

- 1d. When $t = 0$, the volume of water in the container is 2.3 m^3 . It is known that the container is never completely full of water during the 4 hour period. [5 marks]

Find the minimum volume of empty space in the container during the 4 hour period.

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Let $f(x) = \frac{1}{\sqrt{2x-1}}$, for $x > \frac{1}{2}$.

- 2a. Find $\int (f(x))^2 dx$. [3 marks]

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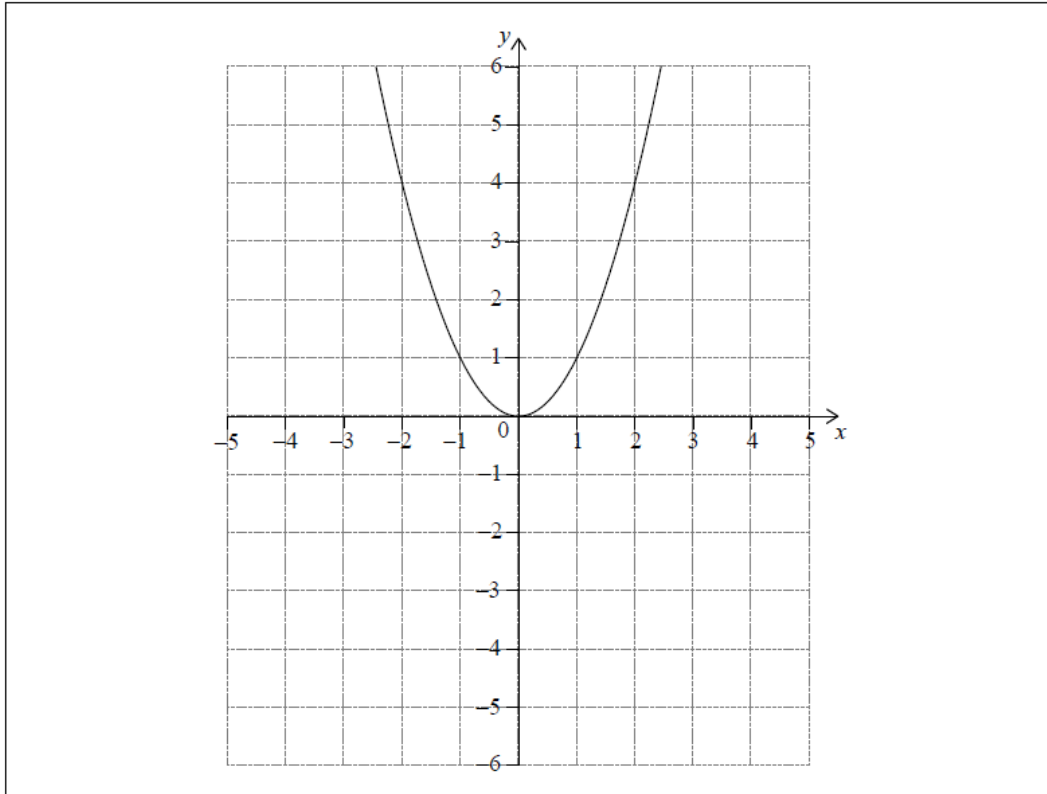
Let $g(x) = -(x - 1)^2 + 5$.

4a. Write down the coordinates of the vertex of the graph of g .

[1 mark]

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Let $f(x) = x^2$. The following diagram shows part of the graph of f .



The graph of g intersects the graph of f at $x = -1$ and $x = 2$.

4b. On the grid above, sketch the graph of g for $-2 \leq x \leq 4$.

[3 marks]

4c. Find the area of the region enclosed by the graphs of f and g .

[3 marks]

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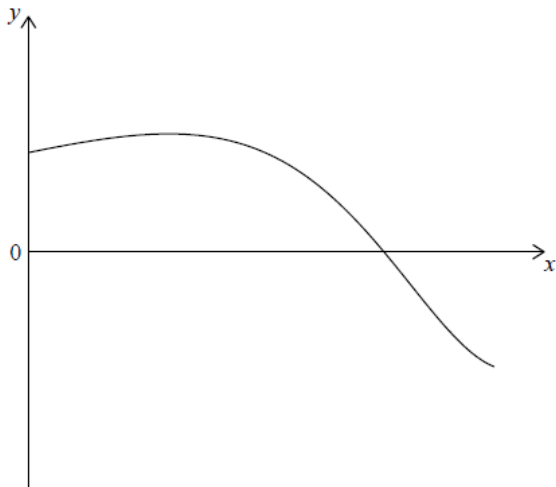
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Let $f(x) = \sin(e^x)$ for $0 \leq x \leq 1.5$. The following diagram shows the graph of f .



5a. Find the x -intercept of the graph of f .

[2 marks]

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5b. The region enclosed by the graph of f , the y -axis and the x -axis is rotated 360° about the x -axis. [3 marks]

Find the volume of the solid formed.

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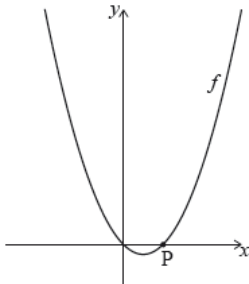
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Let $f(x) = x^2 - x$, for $x \in \mathbb{R}$. The following diagram shows part of the graph of f .

diagram not to scale



The graph of f crosses the x -axis at the origin and at the point $P(1, 0)$.

6a. Show that $f'(1) = 1$.

[3 marks]

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The line L is the normal to the graph of f at P .

6b. Find the equation of L in the form $y = ax + b$.

[3 marks]

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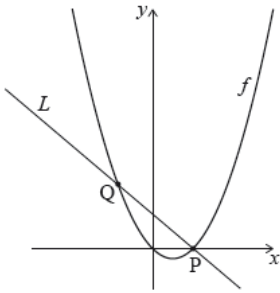
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The line L intersects the graph of f at another point Q , as shown in the following diagram.

diagram not to scale



6c. Find the x -coordinate of Q .

[4 marks]

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6d. Find the area of the region enclosed by the graph of f and the line L .

[6 marks]

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Let $f(x) = 6 - \ln(x^2 + 2)$, for $x \in \mathbb{R}$. The graph of f passes through the point $(p, 4)$, where $p > 0$.

7a. Find the value of p .

[2 marks]

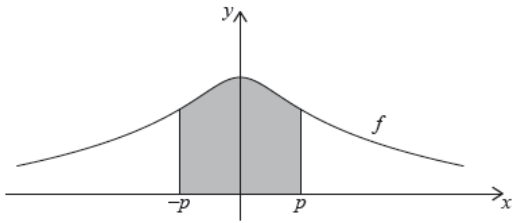
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7b. The following diagram shows part of the graph of f .

[3 marks]



The region enclosed by the graph of f , the x -axis and the lines $x = -p$ and $x = p$ is rotated 360° about the x -axis. Find the volume of the solid formed.

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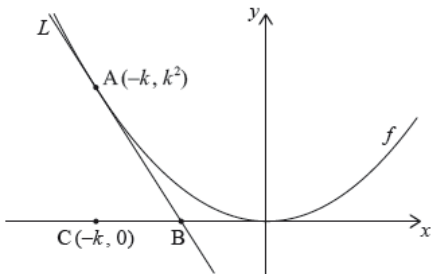
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Let $f(x) = x^2$. The following diagram shows part of the graph of f .

diagram not to scale



The line L is the tangent to the graph of f at the point $A(-k, k^2)$, and intersects the x -axis at point B . The point C is $(-k, 0)$.

8a. Write down $f'(x)$.

[1 mark]

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8d. Find the area of triangle ABC, giving your answer in terms of k .

[2 marks]

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Let $f(x) = xe^{-x}$ and $g(x) = -3f(x) + 1$.

The graphs of f and g intersect at $x = p$ and $x = q$, where $p < q$.

10a. Find the value of p and of q .

[3 marks]

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10b. Hence, find the area of the region enclosed by the graphs of f and g .

[3 marks]

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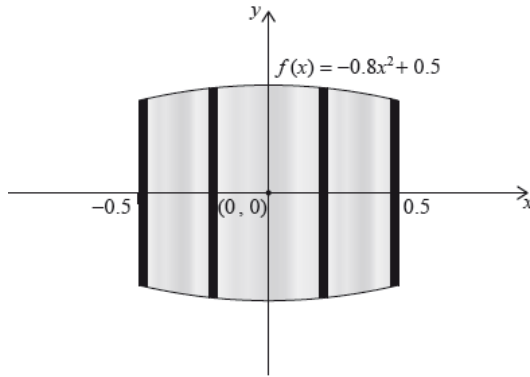
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All lengths in this question are in metres.

Let $f(x) = -0.8x^2 + 0.5$, for $-0.5 \leq x \leq 0.5$. Mark uses $f(x)$ as a model to create a barrel. The region enclosed by the graph of f , the x -axis, the line $x = -0.5$ and the line $x = 0.5$ is rotated 360° about the x -axis. This is shown in the following diagram.



11a. Use the model to find the volume of the barrel.

[3 marks]

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11b. The empty barrel is being filled with water. The volume $V \text{ m}^3$ of water in the barrel after t minutes is given by $V = 0.8(1 - e^{-0.1t})$. How long will it take for the barrel to be half-full? [3 marks]

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12c. Let A_T be the area of the triangle OPQ. Given that $A_T = kA_R$, find the value of k . [4 marks]

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Let $f(x) = x^2$ and $g(x) = 3\ln(x + 1)$, for $x > -1$.

13a. Solve $f(x) = g(x)$. [3 marks]

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13b. Find the area of the region enclosed by the graphs of f and g . [3 marks]

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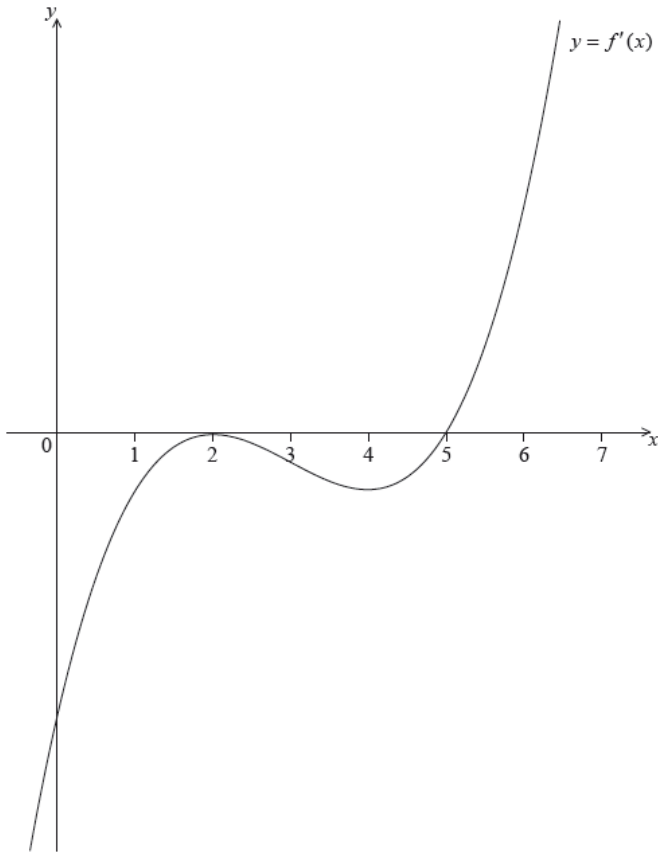
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Let $y = f(x)$, for $-0.5 \leq x \leq 6.5$. The following diagram shows the graph of f' , the derivative of f .



The graph of f' has a local maximum when $x = 2$, a local minimum when $x = 4$, and it crosses the x -axis at the point $(5, 0)$.

14a. Explain why the graph of f has a local minimum when $x = 5$.

[2 marks]

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14b. Find the set of values of x for which the graph of f is concave down.

[2 marks]

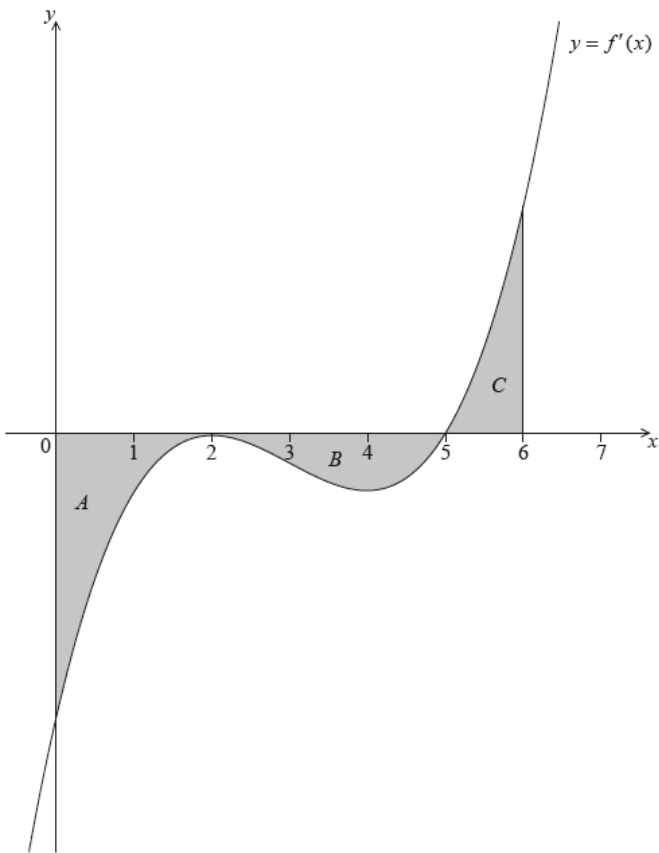
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14c. The following diagram shows the shaded regions A , B and C .

[5 marks]



The regions are enclosed by the graph of f' , the x -axis, the y -axis, and the line $x = 6$.

The area of region A is 12, the area of region B is 6.75 and the area of region C is 6.75.

Given that $f(0) = 14$, find $f(6)$.

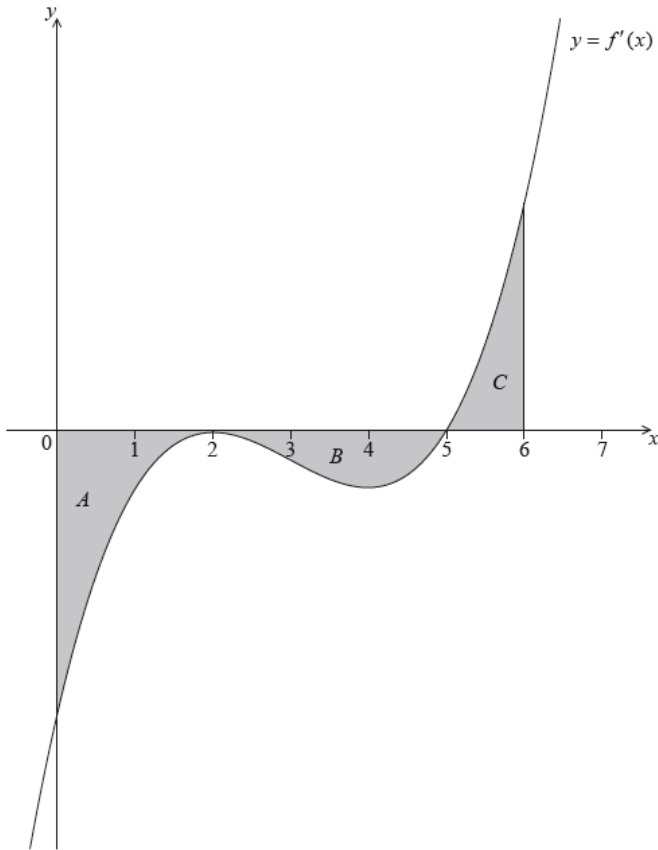
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14d. The following diagram shows the shaded regions A , B and C .

[6 marks]



The regions are enclosed by the graph of f' , the x -axis, the y -axis, and the line $x = 6$.

The area of region A is 12, the area of region B is 6.75 and the area of region C is 6.75.

Let $g(x) = (f(x))^2$. Given that $f'(6) = 16$, find the equation of the tangent to the graph of g at the point where $x = 6$.

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15. Let $f'(x) = 6x^2 - 5$. Given that $f(2) = -3$, find $f(x)$.

[6 marks]

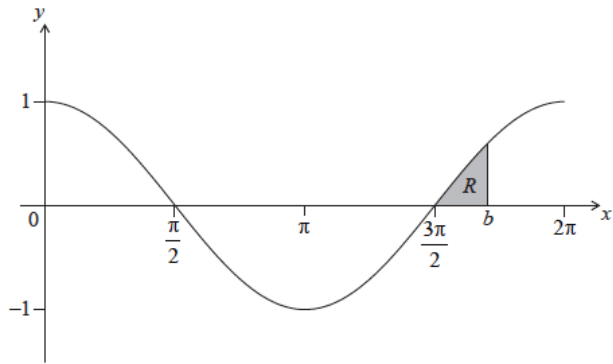
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16. Let $f(x) = \cos x$, for $0 \leq x \leq 2\pi$. The following diagram shows the graph of f . [8 marks]

There are
 x -intercepts at $x = \frac{\pi}{2}, \frac{3\pi}{2}$.



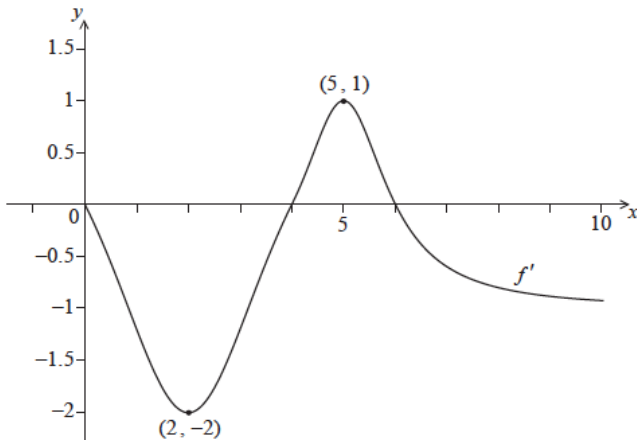
The shaded region R is enclosed by the graph of f , the line $x = b$, where $b > \frac{3\pi}{2}$, and the x -axis. The area of R is $\left(1 - \frac{\sqrt{3}}{2}\right)$. Find the value of b .

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Consider a function f , for $0 \leq x \leq 10$. The following diagram shows the graph of f' , the derivative of f .



The graph of f' passes through $(2, -2)$ and $(5, 1)$, and has x -intercepts at 0, 4 and 6.

17a. The graph of f has a local maximum point when $x = p$. State the value of p , and justify your answer. [3 marks]

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17b. Write down $f'(2)$. [1 mark]

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17c. Let $g(x) = \ln(f(x))$ and $f(2) = 3$.

[4 marks]

Find $g'(2)$.

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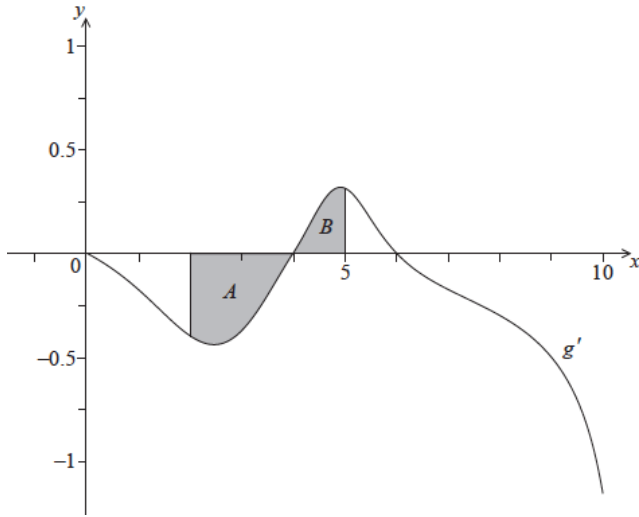
17d. Verify that $\ln 3 + \int_2^a g'(x)dx = g(a)$, where $0 \leq a \leq 10$.

[4 marks]

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17e. The following diagram shows the graph of g' , the derivative of g .

[4 marks]



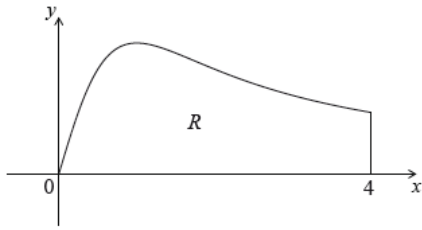
The shaded region A is enclosed by the curve, the x -axis and the line $x = 2$, and has area 0.66 units^2 .

The shaded region B is enclosed by the curve, the x -axis and the line $x = 5$, and has area 0.21 units^2 .

Find $g(5)$.

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18. The following diagram shows the graph of $f(x) = \frac{x}{x^2+1}$, for $0 \leq x \leq 4$, and the line $x = 4$. [6 marks]
 $x = 4$.



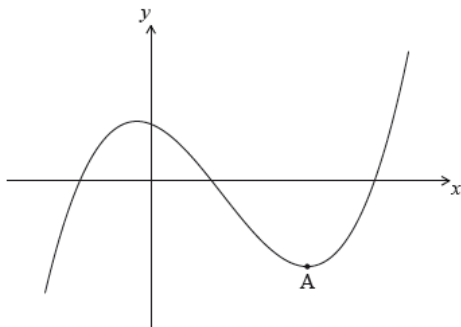
Let R be the region enclosed by the graph of f , the x -axis and the line $x = 4$.
 Find the area of R .

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The following diagram shows the graph of a function f . There is a local minimum point at A , where $x > 0$.



The derivative of f is given by $f'(x) = 3x^2 - 8x - 3$.

- 19a. Find the x -coordinate of A . [5 marks]

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19b. The y -intercept of the graph is at $(0, 6)$. Find an expression for $f(x)$. [6 marks]

The graph of a function g is obtained by reflecting the graph of f in the y -axis, followed by a translation of $\begin{pmatrix} m \\ n \end{pmatrix}$.

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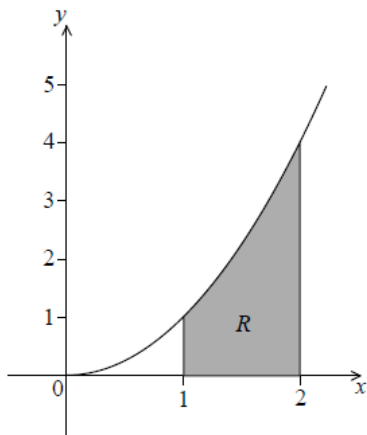
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Let
 $f(x) = x^2$.

20a. Find $\int_1^2 (f(x))^2 dx$. [4 marks]

20b. The following diagram shows part of the graph of f . [2 marks]



The shaded region R is enclosed by the graph of f , the x -axis and the lines $x = 1$ and $x = 2$.
 Find the volume of the solid formed when R is revolved 360° about the x -axis.

21. Let $\int_\pi^a \cos 2x dx = \frac{1}{2}$, where $\pi < a < 2\pi$. Find the value of a . [7 marks]

Let
 $f(x) = \frac{2x}{x^2+5}$.

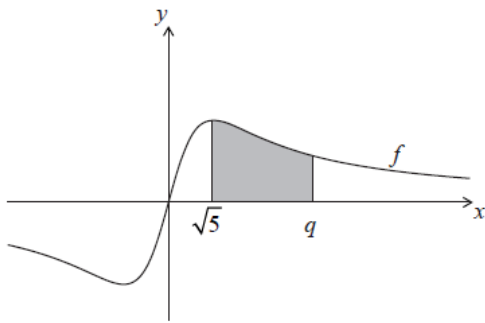
22a. Use the quotient rule to show that $f'(x) = \frac{10-2x^2}{(x^2+5)^2}$. [4 marks]

22b. Find $\int \frac{2x}{x^2+5} dx$.

[4 marks]

22c. The following diagram shows part of the graph of f .

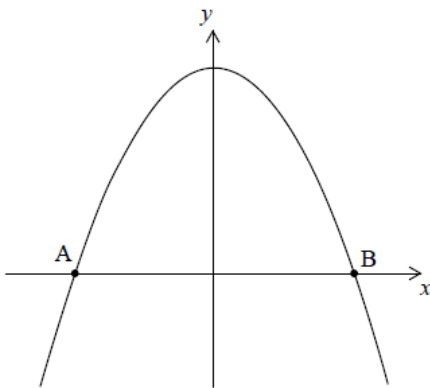
[7 marks]



The shaded region is enclosed by the graph of f , the x -axis, and the lines $x = \sqrt{5}$ and $x = q$. This region has an area of $\ln 7$. Find the value of q .

Let

$f(x) = 5 - x^2$. Part of the graph of f is shown in the following diagram.



The graph crosses the x -axis at the points A and B.

23a. Find the x -coordinate of A and of B.

[3 marks]

23b. The region enclosed by the graph of f and the x -axis is revolved 360° about the x -axis.

[3 marks]

Find the volume of the solid formed.

Let

$$f(x) = \frac{(\ln x)^2}{2}, \text{ for } x > 0.$$

24a. Show that $f'(x) = \frac{\ln x}{x}$.

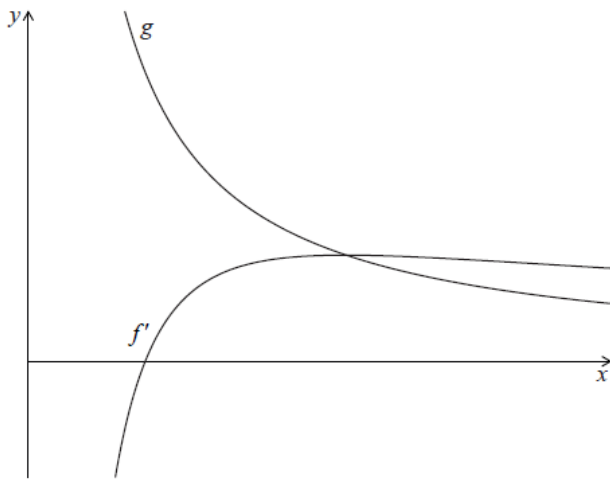
[2 marks]

24b. There is a minimum on the graph of f . Find the x -coordinate of this minimum.

[3 marks]

Let

$g(x) = \frac{1}{x}$. The following diagram shows parts of the graphs of f' and g .



The graph of f' has an x -intercept at $x = p$.

24c. Write down the value of p .

[2 marks]

24d. The graph of g intersects the graph of f' when $x = q$.

[3 marks]

Find the value of q .

24e. The graph of g intersects the graph of f' when $x = q$.

[5 marks]

Let R be the region enclosed by the graph of f' , the graph of g and the line $x = p$.

Show that the area of R is $\frac{1}{2}$.

Consider a function

$f(x)$ such that

$$\int_1^6 f(x) dx = 8.$$

25a. Find $\int_1^6 2f(x) dx$.

[2 marks]

25b. Find $\int_1^6 (f(x) + 2) dx$.

[4 marks]

Let

$$f(x) = (x - 1)(x - 4).$$

26a. Find the x -intercepts of the graph of f .

[3 marks]

26b. The region enclosed by the graph of f and the x -axis is rotated 360° about the x -axis.

[3 marks]

Find the volume of the solid formed.

27. Let $f(x) = \int \frac{12}{2x-5} dx$, $x > \frac{5}{2}$. The graph of f passes through $(4, 0)$.

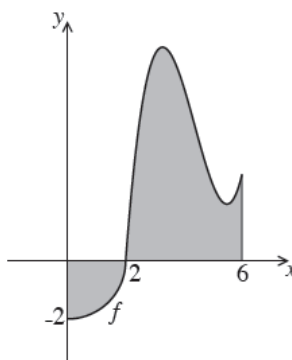
[6 marks]

Find $f(x)$.

The following is the graph of a function

f , for

$$0 \leq x \leq 6.$$



The first part of the graph is a quarter circle of radius 2 with centre at the origin.

28a. (a) Find $\int_0^2 f(x) dx$.

[7 marks]

(b) The shaded region is enclosed by the graph of f , the x -axis, the y -axis and the line $x = 6$. The area of this region is 3π .

Find $\int_2^6 f(x) dx$.

28b. Find $\int_0^2 f(x) dx$.

[4 marks]

28c. The shaded region is enclosed by the graph of f , the x -axis, the y -axis and the line $x = 6$. The area of this region is 3π .

[3 marks]

Find $\int_2^6 f(x) dx$.

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