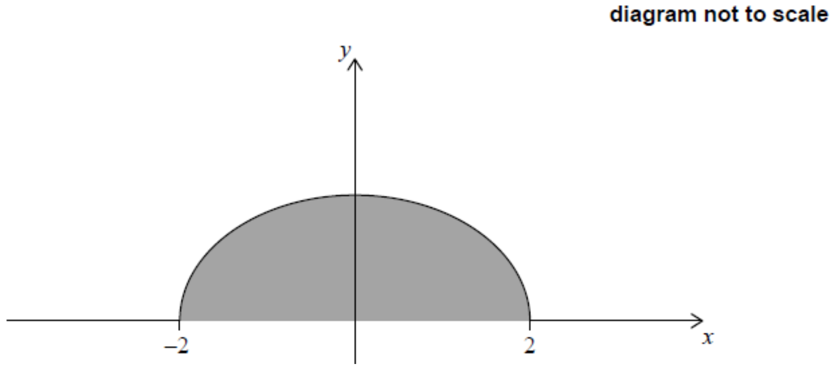
# DefiniteIntegrals *[260 marks]*

**All lengths in this question are in metres.**

Consider the function $f(x) = \sqrt{\frac{4-x^2}{8}}$, for $-2 \le x \le 2$. In the following diagram, the shaded region is enclosed by the graph of $f$ and the $x$-axis.




A container can be modelled by rotating this region by 360˚ about the $x$-axis.

1a. Find the volume of the container. *[3 marks]*

# Markscheme

attempt to substitute correct limits or the function into formula involving $f^2$ ***(M1)***

*eg* $\pi\int_{-2}^{2} y^2 \, dy, \ \pi\int\left(\sqrt{\frac{4-x^2}{8}}\right)^2 dx$

4.18879

volume = 4.19, $\frac{4}{3}\pi$ (exact) (m$^3$) ***A2 N3***

**Note:** If candidates have their GDC incorrectly set in degrees, award ***M*** marks where appropriate, but no ***A*** marks may be awarded. Answers from degrees are *p* = 13.1243 and *q* = 26.9768 in (b)(i) and 12.3130 or 28.3505 in (b)(ii).

***[3 marks]***

Water can flow in and out of the container.

The volume of water in the container is given by the function $g(t)$, for $0 \leq t \leq 4$, where $t$ is measured in hours and $g(t)$ is measured in $m^3$. The rate of change of the volume of water in the container is given by $g'(t) = 0.9 - 2.5\cos(0.4t^2)$.

The volume of water in the container is increasing only when $p < t < q$.

1b. Find the value of $p$ and of $q$. *[3 marks]*

## Markscheme

recognizing the volume increases when $g'$ is positive ***(M1)***

*eg* $g'(t) > 0$, sketch of graph of $g'$ indicating correct interval

1.73387, 3.56393

$p = 1.73$, $p = 3.56$ ***A1A1 N3***

***[3 marks]***

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1c. During the interval $p < t < q$, he volume of water in the container increases by $k$ $m^3$. Find the value of $k$. *[3 marks]*

## Markscheme

valid approach to find change in volume ***(M1)***

*eg* $g(q) - g(p)$, $\int_p^q g'(t)\,dt$

3.74541

total amount = 3.75 ($m^3$) ***A2 N3***

***[3 marks]***

1d. When $t = 0$, the volume of water in the container is 2.3 $m^3$. It is known that the container is never completely full of water during the 4 hour period. *[5 marks]*

Find the minimum volume of empty space in the container during the 4 hour period.

## Markscheme

**Note:** There may be slight differences in the final answer, depending on which values candidates carry through from previous parts. Accept answers that are consistent with correct working.

recognizing when the volume of water is a maximum ***(M1)***

*eg* maximum when $t = q$, $\int_0^q g'(t)\,\mathrm{d}t$

valid approach to find maximum volume of water ***(M1)***

*eg* $2.3 + \int_0^q g'(t)\,\mathrm{d}t,\ 2.3 + \int_0^p g'(t)\,\mathrm{d}t + 3.74541,$ 3.85745

correct expression for the difference between volume of container and maximum value ***(A1)***

*eg* $4.18879 - \left(2.3 + \int_0^q g'(t)\,\mathrm{d}t\right),$ 4.19 – 3.85745

0.331334

0.331 ($m^3$) ***A2 N3***

***[5 marks]***

Let $f(x) = \frac{1}{\sqrt{2x-1}}$, for $x > \frac{1}{2}$.

2a. Find $\int (f(x))^2 \mathrm{d}x$. *[3 marks]*

## Markscheme

correct working ***(A1)***

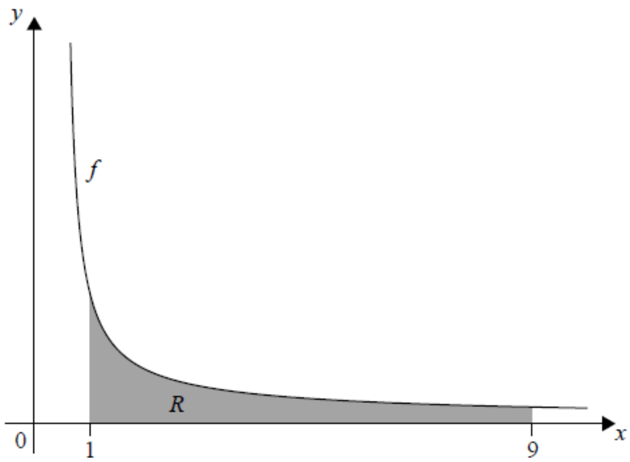
eg $\int \frac{1}{2x-1}\mathrm{d}x,\ \int (2x-1)^{-1},\ \frac{1}{2x-1},\ \int \left(\frac{1}{\sqrt{u}}\right)^2 \frac{\mathrm{d}u}{2}$

$\int (f(x))^2 \mathrm{d}x = \frac{1}{2}\ln(2x-1) + c$ ***A2 N3***

**Note:** Award ***A1*** for $\frac{1}{2}\ln(2x-1)$.

***[3 marks]***

2b. Part of the graph of $f$ is shown in the following diagram. *[4 marks]*




The shaded region $R$ is enclosed by the graph of $f$, the $x$-axis, and the lines $x = 1$ and $x = 9$.
Find the volume of the solid formed when $R$ is revolved 360° about the $x$-axis.

# Markscheme

attempt to substitute either limits or the function into formula involving $f^2$ (accept absence of $\pi$ / d$x$) ***(M1)***

*eg* $\int_1^9 y^2 \mathrm{d}x,\ \pi\int\left(\frac{1}{\sqrt{2x-1}}\right)^2 \mathrm{d}x,\ \left[\frac{1}{2}\ln(2x-1)\right]_1^9$

substituting limits into **their** integral and subtracting (in any order) ***(M1)***

*eg* $\frac{\pi}{2}(\ln(17) - \ln(1)),\ \pi\left(0 - \frac{1}{2}\ln(2\times 9 - 1)\right)$

correct working involving calculating a log value or using log law ***(A1)***

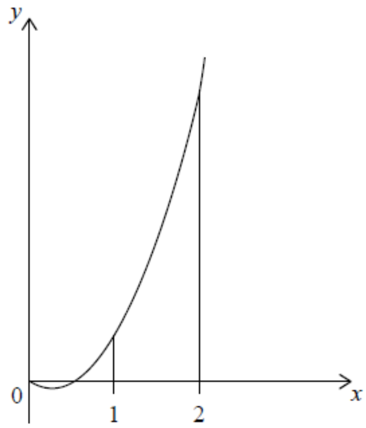
*eg* $\ln(1) = 0,\ \ln\left(\frac{17}{1}\right)$

$\frac{\pi}{2}\ln 17 \quad \left(\text{accept } \pi\ln\sqrt{17}\right)$ ***A1 N3***

**Note:** Full ***FT*** may be awarded as normal, from their incorrect answer in part (a), however, do not award the final two ***A*** marks unless they involve logarithms.

***[4 marks]***

Let $f(x) = 6x^2 - 3x$. The graph of $f$ is shown in the following diagram.




3a. Find $\int (6x^2 - 3x)\,dx$. *[2 marks]*

## Markscheme

$2x^3 - \frac{3x^2}{2} + c$ $\left(\text{accept } \frac{6x^3}{3} - \frac{3x^2}{2} + c\right)$ ***A1A1 N2***

**Notes:** Award ***A1A0*** for both correct terms if $+c$ is omitted.
Award ***A1A0*** for one correct term *eg* $2x^3 + c$.
Award ***A1A0*** if both terms are correct, but candidate attempts further working to solve for $c$.

***[2 marks]***

3b. Find the area of the region enclosed by the graph of $f$, the $x$-axis and the lines $x = 1$ and $x = 2$. *[4 marks]*

## Markscheme

substitution of limits or function ***(A1)***

*eg* $\int_1^2 f(x)\,dx$, $\left[2x^3 - \frac{3x^2}{2}\right]_1^2$

substituting limits into their integrated function and subtracting ***(M1)***

*eg* $\frac{6 \times 2^3}{3} - \frac{3 \times 2^2}{2} - \left(\frac{6 \times 1^3}{3} + \frac{3 \times 1^2}{2}\right)$

**Note:** Award ***M0*** if substituted into original function.

correct working ***(A1)***

*eg* $\frac{6 \times 8}{3} - \frac{3 \times 4}{2} - \frac{6 \times 1}{3} + \frac{3 \times 1}{2}$, $(16 - 6) - \left(2 - \frac{3}{2}\right)$

$\frac{19}{2}$ ***A1 N3***

***[4 marks]***

Let $g(x) = -(x - 1)^2 + 5$.

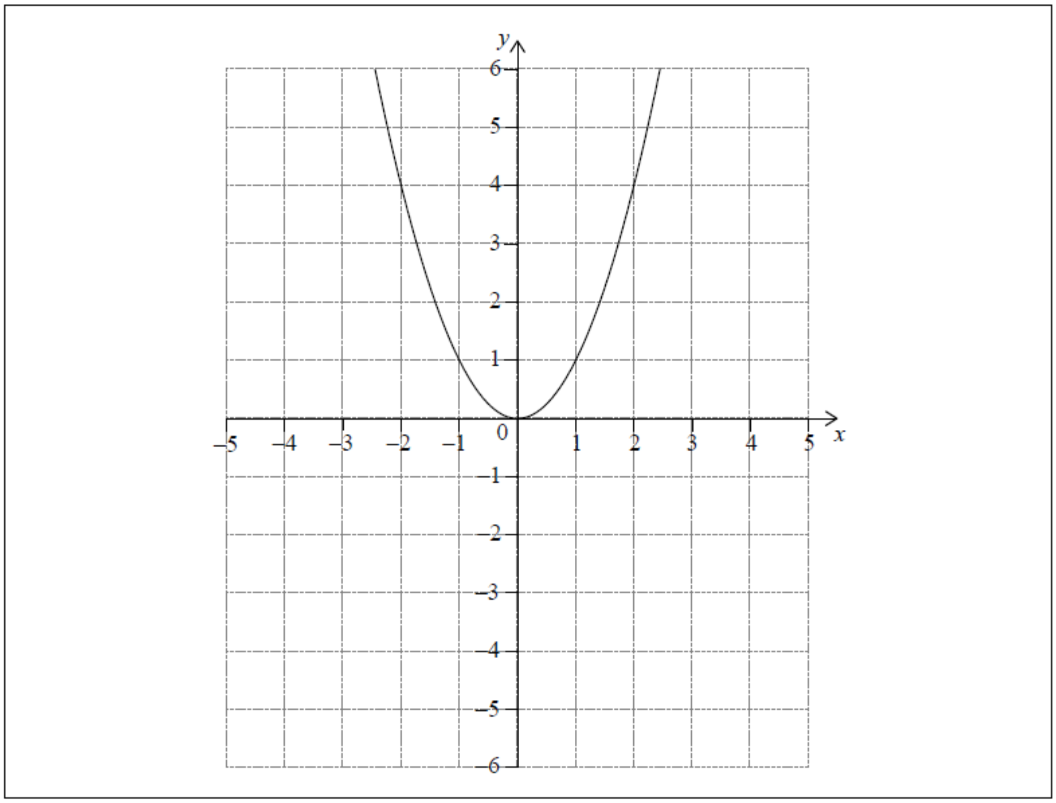
4a. Write down the coordinates of the vertex of the graph of $g$. *[1 mark]*

# Markscheme

(1,5) (exact) ***A1 N1***

***[1 mark]***

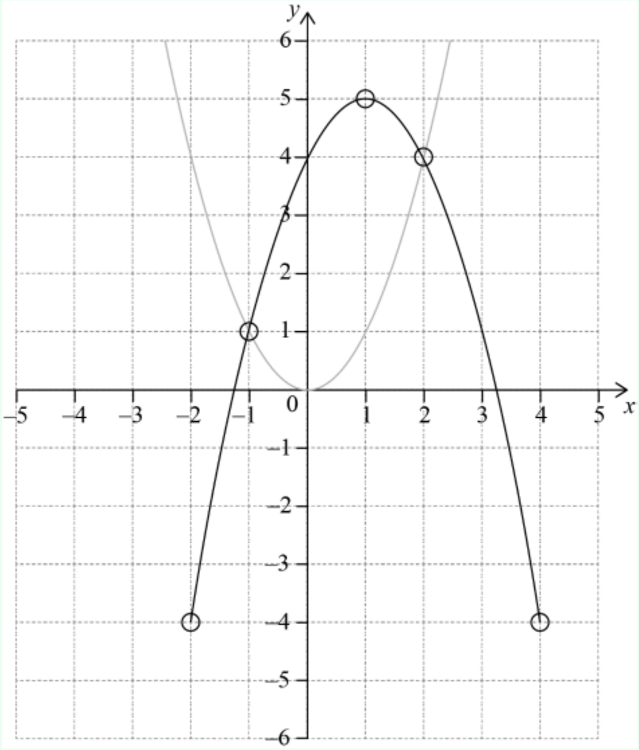
Let $f(x) = x^2$. The following diagram shows part of the graph of $f$.




The graph of $g$ intersects the graph of $f$ at $x = -1$ and $x = 2$.

4b. On the grid above, sketch the graph of g for $-2 \leq x \leq 4$. *[3 marks]*

# Markscheme




***A1A1A1 N3***

**Note:** The shape must be a concave-down parabola.
Only if the shape is correct, award the following for points in circles:
***A1*** for vertex,
***A1*** for correct intersection points,
***A1*** for correct endpoints.

***[3 marks]***

4c. Find the area of the region enclosed by the graphs of *f* and *g*. *[3 marks]*

# Markscheme

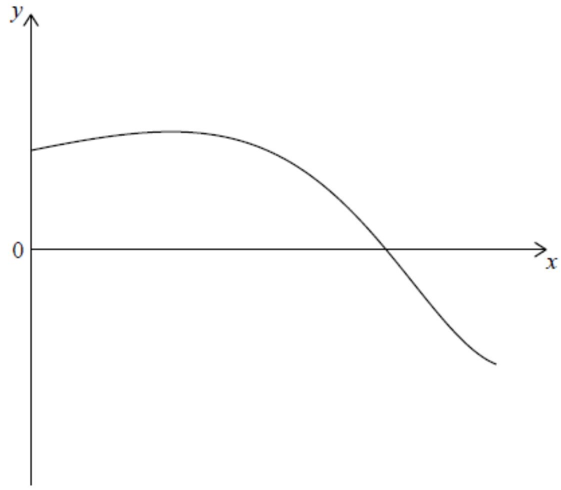
integrating and subtracting functions (in any order) ***(M1)***
*eg* $\int f - g$

correct substitution of limits or functions (accept missing d *x*, but do not accept any errors, including extra bits) ***(A1)***
eg $\int_{-1}^{2} g - f,\ \int -(x-1)^2 + 5 - x^2$

area = 9 (exact) ***A1 N2***

***[3 marks]***

Let $f(x) = \sin(e^x)$ for $0 \le x \le 1.5$. The following diagram shows the graph of $f$.




5a. Find the *x*-intercept of the graph of $f$. *[2 marks]*

# Markscheme

valid approach *(M1)*
*eg* $f(x) = 0,\ e^x = 180$ or $0\ldots$

1.14472

$x = \ln\pi$ (exact), 1.14 ***A1 N2***

***[2 marks]***

5b. The region enclosed by the graph of $f$, the *y*-axis and the *x*-axis is rotated 360° about the *x*-axis. *[3 marks]*

Find the volume of the solid formed.

# Markscheme

attempt to substitute either their **limits** or the function into formula involving $f^2$. ***(M1)***

*eg* $\int_0^{1.14} f^2,\ \pi\int(\sin(e^x))^2 dx,\ 0.795135$

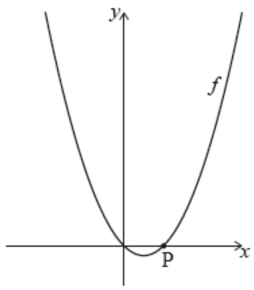
2.49799

volume = 2.50 ***A2 N3***

***[3 marks]***

Let $f(x) = x^2 - x$, for $x \in \mathbb{R}$. The following diagram shows part of the graph of $f$.

**diagram not to scale**




The graph of $f$ crosses the $x$-axis at the origin and at the point $\mathrm{P}(1,\ 0)$.

6a. Show that $f'(1) = 1$. *[3 marks]*

## Markscheme

$f'(x) = 2x - 1$ ***A1A1***

correct substitution ***A1***

*eg* $2(1) - 1,\ 2 - 1$

$f'(1) = 1$ ***AG*** ***N0***

***[3 marks]***

The line *L* is the normal to the graph of *f* at P.

6b. Find the equation of $L$ in the form $y = ax + b$. *[3 marks]*

## Markscheme

correct approach to find the gradient of the normal ***(A1)***

*eg* $\frac{-1}{f'(1)},\ m_1 m_2 = -1,\ \text{slope} = -1$

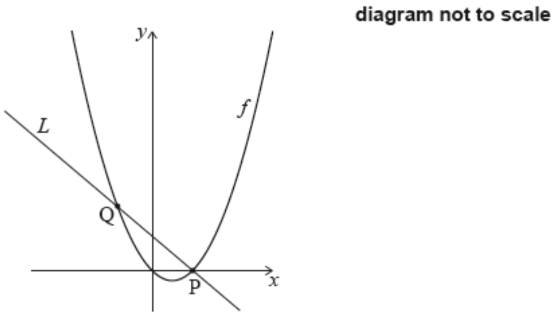
attempt to substitute correct normal gradient and coordinates into equation of a line ***(M1)***

*eg* $y - 0 = -1(x - 1),\ 0 = -1 + b,\ b = 1,\ L = -x + 1$

$y = -x + 1$ ***A1*** ***N2***

***[3 marks]***

The line $L$ intersects the graph of $f$ at another point Q, as shown in the following diagram.




6c. Find the $x$-coordinate of Q. *[4 marks]*

# Markscheme

equating expressions ***(M1)***

*eg* $f(x) = L, \ -x + 1 = x^2 - x$

correct working (must involve combining terms) ***(A1)***

*eg* $x^2 - 1 = 0, \ x^2 = 1, \ x = 1$

$x = -1 \quad (\text{accept } Q(-1, \ 2))$ ***A2 N3***

***[4 marks]***

6d. Find the area of the region enclosed by the graph of $f$ and the line $L$. *[6 marks]*

# Markscheme

valid approach ***(M1)***

*eg* $\int L - f, \ \int_{-1}^{1}(1 - x^2)\mathrm{d}x$, splitting area into triangles and integrals

correct integration ***(A1)(A1)***

*eg* $\left[x - \frac{x^3}{3}\right]_{-1}^{1}, \ -\frac{x^3}{3} - \frac{x^2}{2} + \frac{x^2}{2} + x$

substituting **their** limits into **their** integrated function and subtracting (in any order) ***(M1)***

*eg* $1 - \frac{1}{3} - \left(-1 - \frac{-1}{3}\right)$

**Note:** Award ***M0*** for substituting into original or differentiated function.

area $= \frac{4}{3}$ ***A2 N3***

***[6 marks]***

Let $f(x) = 6 - \ln(x^2 + 2)$, for $x \in \mathbb{R}$. The graph of $f$ passes through the point $(p,\ 4)$, where $p > 0$.

7a. Find the value of $p$. *[2 marks]*

# Markscheme

valid approach ***(M1)***

*eg* $f(p) = 4$, intersection with $y = 4,\ \pm 2.32$

2.32143

$p = \sqrt{e^2 - 2}$ (exact), 2.32 ***A1 N2***

***[2 marks]***

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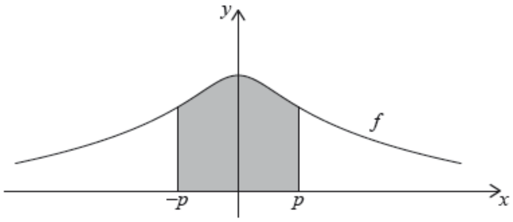
7b. The following diagram shows part of the graph of $f$. *[3 marks]*




The region enclosed by the graph of $f$, the $x$-axis and the lines $x = -p$ and $x = p$ is rotated 360° about the $x$-axis. Find the volume of the solid formed.

# Markscheme

attempt to substitute **either their** limits **or** the function into volume formula (must involve $f^2$, accept reversed limits and absence of $\pi$ and/or $dx$, but do not accept any other errors) ***(M1)***

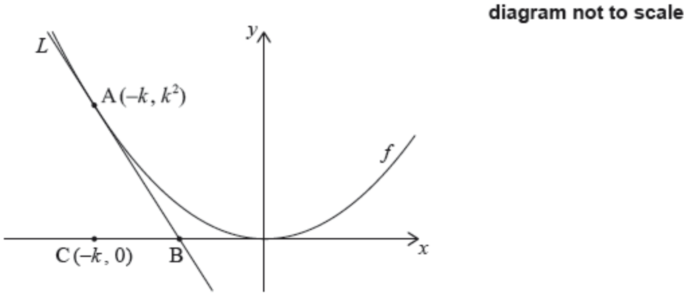
*eg* $\int_{-2.32}^{2.32} f^2,\ \pi \int \left(6 - \ln(x^2 + 2)\right)^2 dx,\ 105.675$

331.989

$\text{volume} = 332$ ***A2 N3***

***[3 marks]***

Let $f(x) = x^2$. The following diagram shows part of the graph of $f$.




The line $L$ is the tangent to the graph of $f$ at the point $\mathrm{A}(-k,\ k^2)$, and intersects the $x$-axis at point B. The point C is $(-k,\ 0)$.

8a. Write down $f'(x)$. *[1 mark]*

# Markscheme

$f'(x) = 2x$ ***A1*** ***N1***

***[1 mark]***

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8b. Find the gradient of $L$. *[2 marks]*

# Markscheme

attempt to substitute $x = -k$ into their derivative ***(M1)***

gradient of $L$ is $-2k$ ***A1*** ***N2***

***[2 marks]***

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8c. Show that the $x$-coordinate of B is $-\frac{k}{2}$. *[5 marks]*

# Markscheme

**METHOD 1**

attempt to substitute coordinates of A and their gradient into equation of a line ***(M1)***

*eg* $k^2 = -2k(-k) + b$

correct equation of $L$ in any form ***(A1)***

*eg* $y - k^2 = -2k(x + k),\ y = -2kx - k^2$

valid approach ***(M1)***

*eg* $y = 0$

correct substitution into $L$ equation ***A1***

*eg* $-k^2 = -2kx - 2k^2,\ 0 = -2kx - k^2$

correct working ***A1***

*eg* $2kx = -k^2$

$x = -\frac{k}{2}$ ***AG*** ***N0***

**METHOD 2**

valid approach ***(M1)***

*eg* $\text{gradient} = \frac{y_2 - y_1}{x_2 - x_1},\ -2k = \frac{\text{rise}}{\text{run}}$

recognizing $y = 0$ at B ***(A1)***

attempt to substitute coordinates of A and B into slope formula ***(M1)***

*eg* $\frac{k^2 - 0}{-k - x},\ \frac{-k^2}{x + k}$

correct equation ***A1***

*eg* $\frac{k^2 - 0}{-k - x} = -2k,\ \frac{-k^2}{x + k} = -2k,\ -k^2 = -2k(x + k)$

correct working ***A1***

*eg* $2kx = -k^2$

$x = -\frac{k}{2}$ ***AG*** ***N0***

***[5 marks]***

8d. Find the area of triangle ABC, giving your answer in terms of $k$. *[2 marks]*

# Markscheme

valid approach to find area of triangle ***(M1)***

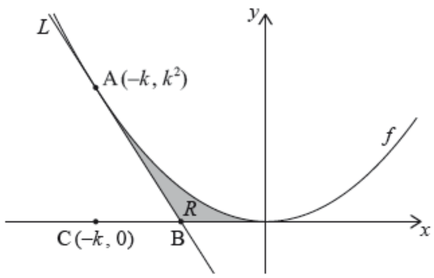
*eg* $\frac{1}{2}(k^2)\left(\frac{k}{2}\right)$

area of $\text{ABC} = \frac{k^3}{4}$ ***A1*** ***N2***

***[2 marks]***

The region $R$ is enclosed by $L$, the graph of $f$, and the $x$-axis. This is shown in the following diagram.

**diagram not to scale**




8e. Given that the area of triangle ABC is $p$ times the area of $R$, find the value of $p$. *[7 marks]*

# Markscheme

**METHOD 1 ($\int f -$ triangle)**

valid approach to find area from $-k$ to 0 ***(M1)***

*eg* $\int_{-k}^{0} x^2 \mathrm{d}x, \ \int_{0}^{-k} f$

correct integration (seen anywhere, even if ***M0*** awarded) ***A1***

*eg* $\frac{x^3}{3}, \ \left[\frac{1}{3}x^3\right]_{-k}^{0}$

substituting **their** limits into **their** integrated function and subtracting ***(M1)***

*eg* $0 - \frac{(-k)^3}{3}$, area from $-k$ to 0 is $\frac{k^3}{3}$

**Note:** Award ***M0*** for substituting into original or differentiated function.

attempt to find area of $R$ ***(M1)***

*eg* $\int_{-k}^{0} f(x)\mathrm{d}x - \text{ triangle}$

correct working for $R$ ***(A1)***

*eg* $\frac{k^3}{3} - \frac{k^3}{4}, \ R = \frac{k^3}{12}$

correct substitution into $\text{triangle} = pR$ ***(A1)***

*eg* $\frac{k^3}{4} = p\left(\frac{k^3}{3} - \frac{k^3}{4}\right), \ \frac{k^3}{4} = p\left(\frac{k^3}{12}\right)$

$p = 3$ ***A1 N2***

**METHOD 2 ($\int (f - L)$)**

valid approach to find area of $R$ ***(M1)***

*eg* $\int_{-k}^{-\frac{k}{2}} x^2 - (-2kx - k^2)\mathrm{d}x + \int_{-\frac{k}{2}}^{0} x^2 \mathrm{d}x, \ \int_{-k}^{-\frac{k}{2}} (f - L) + \int_{-\frac{k}{2}}^{0} f$

correct integration (seen anywhere, even if ***M0*** awarded) ***A2***

*eg* $\frac{x^3}{3} + kx^2 + k^2x, \ \left[\frac{x^3}{3} + kx^2 + k^2x\right]_{-k}^{-\frac{k}{2}} + \left[\frac{x^3}{3}\right]_{-\frac{k}{2}}^{0}$

substituting **their** limits into **their** integrated function and subtracting ***(M1)***

*eg*

$$\left(\frac{\left(-\frac{k}{2}\right)^3}{3}+k\left(-\frac{k}{2}\right)^2+k^2\left(-\frac{k}{2}\right)\right)-\left(\frac{(-k)^3}{3}+k(-k)^2+k^2(-k)\right)+(0)-\left(\frac{\left(-\frac{k}{2}\right)^3}{3}\right)$$

**Note:** Award ***M0*** for substituting into original or differentiated function.

correct working for $R$ ***(A1)***

*eg* $\frac{k^3}{24}+\frac{k^3}{24},\ -\frac{k^3}{24}+\frac{k^3}{4}-\frac{k^3}{2}+\frac{k^3}{3}-k^3+k^3+\frac{k^3}{24},\ R=\frac{k^3}{12}$

correct substitution into $\text{triangle}=pR$ ***(A1)***

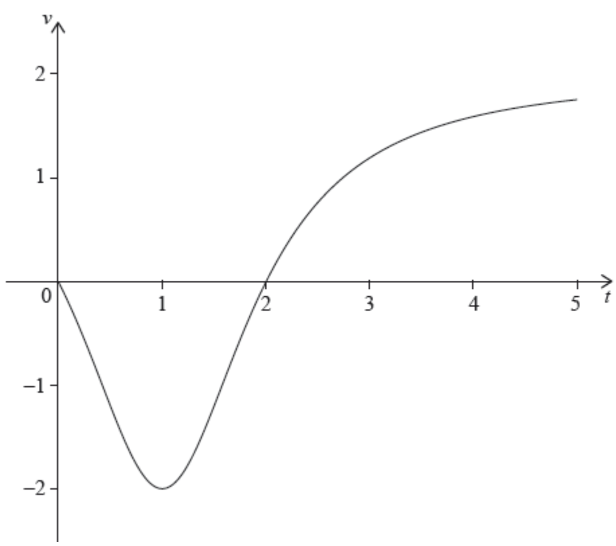
*eg* $\frac{k^3}{4}=p\left(\frac{k^3}{24}+\frac{k^3}{24}\right),\ \frac{k^3}{4}=p\left(\frac{k^3}{12}\right)$

$p=3$ ***A1*** ***N2***

***[7 marks]***

9. **Note: In this question, distance is in metres and time is in seconds.** *[6 marks]*

A particle moves along a horizontal line starting at a fixed point A. The velocity $v$ of the particle, at time $t$, is given by $v(t)=\frac{2t^2-4t}{t^2-2t+2}$, for $0\leqslant t\leqslant 5$. The following diagram shows the graph of $v$




There are $t$-intercepts at $(0,\ 0)$ and $(2,\ 0)$.

Find the maximum distance of the particle from A during the time $0\leqslant t\leqslant 5$ and justify your answer.

# Markscheme

**METHOD 1 (displacement)**

recognizing $s = \int v\mathrm{d}t$ ***(M1)***

consideration of displacement at $t = 2$ **and** $t = 5$ (seen anywhere) ***M1***

*eg* $\int_0^2 v$ and $\int_0^5 v$

**Note:** Must have both for any further marks.

correct displacement at $t = 2$ and $t = 5$ (seen anywhere) ***A1A1***

$-2.28318$ (accept 2.28318), 1.55513

valid reasoning comparing correct displacements ***R1***

*eg* $|-2.28| > |1.56|$, more left than right

2.28 (m) ***A1*** ***N1***

**Note:** Do not award the final ***A1*** without the ***R1***.

**METHOD 2 (distance travelled)**

recognizing distance $= \int |v|\,\mathrm{d}t$ ***(M1)***

consideration of distance travelled from $t = 0$ to 2 **and** $t = 2$ to 5 (seen anywhere) ***M1***

*eg* $\int_0^2 v$ and $\int_2^5 v$

**Note:** Must have both for any further marks

correct distances travelled (seen anywhere) ***A1A1***

2.28318, (accept $-2.28318$), 3.83832

valid reasoning comparing correct distance values ***R1***

*eg* $3.84 - 2.28 < 2.28,\ 3.84 < 2 \times 2.28$

2.28 (m) ***A1*** ***N1***

**Note:** Do not award the final ***A1*** without the ***R1***.

***[6 marks]***

Let $f(x) = x\mathrm{e}^{-x}$ and $g(x) = -3f(x) + 1$.

The graphs of $f$ and $g$ intersect at $x = p$ and $x = q$, where $p < q$.

10a. Find the value of $p$ and of $q$. *[3 marks]*

# Markscheme

valid attempt to find the intersection ***(M1)***

*eg* $f = g$, sketch, one correct answer

$p = 0.357402,\ q = 2.15329$

$p = 0.357,\ q = 2.15$ ***A1A1*** ***N3***

***[3 marks]***

10b. Hence, find the area of the region enclosed by the graphs of $f$ and $g$. *[3 marks]*

# Markscheme

attempt to set up an integral involving subtraction (in any order) ***(M1)***

*eg* $\int_p^q [f(x) - g(x)]\,dx,\ \int_p^q f(x)dx - \int_p^q g(x)dx$

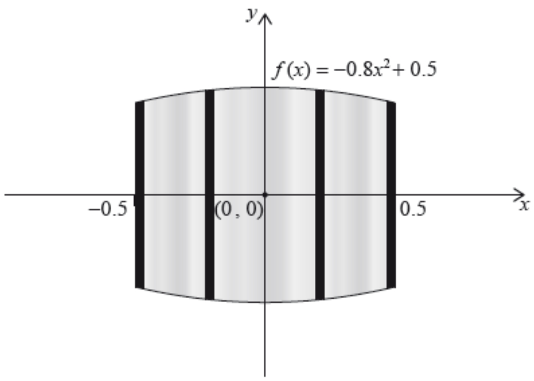
0.537667

$\text{area} = 0.538$ ***A2*** ***N3***

***[3 marks]***

**All lengths in this question are in metres.**

Let $f(x) = -0.8x^2 + 0.5$, for $-0.5 \leqslant x \leqslant 0.5$. Mark uses $f(x)$ as a model to create a barrel. The region enclosed by the graph of $f$, the $x$-axis, the line $x = -0.5$ and the line $x = 0.5$ is rotated 360° about the $x$-axis. This is shown in the following diagram.




11a. Use the model to find the volume of the barrel. *[3 marks]*

# Markscheme

attempt to substitute correct limits or the function into the formula involving $y^2$

*eg* $\pi \int_{-0.5}^{0.5} y^2 \mathrm{d}x,\ \pi \int \left(-0.8x^2 + 0.5\right)^2 \mathrm{d}x$

0.601091

volume $= 0.601\ (\mathrm{m}^3)$ ***A2*** ***N3***

***[3 marks]***

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11b. The empty barrel is being filled with water. The volume $V\ \mathrm{m}^3$ of water in the barrel after $t$ minutes is given by $V = 0.8(1 - \mathrm{e}^{-0.1t})$. How long will it take for the barrel to be half-full? *[3 marks]*

# Markscheme

attempt to equate half **their** volume to $V$ ***(M1)***

*eg* $0.30055 = 0.8(1 - \mathrm{e}^{-0.1t})$, graph

4.71104

4.71 (minutes) ***A2*** ***N3***

***[3 marks]***

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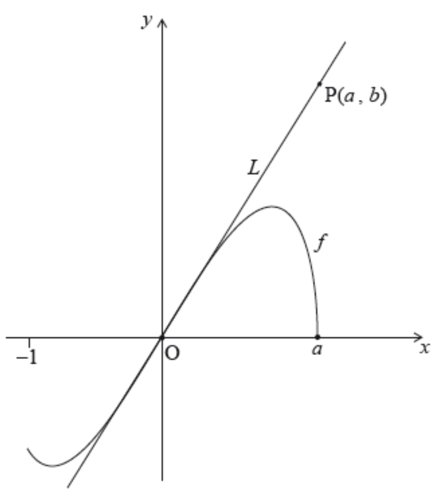
The following diagram shows the graph of $f(x) = 2x\sqrt{a^2 - x^2}$, for $-1 \leqslant x \leqslant a$, where $a > 1$.




The line $L$ is the tangent to the graph of $f$ at the origin, O. The point $\mathrm{P}(a,\ b)$ lies on $L$.

12a. (i) Given that $f'(x) = \frac{2a^2 - 4x^2}{\sqrt{a^2 - x^2}}$, for $-1 \leqslant x < a$, find the equation of $L$. *[6 marks]*

(ii) Hence or otherwise, find an expression for $b$ in terms of $a$.

# Markscheme

(i) recognizing the need to find the gradient when $x = 0$ (seen anywhere) ***R1***

*eg* $f'(0)$

correct substitution ***(A1)***

$f'(0) = \frac{2a^2 - 4(0)}{\sqrt{a^2 - 0}}$

$f'(0) = 2a$ ***(A1)***

correct equation with gradient $2a$ (do not accept equations of the form $L = 2ax$) ***A1 N3***

*eg* $y = 2ax,\ y - b = 2a(x - a),\ y = 2ax - 2a^2 + b$

(ii) **METHOD 1**

attempt to substitute $x = a$ into **their** equation of $L$ ***(M1)***

*eg* $y = 2a \times a$

$b = 2a^2$ ***A1 N2***

**METHOD 2**

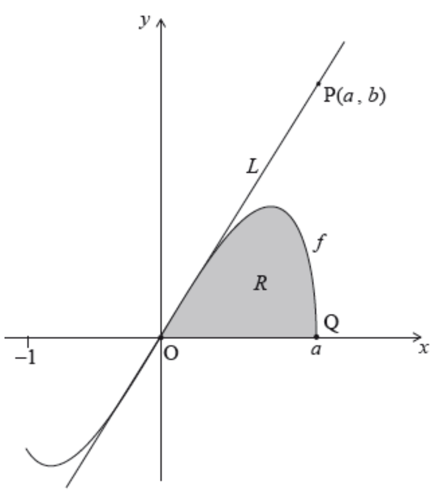
equating gradients ***(M1)***

*eg* $\frac{b}{a} = 2a$

$b = 2a^2$ ***A1 N2***

***[6 marks]***

The point $\mathrm{Q}(a,\ 0)$ lies on the graph of $f$. Let $R$ be the region enclosed by the graph of $f$ and the $x$-axis. This information is shown in the following diagram.




Let $A_R$ be the area of the region $R$.

12b. Show that $A_R = \frac{2}{3}a^3$. *[6 marks]*

# Markscheme

**METHOD 1**

recognizing that area $= \int_0^a f(x)\mathrm{d}x$ (seen anywhere) ***R1***

valid approach using substitution or inspection ***(M1)***

*eg* $\int 2x\sqrt{u}\mathrm{d}x,\ u = a^2 - x^2,\ \mathrm{d}u = -2x\mathrm{d}x,\ \frac{2}{3}(a^2 - x^2)^{\frac{3}{2}}$

correct working ***(A1)***

*eg* $\int 2x\sqrt{a^2 - x^2}\mathrm{d}x = \int -\sqrt{u}\mathrm{d}u$

$\int -\sqrt{u}\mathrm{d}u = -\frac{u^{\frac{3}{2}}}{\frac{3}{2}}$ ***(A1)***

$\int f(x)\mathrm{d}x = -\frac{2}{3}(a^2 - x^2)^{\frac{3}{2}} + c$ ***(A1)***

substituting limits and subtracting ***A1***

*eg* $A_R = -\frac{2}{3}(a^2 - a^2)^{\frac{3}{2}} + \frac{2}{3}(a^2 - 0)^{\frac{3}{2}},\ \frac{2}{3}(a^2)^{\frac{3}{2}}$

$A_R = \frac{2}{3}a^3$ ***AG*** ***N0***

**METHOD 2**

recognizing that area $= \int_0^a f(x)\mathrm{d}x$ (seen anywhere) ***R1***

valid approach using substitution or inspection ***(M1)***

*eg* $\int 2x\sqrt{u}\mathrm{d}x,\ u = a^2 - x^2,\ \mathrm{d}u = -2x\mathrm{d}x,\ \frac{2}{3}(a^2 - x^2)^{\frac{3}{2}}$

correct working ***(A1)***

*eg* $\int 2x\sqrt{a^2 - x^2}\mathrm{d}x = \int -\sqrt{u}\mathrm{d}u$

$\int -\sqrt{u}\mathrm{d}u = -\frac{u^{\frac{3}{2}}}{\frac{3}{2}}$ ***(A1)***

new limits for *u* (even if integration is incorrect) ***(A1)***

*eg* $u = 0 \text{ and } u = a^2,\ \int_0^{a^2} u^{\frac{1}{2}}\mathrm{d}u,\ \left[-\frac{2}{3}u^{\frac{3}{2}}\right]_{a^2}^{0}$

substituting limits and subtracting ***A1***

*eg* $A_R = -\left(0 - \frac{2}{3}a^3\right),\ \frac{2}{3}(a^2)^{\frac{3}{2}}$

$A_R = \frac{2}{3}a^3$ ***AG*** ***N0***

***[6 marks]***

12c. Let $A_T$ be the area of the triangle OPQ. Given that $A_T = kA_R$, find the value of $k$. *[4 marks]*

# Markscheme

**METHOD 1**

valid approach to find area of triangle ***(M1)***

*eg* $\frac{1}{2}(\mathrm{OQ})(\mathrm{PQ}),\ \frac{1}{2}ab$

correct substitution into formula for $A_T$ (seen anywhere) ***(A1)***

*eg* $A_T = \frac{1}{2} \times a \times 2a^2,\ a^3$

valid attempt to find $k$ (must be in terms of $a$) ***(M1)***

*eg* $a^3 = k\frac{2}{3}a^3,\ k = \frac{a^3}{\frac{2}{3}a^3}$

$k = \frac{3}{2}$ ***A1 N2***

**METHOD 2**

valid approach to find area of triangle ***(M1)***

*eg* $\int_0^a (2ax)\mathrm{d}x$

correct working ***(A1)***

*eg* $[ax^2]_0^a,\ a^3$

valid attempt to find $k$ (must be in terms of $a$) ***(M1)***

*eg* $a^3 = k\frac{2}{3}a^3,\ k = \frac{a^3}{\frac{2}{3}a^3}$

$k = \frac{3}{2}$ ***A1 N2***

***[4 marks]***

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Let $f(x) = x^2$ and $g(x) = 3\ln(x+1)$, for $x > -1$.

13a. Solve $f(x) = g(x)$. *[3 marks]*

# Markscheme

valid approach ***(M1)***

*eg* sketch

0, 1.73843

$x = 0,\ x = 1.74\ (\text{accept } (0,\ 0) \text{ and } (1.74,\ 3.02))$ ***A1A1 N3***

***[3 marks]***

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13b. Find the area of the region enclosed by the graphs of $f$ and $g$. *[3 marks]*

# Markscheme

integrating and subtracting functions (in any order) ***(M1)***

*eg* $\int g - f$

correct substitution of their limits **or** function (accept missing $\mathrm{d}x$)

***(A1)***

*eg* $\int_0^{1.74} g - f,\ \int 3\ln(x+1) - x^2$

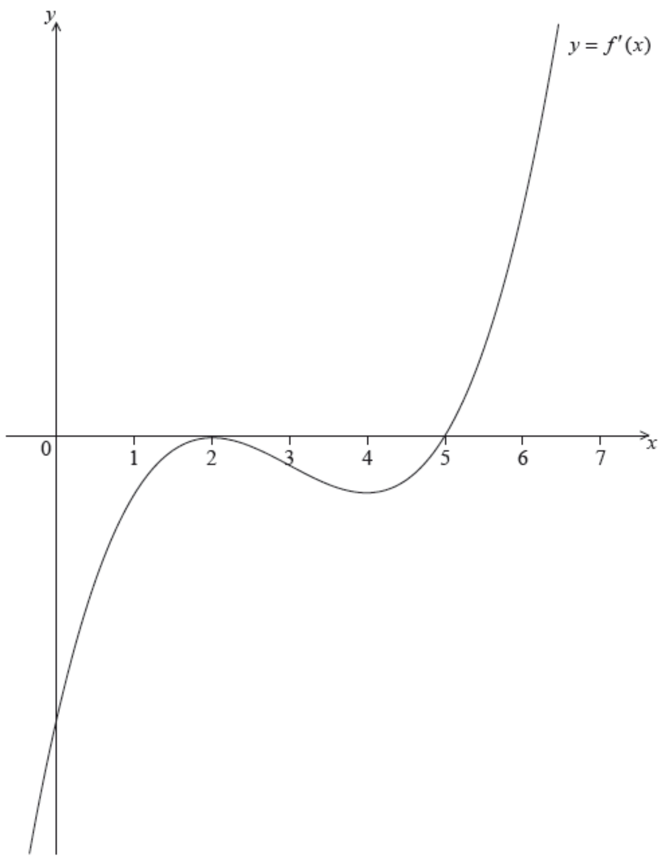
**Note:** Do not award ***A1*** if there is an error in the substitution.

1.30940

1.31 ***A1*** ***N3***

***[3 marks]***

Let $y = f(x)$, for $-0.5 \le x \le 6.5$. The following diagram shows the graph of $f'$, the derivative of $f$.




The graph of $f'$ has a local maximum when $x = 2$, a local minimum when $x = 4$, and it crosses the
$x$-axis at the point $(5,\ 0)$.

14a. Explain why the graph of $f$ has a local minimum when $x = 5$. *[2 marks]*

## Markscheme

**METHOD 1**

$f'(5) = 0$ ***(A1)***

valid reasoning including reference to the graph of $f'$ ***R1***

*eg* $f'$ changes sign from negative to positive at $x = 5$, labelled sign chart for $f'$

so $f$ has a local minimum at $x = 5$ ***AG*** ***N0***

**Note:** It must be clear that any description is referring to the graph of $f'$, simply giving the conditions for a minimum without relating them to $f'$ does not gain the ***R1***.

**METHOD 2**

$f'(5) = 0$ ***A1***

valid reasoning referring to second derivative ***R1***

*eg* $f''(5) > 0$

so $f$ has a local minimum at $x = 5$ ***AG*** ***N0***

***[2 marks]***

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14b. Find the set of values of $x$ for which the graph of $f$ is concave down. *[2 marks]*

## Markscheme

attempt to find relevant interval ***(M1)***

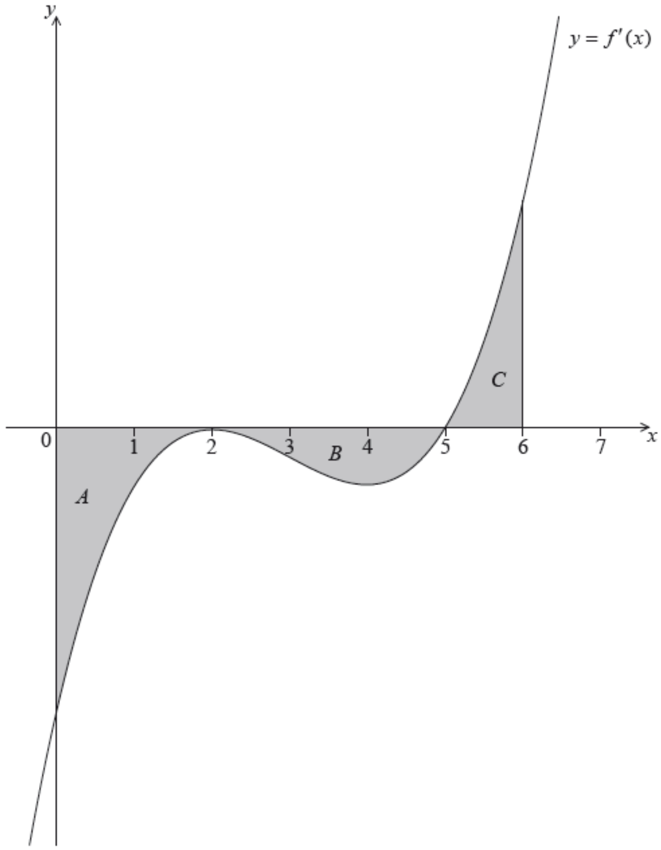
*eg* $f'$ is decreasing, gradient of $f'$ is negative, $f'' < 0$

$2 < x < 4$ (accept “between 2 and 4”) ***A1*** ***N2***

**Notes:** If no other working shown, award ***M1A0*** for incorrect inequalities such as $2 \leq x \leq 4$, or “from 2 to 4”

***[2 marks]***

14c. The following diagram shows the shaded regions $A$, $B$ and $C$. *[5 marks]*




The regions are enclosed by the graph of $f'$, the $x$-axis, the $y$-axis, and the line $x = 6$.

The area of region $A$ is 12, the area of region $B$ is 6.75 and the area of region $C$ is 6.75.

Given that $f(0) = 14$, find $f(6)$.

# Markscheme

**METHOD 1 (one integral)**

correct application of Fundamental Theorem of Calculus ***(A1)***

*eg* $\int_0^6 f'(x)\mathrm{d}x = f(6) - f(0),\ f(6) = 14 + \int_0^6 f'(x)\mathrm{d}x$

attempt to link definite integral with areas ***(M1)***

*eg* $\int_0^6 f'(x)\mathrm{d}x = -12 - 6.75 + 6.75,\ \int_0^6 f'(x)\mathrm{d}x = \text{Area } A + \text{Area } B + \text{ Area } C$

correct value for $\int_0^6 f'(x)\mathrm{d}x$ ***(A1)***

*eg* $\int_0^6 f'(x)\mathrm{d}x = -12$

correct working ***A1***

*eg* $f(6) - 14 = -12,\ f(6) = -12 + f(0)$

$f(6) = 2$ ***A1 N3***

**METHOD 2 (more than one integral)**

correct application of Fundamental Theorem of Calculus ***(A1)***

*eg* $\int_0^2 f'(x)\mathrm{d}x = f(2) - f(0),\ f(2) = 14 + \int_0^2 f'(x)$

attempt to link definite integrals with areas ***(M1)***

*eg* $\int_0^2 f'(x)\mathrm{d}x = 12,\ \int_2^5 f'(x)\mathrm{d}x = -6.75,\ \int_0^6 f'(x) = 0$

correct values for integrals ***(A1)***

*eg* $\int_0^2 f'(x)\mathrm{d}x = -12,\ \int_5^2 f'(x)\mathrm{d}x = 6.75,\ f(6) - f(2) = 0$

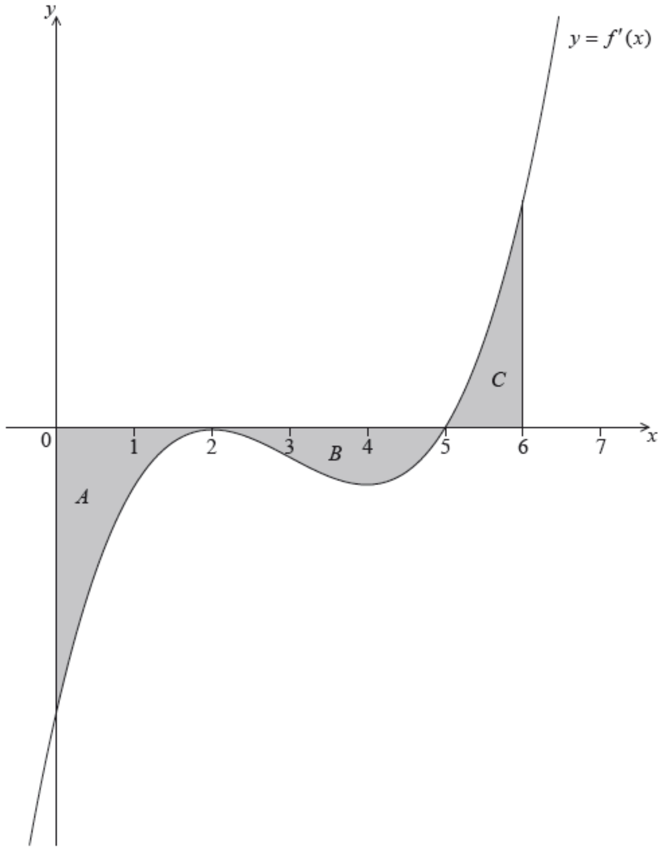
one correct intermediate value ***A1***

*eg* $f(2) = 2,\ f(5) = -4.75$

$f(6) = 2$ ***A1 N3***

***[5 marks]***

14d. The following diagram shows the shaded regions $A$, $B$ and $C$. *[6 marks]*




The regions are enclosed by the graph of $f'$, the *x*-axis, the *y*-axis, and the line $x = 6$.

The area of region $A$ is 12, the area of region $B$ is 6.75 and the area of region $C$ is 6.75.

Let $g(x) = (f(x))^2$. Given that $f'(6) = 16$, find the equation of the tangent to the graph of $g$ at the point where $x = 6$.

# Markscheme

correct calculation of $g(6)$ (seen anywhere) ***A1***

*eg* $2^2,\ g(6) = 4$

choosing chain rule or product rule ***(M1)***

*eg* $g'(f(x))\,f'(x),\ \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \times \frac{\mathrm{d}u}{\mathrm{d}x},\ f(x)f'(x) + f'(x)f(x)$

correct derivative ***(A1)***

*eg* $g'(x) = 2f(x)f'(x),\ f(x)f'(x) + f'(x)f(x)$

correct calculation of $g'(6)$ (seen anywhere) ***A1***

*eg* $2(2)(16),\ g'(6) = 64$

attempt to substitute **their** values of $g'(6)$ and $g(6)$ (in any order) into equation of a line ***(M1)***

*eg* $2^2 = (2 \times 2 \times 16)6 + b,\ y - 6 = 64(x - 4)$

correct equation in any form ***A1*** ***N2***

*eg* $y - 4 = 64(x - 6),\ y = 64x - 380$

***[6 marks]***

***[Total 15 marks]***

15. Let $f'(x) = 6x^2 - 5$. Given that $f(2) = -3$, find $f(x)$. *[6 marks]*

# Markscheme

evidence of antidifferentiation ***(M1)***

*eg* $f = \int f'$

correct integration (accept absence of $C$) ***(A1)(A1)***

$f(x) = \frac{6x^3}{3} - 5x + C,\ 2x^3 - 5x$

attempt to substitute $(2,\ -3)$ into **their** integrated expression (must have $C$) ***M1***

*eg* $2(2)^3 - 5(2) + C = -3,\ 16 - 10 + C = -3$

**Note:** Award ***M0*** if substituted into original or differentiated function.

correct working to find $C$ ***(A1)***

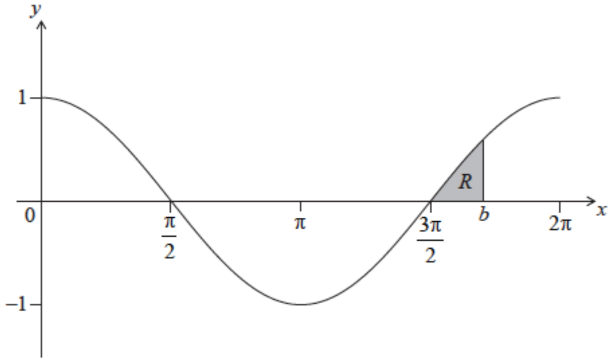
*eg* $16 - 10 + C = -3,\ 6 + C = -3,\ C = -9$

$f(x) = 2x^3 - 5x - 9$ ***A1*** ***N4***

***[6 marks]***

16. Let $f(x) = \cos x$, for $0 \leq x \leq 2\pi$. The following diagram shows the graph of $f$. *[8 marks]*

There are
$x$-intercepts at $x = \frac{\pi}{2}, \frac{3\pi}{2}$.




The shaded region $R$ is enclosed by the graph of $f$, the line $x = b$, where $b > \frac{3\pi}{2}$, and the $x$-axis. The area of $R$ is $\left(1 - \frac{\sqrt{3}}{2}\right)$. Find the value of $b$.

# Markscheme

attempt to set up integral (accept missing or incorrect limits and missing $\mathrm{d}x$) ***M1***

*eg* $\int_{\frac{3\pi}{2}}^{b} \cos x \mathrm{d}x, \int_{a}^{b} \cos x \mathrm{d}x, \int_{\frac{3\pi}{2}}^{b} f \mathrm{d}x, \int \cos x$

correct integration (accept missing or incorrect limits) ***(A1)***

*eg* $[\sin x]_{\frac{3\pi}{2}}^{b}, \sin x$

substituting correct limits into **their** integrated function and subtracting (in any order) ***(M1)***

*eg* $\sin b - \sin\left(\frac{3\pi}{2}\right), \sin\left(\frac{3\pi}{2}\right) - \sin b$

$\sin\left(\frac{3\pi}{2}\right) = -1$ (seen anywhere) ***(A1)***

setting **their** result from an integrated function equal to $\left(1 - \frac{\sqrt{3}}{2}\right)$ ***M1***

*eg* $\sin b = -\frac{\sqrt{3}}{2}$

evaluating $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$ or $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$ ***(A1)***

*eg* $b = \frac{\pi}{3}, -60°$

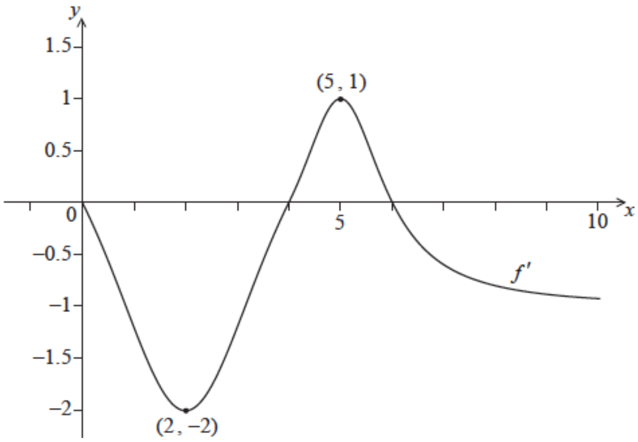
identifying correct value ***(A1)***

*eg* $2\pi - \frac{\pi}{3}, 360 - 60$

$b = \frac{5\pi}{3}$ ***A1*** ***N3***

***[8 marks]***

Consider a function $f$, for $0 \leq x \leq 10$. The following diagram shows the graph of $f'$, the derivative of $f$.




The graph of $f'$ passes through $(2,\ -2)$ and $(5,\ 1)$, and has $x$-intercepts at $0$, $4$ and $6$.

17a. The graph of $f$ has a local maximum point when $x = p$. State the value of $p$, and justify your answer. *[3 marks]*

# Markscheme

$p = 6$ ***A1 N1***

recognizing that turning points occur when $f'(x) = 0$ ***R1 N1***

eg correct sign diagram

$f'$ changes from positive to negative at $x = 6$ ***R1 N1***

***[3 marks]***

17b. Write down $f'(2)$. *[1 mark]*

# Markscheme

$f'(2) = -2$ ***A1 N1***

***[1 mark]***

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17c. Let $g(x) = \ln(f(x))$ and $f(2) = 3$. *[4 marks]*

Find $g'(2)$.

# Markscheme

attempt to apply chain rule ***(M1)***

*eg* $\ln(x)' \times f'(x)$

correct expression for $g'(x)$ ***(A1)***

*eg* $g'(x) = \frac{1}{f(x)} \times f'(x)$

substituting $x = 2$ into **their** $g'$ ***(M1)***

*eg* $\frac{f'(2)}{f(2)}$

$-0.666667$

$g'(2) = -\frac{2}{3}\ (\text{exact}),\ -0.667$ ***A1*** ***N3***

***[4 marks]***

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17d. Verify that $\ln 3 + \int_2^a g'(x)\mathrm{d}x = g(a)$, where $0 \le a \le 10$. *[4 marks]*

# Markscheme

evidence of integrating $g'(x)$ ***(M1)***

eg $g(x)|_2^a,\ g(x)|_a^2$

applying the fundamental theorem of calculus (seen anywhere) ***R1***

eg $\int_2^a g'(x) = g(a) - g(2)$

correct substitution into integral ***(A1)***

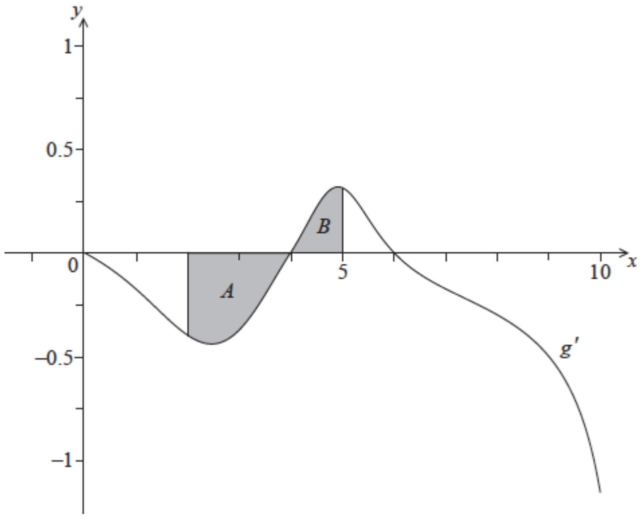
eg $\ln 3 + g(a) - g(2),\ \ln 3 + g(a) - \ln(f(2))$

$\ln 3 + g(a) - \ln 3$ ***A1***

$\ln 3 + \int_2^a g'(x) = g(a)$ ***AG*** ***N0***

***[4 marks]***

17e. The following diagram shows the graph of $g'$, the derivative of $g$. *[4 marks]*




The shaded region $A$ is enclosed by the curve, the $x$-axis and the line $x = 2$, and has area $0.66 \text{ units}^2$.

The shaded region $B$ is enclosed by the curve, the $x$-axis and the line $x = 5$, and has area $0.21 \text{ units}^2$.

Find $g(5)$.

# Markscheme

**METHOD 1**

substituting $a = 5$ into the formula for $g(a)$ ***(M1)***

*eg* $\int_2^5 g'(x)\mathrm{d}x,\ g(5) = \ln 3 + \int_2^5 g'(x)\mathrm{d}x \quad (\text{do not accept only } g(5))$

attempt to substitute areas ***(M1)***

*eg* $\ln 3 + 0.66 - 0.21,\ \ln 3 + 0.66 + 0.21$

correct working

*eg* $g(5) = \ln 3 + (-0.66 + 0.21)$ ***(A1)***

$0.648612$

$g(5) = \ln 3 - 0.45$ (exact), $0.649$ ***A1 N3***

**METHOD 2**

attempt to set up an equation for one shaded region ***(M1)***

*eg* $\int_4^5 g'(x)\mathrm{d}x = 0.21,\ \int_2^4 g'(x)\mathrm{d}x = -0.66,\ \int_2^5 g'(x)\mathrm{d}x = -0.45$

two correct equations ***(A1)***

*eg* $g(5) - g(4) = 0.21,\ g(2) - g(4) = 0.66$

combining equations to eliminate $g(4)$ ***(M1)***

*eg* $g(5) - [\ln 3 - 0.66] = 0.21$

$0.648612$

$g(5) = \ln 3 - 0.45$ (exact), $0.649$ ***A1 N3***

**METHOD 3**

attempt to set up a definite integral ***(M1)***

*eg* $\int_2^5 g'(x)\mathrm{d}x = -0.66 + 0.21,\ \int_2^5 g'(x)\mathrm{d}x = -0.45$

correct working ***(A1)***

*eg* $g(5) - g(2) = -0.45$

correct substitution ***(A1)***

*eg* $g(5) - \ln 3 = -0.45$

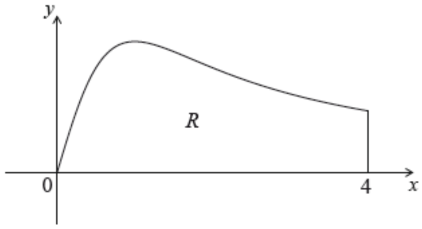
$0.648612$

$g(5) = \ln 3 - 0.45$ (exact), $0.649$ ***A1 N3***

***[4 marks]***

***Total [16 marks]***

18. The following diagram shows the graph of $f(x) = \frac{x}{x^2+1}$, for $0 \leq x \leq 4$, and the line $x = 4$. *[6 marks]*




Let $R$ be the region enclosed by the graph of $f$, the $x$-axis and the line $x = 4$.

Find the area of $R$.

# Markscheme

substitution of limits or function ***(A1)***

*eg* $A = \int_0^4 f(x),\ \int \frac{x}{x^2+1} \mathrm{d}x$

correct integration by substitution/inspection ***A2***

$\frac{1}{2}\ln(x^2+1)$

substituting limits into **their** integrated function and subtracting (in any order) ***(M1)***

*eg* $\frac{1}{2}\left(\ln(4^2+1) - \ln(0^2+1)\right)$

correct working ***A1***

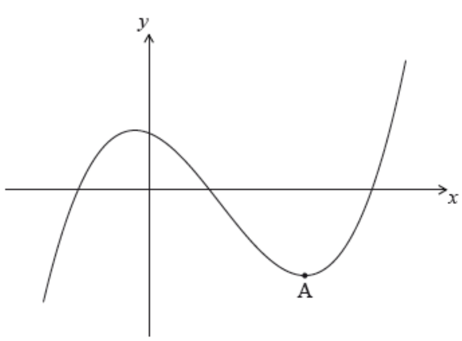
*eg* $\frac{1}{2}\left(\ln(4^2+1) - \ln(0^2+1)\right)$, $\frac{1}{2}(\ln(17) - \ln(1))$, $\frac{1}{2}\ln 17 - 0$

$A = \frac{1}{2}\ln(17)$ ***A1*** ***N3***

**Note:** Exception to ***FT*** rule. Allow full ***FT*** on incorrect integration involving a $\ln$ function.

***[6 marks]***

The following diagram shows the graph of a function $f$. There is a local minimum point at $A$, where $x > 0$.




The derivative of $f$ is given by $f'(x) = 3x^2 - 8x - 3$.

19a. Find the $x$-coordinate of $A$. *[5 marks]*

## Markscheme

recognizing that the local minimum occurs when $f'(x) = 0$ ***(M1)***

valid attempt to solve $3x^2 - 8x - 3 = 0$ ***(M1)***

*eg* factorization, formula

correct working ***A1***

$(3x + 1)(x - 3),\ x = \frac{8 \pm \sqrt{64+36}}{6}$

$x = 3$ ***A2*** ***N3***

**Note:** Award ***A1*** if both values $x = \frac{-1}{3},\ x = 3$ are given.

***[5 marks]***

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19b. The $y$-intercept of the graph is at $(0, 6)$. Find an expression for $f(x)$. *[6 marks]*

The graph of a function $g$ is obtained by reflecting the graph of $f$ in the $y$-axis, followed by a translation of $\begin{pmatrix} m \\ n \end{pmatrix}$.

## Markscheme

valid approach ***(M1)***

$f(x) = \int f'(x) \mathrm{d}x$

$f(x) = x^3 - 4x^2 - 3x + c$ (do not penalize for missing "$+c$") ***A1A1A1***

$c = 6$ ***(A1)***

$f(x) = x^3 - 4x^2 - 3x + 6$ ***A1*** ***N6***

***[6 marks]***

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Let
$f(x) = x^2$.

20a. Find $\int_1^2 (f(x))^2 \mathrm{d}x$. *[4 marks]*

## Markscheme

substituting for $(f(x))^2$ (may be seen in integral) ***A1***

*eg* $(x^2)^2, x^4$

correct integration, $\int x^4 \mathrm{d}x = \frac{1}{5}x^5$ ***(A1)***

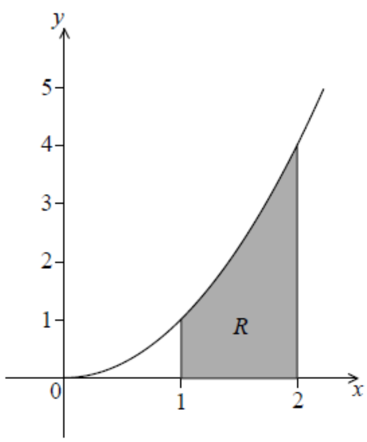
substituting limits into **their integrated** function and subtracting (in any order) ***(M1)***

*eg* $\frac{2^5}{5} - \frac{1}{5}, \frac{1}{5}(1-4)$

$\int_1^2 (f(x))^2 \mathrm{d}x = \frac{31}{5} (= 6.2)$ ***A1 N2***

***[4 marks]***

20b. The following diagram shows part of the graph of $f$. *[2 marks]*




The shaded region $R$ is enclosed by the graph of $f$, the $x$-axis and the lines $x = 1$ and $x = 2$.

Find the volume of the solid formed when $R$ is revolved $360°$ about the $x$-axis.

## Markscheme

attempt to substitute limits or function into formula involving $f^2$ ***(M1)***

*eg* $\int_1^2 (f(x))^2 \mathrm{d}x, \pi \int x^4 \mathrm{d}x$

$\frac{31}{5}\pi \ (= 6.2\pi)$ ***A1 N2***

***[2 marks]***

21. Let $\int_\pi^a \cos 2x \mathrm{d}x = \frac{1}{2}$, where $\pi < a < 2\pi$. Find the value of $a$. *[7 marks]*

## Markscheme

correct integration (ignore absence of limits and "$+C$")     ***(A1)***

*eg*   $\frac{\sin(2x)}{2}$, $\int_{\pi}^{a}\cos 2x = \left[\frac{1}{2}\sin(2x)\right]_{\pi}^{a}$

substituting limits into **their** integrated function and subtracting (in any order)     ***(M1)***

*eg*   $\frac{1}{2}\sin(2a) - \frac{1}{2}\sin(2\pi)$, $\sin(2\pi) - \sin(2a)$

$\sin(2\pi) = 0$     ***(A1)***

setting **their** result from an integrated function equal to $\frac{1}{2}$     ***M1***

*eg*   $\frac{1}{2}\sin 2a = \frac{1}{2}$, $\sin(2a) = 1$

recognizing $\sin^{-1}1 = \frac{\pi}{2}$     ***(A1)***

*eg*   $2a = \frac{\pi}{2}$, $a = \frac{\pi}{4}$

correct value     ***(A1)***

*eg*   $\frac{\pi}{2} + 2\pi$, $2a = \frac{5\pi}{2}$, $a = \frac{\pi}{4} + \pi$

$a = \frac{5\pi}{4}$     ***A1   N3***

***[7 marks]***

Let
$f(x) = \frac{2x}{x^2+5}$.

22a. Use the quotient rule to show that $f'(x) = \frac{10-2x^2}{(x^2+5)^2}$.     *[4 marks]*

## Markscheme

derivative of $2x$ is $2$ (must be seen in quotient rule)     ***(A1)***

derivative of $x^2+5$ is $2x$ (must be seen in quotient rule)     ***(A1)***

correct substitution into quotient rule     ***A1***

*eg*   $\frac{(x^2+5)(2)-(2x)(2x)}{(x^2+5)^2}$, $\frac{2(x^2+5)-4x^2}{(x^2+5)^2}$

correct working which clearly leads to given answer     ***A1***

*eg*   $\frac{2x^2+10-4x^2}{(x^2+5)^2}$, $\frac{2x^2+10-4x^2}{x^4+10x^2+25}$

$f'(x) = \frac{10-2x^2}{(x^2+5)^2}$     ***AG   N0***

***[4 marks]***

22b. Find $\int \frac{2x}{x^2+5}\mathrm{d}x$.     *[4 marks]*

## Markscheme

valid approach using substitution or inspection ***(M1)***

*eg* $u = x^2 + 5,\ \mathrm{d}u = 2x\mathrm{d}x,\ \frac{1}{2}\ln(x^2+5)$

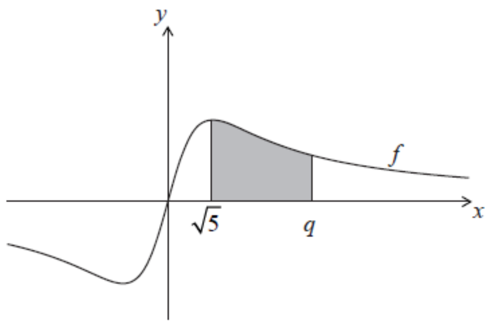
$\int \frac{2x}{x^2+5}\mathrm{d}x = \int \frac{1}{u}\mathrm{d}u$ ***(A1)***

$\int \frac{1}{u}\mathrm{d}u = \ln u + c$ ***(A1)***

$\ln(x^2+5) + c$ ***A1 N4***

***[4 marks]***

22c. The following diagram shows part of the graph of $f$. *[7 marks]*




The shaded region is enclosed by the graph of $f$, the $x$-axis, and the lines $x = \sqrt{5}$ and $x = q$. This region has an area of $\ln 7$. Find the value of $q$.

## Markscheme

correct expression for area ***(A1)***

*eg* $\left[\ln\left(x^2+5\right)\right]_{\sqrt{5}}^{q}$, $\int_{\sqrt{5}}^{q} \frac{2x}{x^2+5} \mathrm{d}x$

substituting limits into **their** integrated function and subtracting (in either order) ***(M1)***

*eg* $\ln(q^2+5) - \ln\left(\sqrt{5}^2 + 5\right)$

correct working ***(A1)***

*eg* $\ln\left(q^2+5\right) - \ln 10$, $\ln \frac{q^2+5}{10}$

equating **their** expression to $\ln 7$ (seen anywhere) ***(M1)***

*eg* $\ln\left(q^2+5\right) - \ln 10 = \ln 7$, $\ln \frac{q^2+5}{10} = \ln 7$, $\ln(q^2+5) = \ln 7 + \ln 10$

correct equation without logs ***(A1)***

*eg* $\frac{q^2+5}{10} = 7$, $q^2+5 = 70$

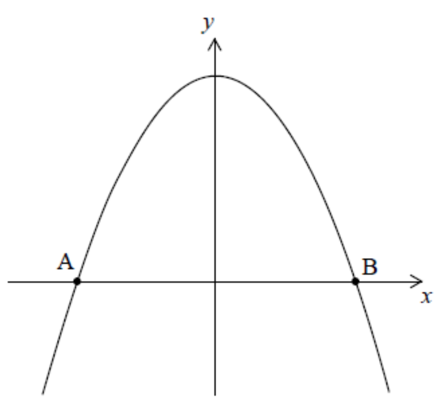
$q^2 = 65$ ***(A1)***

$q = \sqrt{65}$ ***A1 N3***

**Note:** Award ***A0*** for $q = \pm\sqrt{65}$.

***[7 marks]***

Let
$f(x) = 5 - x^2$. Part of the graph of
$f$ is shown in the following diagram.




The graph crosses the
$x$-axis at the points
$\mathrm{A}$ and
$\mathrm{B}$.

23a. Find the $x$-coordinate of $\mathrm{A}$ and of $\mathrm{B}$. *[3 marks]*

## Markscheme

recognizing $f(x) = 0$ ***(M1)***

*eg* $f = 0,\ x^2 = 5$

$x = \pm 2.23606$

$x = \pm\sqrt{5}\ (\text{exact}),\ x = \pm 2.24$ ***A1A1*** ***N3***

***[3 marks]***

---

23b. The region enclosed by the graph of $f$ and the $x$-axis is revolved $360°$ about the $x$-axis. *[3 marks]*

Find the volume of the solid formed.

## Markscheme

attempt to substitute either limits or the function into formula involving $f^2$ ***(M1)***

*eg* $\pi\int\left(5 - x^2\right)^2 \mathrm{d}x,\ \pi\int_{-2.24}^{2.24}\left(x^4 - 10x^2 + 25\right),\ 2\pi\int_0^{\sqrt{5}} f^2$

$187.328$

volume $= 187$ ***A2*** ***N3***

***[3 marks]***

Let $f(x) = \frac{(\ln x)^2}{2}$, for $x > 0$.

24a. Show that $f'(x) = \frac{\ln x}{x}$. *[2 marks]*

## Markscheme

**METHOD 1**

correct use of chain rule ***A1A1***

*eg* $\frac{2\ln x}{2} \times \frac{1}{x}, \frac{2\ln x}{2x}$

**Note:** Award ***A1*** for $\frac{2\ln x}{2x}$, ***A1*** for $\times\frac{1}{x}$.

$f'(x) = \frac{\ln x}{x}$ ***AG N0***

***[2 marks]***

**METHOD 2**

correct substitution into quotient rule, with derivatives seen ***A1***

*eg* $\frac{2\times 2\ln x\times\frac{1}{x} - 0\times(\ln x)^2}{4}$

correct working ***A1***

*eg* $\frac{4\ln x\times\frac{1}{x}}{4}$

$f'(x) = \frac{\ln x}{x}$ ***AG N0***

***[2 marks]***

24b. There is a minimum on the graph of $f$. Find the $x$-coordinate of this minimum. *[3 marks]*

## Markscheme

setting derivative $= 0$ ***(M1)***

*eg* $f'(x) = 0,\ \frac{\ln x}{x} = 0$

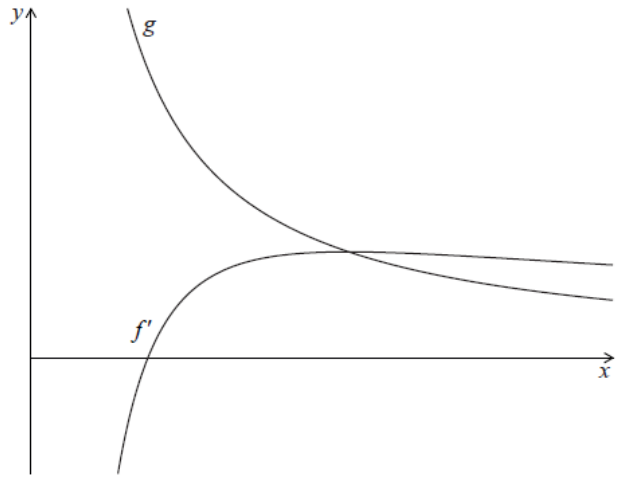
correct working ***(A1)***

*eg* $\ln x = 0,\ x = \mathrm{e}^0$

$x = 1$ ***A1 N2***

***[3 marks]***

Let
$g(x) = \frac{1}{x}$. The following diagram shows parts of the graphs of
$f'$ and *g*.




The graph of
$f'$ has an *x*-intercept at
$x = p$.

24c. Write down the value of $p$. *[2 marks]*

# Markscheme

intercept when $f'(x) = 0$ ***(M1)***

$p = 1$ ***A1 N2***

***[2 marks]***

---

24d. The graph of $g$ intersects the graph of $f'$ when $x = q$. *[3 marks]*

Find the value of $q$.

# Markscheme

equating functions ***(M1)***

*eg* $f' = g, \frac{\ln x}{x} = \frac{1}{x}$

correct working ***(A1)***

*eg* $\ln x = 1$

$q = \mathrm{e} \quad (\text{accept } x = \mathrm{e})$ ***A1 N2***

***[3 marks]***

24e. The graph of $g$ intersects the graph of $f'$ when $x = q$. *[5 marks]*

Let $R$ be the region enclosed by the graph of $f'$, the graph of $g$ and the line $x = p$.

Show that the area of $R$ is $\frac{1}{2}$.

# Markscheme

evidence of integrating and subtracting functions (in any order, seen anywhere) ***(M1)***

*eg* $\int_q^e \left(\frac{1}{x} - \frac{\ln x}{x}\right) \mathrm{d}x,\ \int f' - g$

correct integration $\ln x - \frac{(\ln x)^2}{2}$ ***A2***

substituting limits into **their** integrated function and subtracting (in any order) ***(M1)***

*eg* $(\ln \mathrm{e} - \ln 1) - \left(\frac{(\ln \mathrm{e})^2}{2} - \frac{(\ln 1)^2}{2}\right)$

**Note:** Do not award ***M1*** if the integrated function has only one term.

correct working ***A1***

*eg* $(1 - 0) - \left(\frac{1}{2} - 0\right),\ 1 - \frac{1}{2}$

$\text{area} = \frac{1}{2}$ ***AG*** ***N0***

**Notes:** Candidates may work with two separate integrals, and only combine them at the end. Award marks in line with the markscheme.

***[5 marks]***

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Consider a function $f(x)$ such that $\int_1^6 f(x)\mathrm{d}x = 8$.

25a. Find $\int_1^6 2f(x)\mathrm{d}x$. *[2 marks]*

# Markscheme

appropriate approach ***(M1)***

*eg* $2\int f(x),\ 2(8)$

$\int_1^6 2f(x)\mathrm{d}x = 16$ ***A1*** ***N2***

***[2 marks]***

25b. Find $\int_1^6 (f(x) + 2)\,\mathrm{d}x$. *[4 marks]*

## Markscheme

appropriate approach ***(M1)***

*eg* $\int f(x) + \int 2,\ 8 + \int 2$

$\int 2\mathrm{d}x = 2x$ (seen anywhere) ***(A1)***

substituting limits into **their** integrated function and subtracting (in any order) ***(M1)***

*eg* $2(6) - 2(1),\ 8 + 12 - 2$

$\int_1^6 (f(x) + 2)\,\mathrm{d}x = 18$ ***A1 N3***

***[4 marks]***

Let
$f(x) = (x - 1)(x - 4).$

26a. Find the $x$-intercepts of the graph of $f$. *[3 marks]*

## Markscheme

valid approach ***(M1)***

*eg* $f(x) = 0$, sketch of parabola showing two $x$-intercepts

$x = 1,\ x = 4\ (\text{accept } (1, 0), (4, 0))$ ***A1A1 N3***

***[3 marks]***

26b. The region enclosed by the graph of $f$ and the $x$-axis is rotated $360°$ about the $x$-axis. *[3 marks]*

Find the volume of the solid formed.

## Markscheme

attempt to substitute either limits or the function into formula involving $f^2$ ***(M1)***

*eg* $\int_1^4 (f(x))^2 \mathrm{d}x,\ \pi \int ((x-1)(x-4))^2$

$\text{volume} = 8.1\pi\ (\text{exact}), 25.4$ ***A2 N3***

***[3 marks]***

27. Let $f(x) = \int \frac{12}{2x-5} \mathrm{d}x$ , $x > \frac{5}{2}$ . The graph of $f$ passes through $(4,\ 0)$ . *[6 marks]*

Find $f(x)$ .

# Markscheme

attempt to integrate which involves $\ln$ ***(M1)***

*eg* $\ln(2x-5)$ , $12\ln 2x-5$ , $\ln 2x$

correct expression (accept absence of $C$)

*eg* $12\ln(2x-5)\frac{1}{2}+C$ , $6\ln(2x-5)$ ***A2***

attempt to substitute (4,0) into **their** integrated ***f*** ***(M1)***

*eg* $0=6\ln(2\times 4-5)$ , $0=6\ln(8-5)+C$

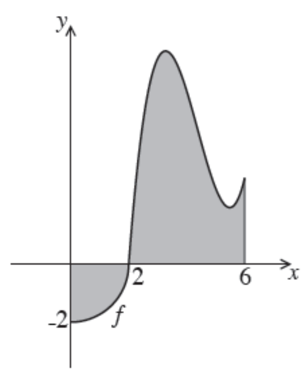
$C=-6\ln 3$ ***(A1)***

$f(x)=6\ln(2x-5)-6\ln 3\ \left(=6\ln\left(\frac{2x-5}{3}\right)\right)$ (accept $6\ln(2x-5)-\ln 3^6$ ) ***A1 N5***

**Note**: Exception to the ***FT*** rule. Allow full ***FT*** on incorrect integration which must involve $\ln$.

***[6 marks]***

The following is the graph of a function
$f$ , for
$0\le x\le 6$ .




The first part of the graph is a quarter circle of radius
2 with centre at the origin.

28a. (a) Find $\int_0^2 f(x)\mathrm{d}x$ . *[7 marks]*

(b) The shaded region is enclosed by the graph of $f$ , the $x$-axis, the $y$-axis and the line $x=6$ . The area of this region is $3\pi$ .

Find $\int_2^6 f(x)\mathrm{d}x$ .

## Markscheme

(a) attempt to find quarter circle area ***(M1)***

*eg* $\frac{1}{4}(4\pi)$ , $\frac{\pi r^2}{4}$ , $\int_0^2 \sqrt{4 - x^2 \mathrm{d}x}$

area of region $= \pi$ ***(A1)***

$\int_0^2 f(x)\mathrm{d}x = -\pi$ ***A2 N3***

***[4 marks]***

(b) attempted set up with both regions ***(M1)***

*eg* $\text{shaded area} - \text{quarter circle}$ , $3\pi - \pi$ , $3\pi - \int_0^2 f = \int_2^6 f$

$\int_2^6 f(x)\mathrm{d}x = 2\pi$ ***A2 N2***

***[3 marks]***

***Total [7 marks]***

28b. Find $\int_0^2 f(x)\mathrm{d}x$ . *[4 marks]*


## Markscheme

attempt to find quarter circle area ***(M1)***

*eg* $\frac{1}{4}(4\pi)$ , $\frac{\pi r^2}{4}$ , $\int_0^2 \sqrt{4 - x^2 \mathrm{d}x}$

area of region $= \pi$ ***(A1)***

$\int_0^2 f(x)\mathrm{d}x = -\pi$ ***A2 N3***

***[4 marks]***


28c. The shaded region is enclosed by the graph of $f$ , the $x$-axis, the $y$-axis and the line $x = 6$ . The area of this region is $3\pi$ . *[3 marks]*

Find $\int_2^6 f(x)\mathrm{d}x$ .

# Markscheme

attempted set up with both regions ***(M1)***

*eg* $\text{shaded area} - \text{quarter circle}$ , $3\pi - \pi$ , $3\pi - \int_0^2 f = \int_2^6 f$

$\int_2^6 f(x)\mathrm{d}x = 2\pi$ ***A2 N2***

***[3 marks]***

***Total [7 marks]***