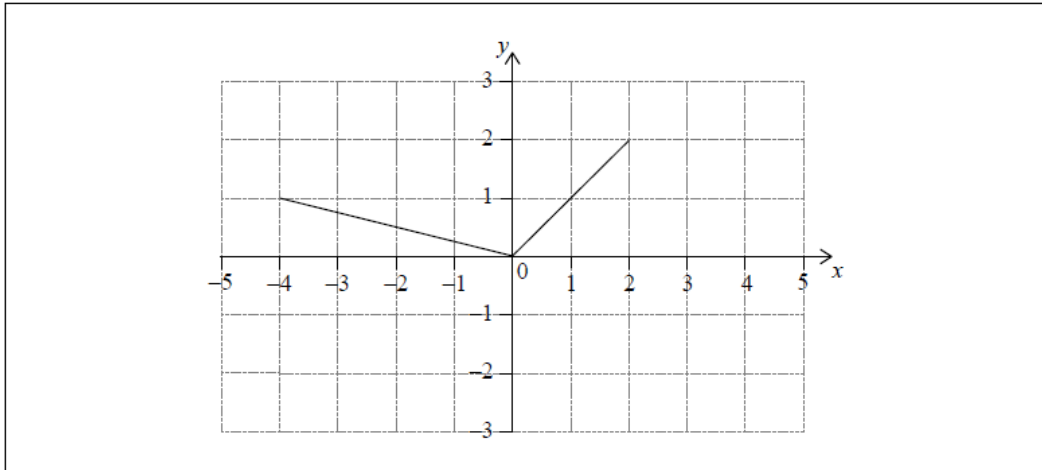
diagram not to scale



The vectors p , q and r are shown on the diagram.

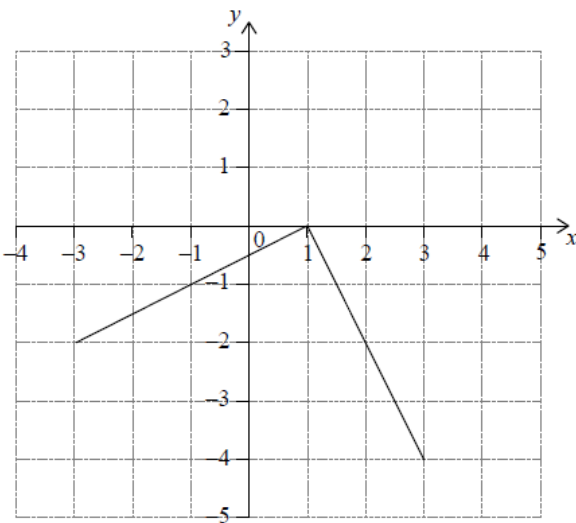
Find $p \cdot (p + q + r)$.

The following diagram shows the graph of a function f , for $-4 \leq x \leq 2$.



7a. On the same axes, sketch the graph of $f(-x)$. [2 marks]

7b. Another function, g , can be written in the form $g(x) = a \times f(x + b)$. The following diagram shows the graph of g . [4 marks]



Write down the value of a and of b .

8. Let $f(x) = px^2 + qx - 4p$, where $p \neq 0$. Find the number of roots for the equation $f(x) = 0$. [7 marks]

Justify your answer.

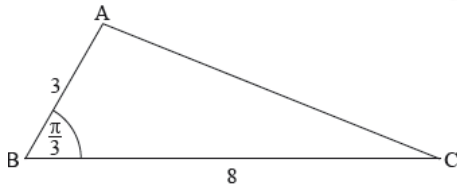
An arithmetic sequence has $u_1 = \log_c(p)$ and $u_2 = \log_c(pq)$, where $c > 1$ and $p, q > 0$.

9a. Show that $d = \log_c(q)$. [2 marks]

9b. Let $p = c^2$ and $q = c^3$. Find the value of $\sum_{n=1}^{20} u_n$. [6 marks]

The following diagram shows triangle ABC, with $AB = 3$ cm, $BC = 8$ cm, and $\hat{ABC} = \frac{\pi}{3}$.

diagram not to scale



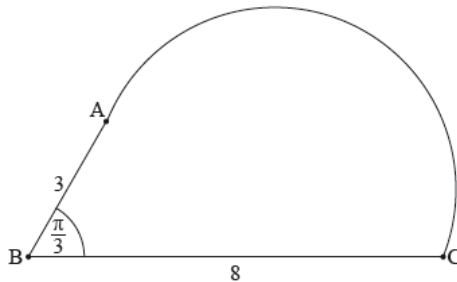
10a. Show that $AC = 7$ cm.

[4 marks]

10b. The shape in the following diagram is formed by adding a semicircle with diameter [AC] to the triangle.

[3 marks]

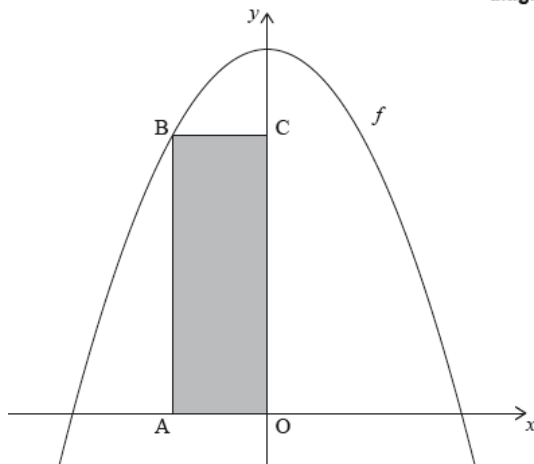
diagram not to scale



Find the exact perimeter of this shape.

11. Let $f(x) = 15 - x^2$, for $x \in \mathbb{R}$. The following diagram shows part of the graph of f and the rectangle OABC, where A is on the negative x -axis, B is on the graph of f , and C is on the y -axis.

diagram not to scale



Find the x -coordinate of A that gives the maximum area of OABC.

12. The following diagram shows triangle PQR.

[6 marks]

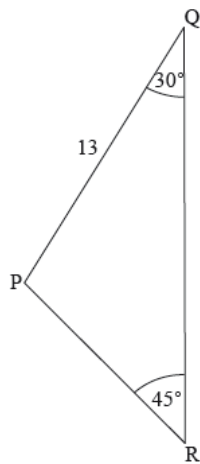


diagram not to scale

$\hat{PQR} = 30^\circ$, $\hat{QRP} = 45^\circ$ and $PQ = 13$ cm.

Find PR.

The first three terms of a geometric sequence are $\ln x^{16}$, $\ln x^8$, $\ln x^4$, for $x > 0$.

13a. Find the common ratio.

[3 marks]

13b.

Solve $\sum_{k=1}^{\infty} 2^{5-k} \ln x = 64$.

[5 marks]

14. Solve $\log_2(2 \sin x) + \log_2(\cos x) = -1$, for $2\pi < x < \frac{5\pi}{2}$.

[7 marks]

Let $\sin \theta = \frac{\sqrt{5}}{3}$, where θ is acute.

15a. Find $\cos \theta$.

[3 marks]

15b. Find $\cos 2\theta$.

[2 marks]

The first two terms of an infinite geometric sequence, in order, are $2\log_2 x$, $\log_2 x$, where $x > 0$.

16a. Find r .

[2 marks]

16b. Show that the sum of the infinite sequence is $4\log_2 x$.

[2 marks]

The first three terms of an arithmetic sequence, in order, are

$\log_2 x$, $\log_2\left(\frac{x}{2}\right)$, $\log_2\left(\frac{x}{4}\right)$, where $x > 0$.

16c. Find d , giving your answer as an integer.

[4 marks]

Let S_{12} be the sum of the first 12 terms of the arithmetic sequence.

16d. Show that $S_{12} = 12\log_2 x - 66$.

[2 marks]

16e. Given that S_{12} is equal to half the sum of the infinite geometric sequence, find x , giving your answer in the form 2^p , where $p \in \mathbb{Q}$.

[3 marks]

Let $f(x) = 3 \sin\left(\frac{\pi}{2}x\right)$, for $0 \leq x \leq 4$.

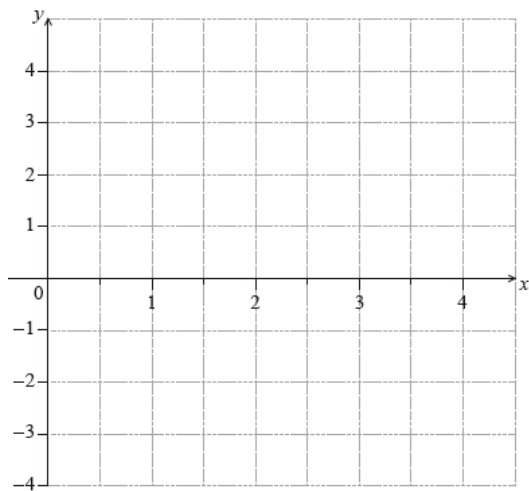
17a. (i) Write down the amplitude of f .

[3 marks]

(ii) Find the period of f .

17b. On the following grid sketch the graph of f .

[4 marks]



Consider $f(x) = x^2 + qx + r$. The graph of f has a minimum value when $x = -1.5$.

The distance between the two zeros of f is 9.

18a. Show that the two zeros are 3 and -6 .

[2 marks]

18b. Find the value of q and of r .

[4 marks]

Let $f'(x) = \frac{6-2x}{6x-x^2}$, for $0 < x < 6$.

The graph of f has a maximum point at P.

19a. Find the x -coordinate of P. [3 marks]

The y -coordinate of P is $\ln 27$.

19b. Find $f(x)$, expressing your answer as a single logarithm. [8 marks]

19c. The graph of f is transformed by a vertical stretch with scale factor $\frac{1}{\ln 3}$. The image of P under this transformation has coordinates (a, b) .

Find the value of a and of b , where $a, b \in \mathbb{N}$.

Let $x = \ln 3$ and $y = \ln 5$. Write the following expressions in terms of x and y .

20a. $\ln\left(\frac{5}{3}\right)$. [2 marks]

20b. $\ln 45$. [4 marks]

Let $f(x) = 6x\sqrt{1-x^2}$, for $-1 \leq x \leq 1$, and $g(x) = \cos(x)$, for $0 \leq x \leq \pi$.

Let $h(x) = (f \circ g)(x)$.

21a. Write $h(x)$ in the form $a \sin(bx)$, where $a, b \in \mathbb{Z}$. [5 marks]

21b. Hence find the range of h . [2 marks]

22. An arithmetic sequence has the first term $\ln a$ and a common difference $\ln 3$. [6 marks]

The 13th term in the sequence is $8 \ln 9$. Find the value of a .

23a. Given that $2^m = 8$ and $2^n = 16$, write down the value of m and of n . [2 marks]

23b. Hence or otherwise solve $8^{2x+1} = 16^{2x-3}$. [4 marks]

Given that $\sin x = \frac{3}{4}$, where x is an obtuse angle,

24a. find the value of $\cos x$; [4 marks]

24b. find the value of $\cos 2x$. [3 marks]

Let $f(x) = x^2 + x - 6$.

25a. Write down the y -intercept of the graph of f . [1 mark]

25b. Solve $f(x) = 0$. [3 marks]

26a. Write the expression $3 \ln 2 - \ln 4$ in the form $\ln k$, where $k \in \mathbb{Z}$. [3 marks]

26b. Hence or otherwise, solve $3 \ln 2 - \ln 4 = -\ln x$. [3 marks]

Let

$f(x) = a(x - h)^2 + k$. The vertex of the graph of f is at

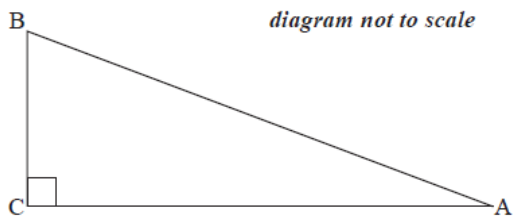
$(2, 3)$ and the graph passes through

$(1, 7)$.

27a. Write down the value of h and of k . [2 marks]

27b. Find the value of a . [3 marks]

The following diagram shows a right-angled triangle, ABC, where $\sin A = \frac{5}{13}$.



28a. Show that $\cos A = \frac{12}{13}$. [2 marks]

28b. Find $\cos 2A$. [3 marks]

Find the value of each of the following, giving your answer as an integer.

29a. $\log_6 36$

[2 marks]

29b. $\log_6 4 + \log_6 9$

[2 marks]

29c. $\log_6 2 - \log_6 12$

[3 marks]

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