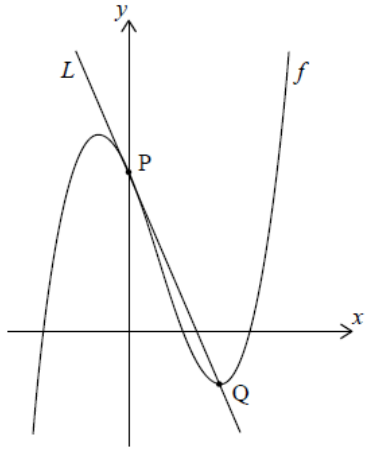


Calculus Review [328 marks]

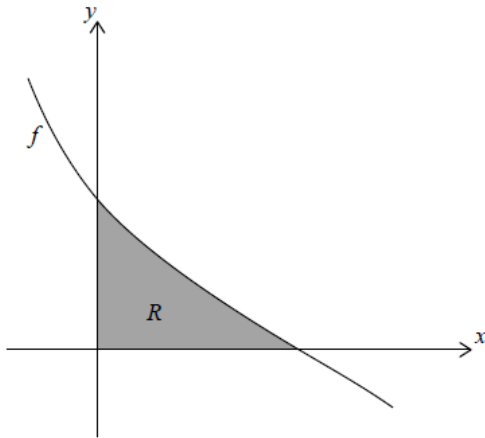
Let $f(x) = x^3 - 2x^2 + ax + 6$. Part of the graph of f is shown in the following diagram.



The graph of f crosses the y -axis at the point P . The line L is tangent to the graph of f at P .

- 1a. Find the coordinates of P . [2 marks]
- 1b. Find $f'(x)$. [2 marks]
- 1c. Hence, find the equation of L in terms of a . [4 marks]
- 1d. The graph of f has a local minimum at the point Q . The line L passes through Q . [8 marks]
Find the value of a .

2. Let $f(x) = \frac{6-2x}{\sqrt{16+6x-x^2}}$. The following diagram shows part of the graph of f . [8 marks]



The region R is enclosed by the graph of f , the x -axis, and the y -axis. Find the area of R .

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Let $f(x) = \frac{1}{\sqrt{2x-1}}$, for $x > \frac{1}{2}$.

3a. Find $\int (f(x))^2 dx$.

[3 marks]

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5c. Find the values of x for which the graph of f is concave-down.

[3 marks]

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Consider a function f . The line L_1 with equation $y = 3x + 1$ is a tangent to the graph of f when $x = 2$

6a. Write down $f'(2)$.

[2 marks]

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6b. Find $f(2)$.

[2 marks]

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6d. Let L_2 be the tangent to the graph of g at P . L_1 intersects L_2 at the point Q .

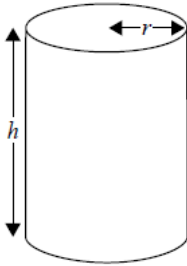
[7 marks]

Find the y-coordinate of Q .

Dotted lines for writing the solution.

A closed cylindrical can with radius r centimetres and height h centimetres has a volume of 20π cm^3 .

diagram not to scale



7a. Express h in terms of r .

[2 marks]

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7c. Given that there is a minimum value for C , find this minimum value in terms of π . [9 marks]

The response area consists of a large rectangle with a solid black border. Inside this rectangle, there are 22 horizontal dotted lines, evenly spaced, extending from the left margin to the right margin, providing a guide for writing the solution.

Let $f(x) = 1 + e^{-x}$ and $g(x) = 2x + b$, for $x \in \mathbb{R}$, where b is a constant.

8a. Find $(g \circ f)(x)$.

[2 marks]

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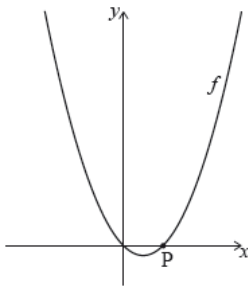
8b. Given that $\lim_{x \rightarrow +\infty} (g \circ f)(x) = -3$, find the value of b .

[4 marks]

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Let $f(x) = x^2 - x$, for $x \in \mathbb{R}$. The following diagram shows part of the graph of f .

diagram not to scale



The graph of f crosses the x -axis at the origin and at the point $P(1, 0)$.

9a. Show that $f'(1) = 1$.

[3 marks]

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The line L is the normal to the graph of f at P .

9b. Find the equation of L in the form $y = ax + b$.

[3 marks]

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10a. Find $\int xe^{x^2-1} dx$.

[4 marks]

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10b. Find $f(x)$, given that $f'(x) = xe^{x^2-1}$ and $f(-1) = 3$.

[3 marks]

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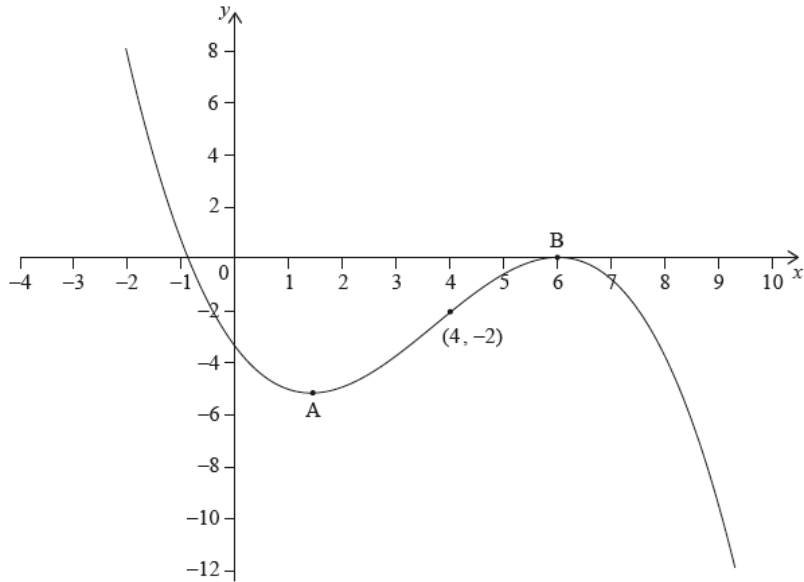
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The following diagram shows the graph of f' , the derivative of f .



The graph of f' has a local minimum at A, a local maximum at B and passes through $(4, -2)$.

The point $P(4, 3)$ lies on the graph of the function, f .

11a. Write down the gradient of the curve of f at P.

[1 mark]

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11b. Find the equation of the normal to the curve of f at P.

[3 marks]

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11c. Determine the concavity of the graph of f when $4 < x < 5$ **and** justify your answer. [2 marks]

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A quadratic function f can be written in the form $f(x) = a(x - p)(x - 3)$. The graph of f has axis of symmetry $x = 2.5$ and y -intercept at $(0, -6)$

12a. Find the value of p .

[3 marks]

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12b. Find the value of a .

[3 marks]

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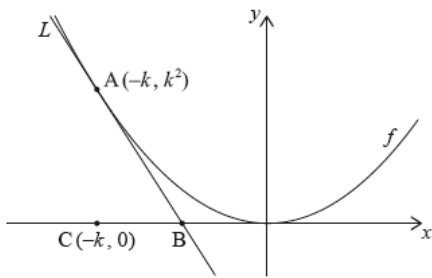
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Let $f(x) = x^2$. The following diagram shows part of the graph of f .

diagram not to scale



The line L is the tangent to the graph of f at the point $A(-k, k^2)$, and intersects the x -axis at point B . The point C is $(-k, 0)$.

13a. Write down $f'(x)$.

[1 mark]

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13b. Find the gradient of L .

[2 marks]

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13c. Show that the x -coordinate of B is $-\frac{k}{2}$.

[5 marks]

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13d. Find the area of triangle ABC, giving your answer in terms of k .

[2 marks]

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14. Let $f'(x) = \frac{3x^2}{(x^3+1)^5}$. Given that $f(0) = 1$, find $f(x)$.

[6 marks]

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Let $f(x) = \cos x$.

15a. (i) Find the first four derivatives of $f(x)$.

[4 marks]

(ii) Find $f^{(19)}(x)$.

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Let $f(x) = \sqrt{4x + 5}$, for $x \geq -1.25$.

16a. Find $f'(1)$.

[4 marks]

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Consider another function g . Let R be a point on the graph of g . The x -coordinate of R is 1. The equation of the tangent to the graph at R is $y = 3x + 6$.

16b. Write down $g'(1)$.

[2 marks]

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16c. Find $g(1)$.

[2 marks]

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16d. Let $h(x) = f(x) \times g(x)$. Find the equation of the tangent to the graph of h at the point where $x = 1$. [7 marks]

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Let $f'(x) = \frac{6-2x}{6x-x^2}$, for $0 < x < 6$.

The graph of f has a maximum point at P.

17a. Find the x -coordinate of P. [3 marks]

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The y -coordinate of P is $\ln 27$.

17b. Find $f(x)$, expressing your answer as a single logarithm.

[8 marks]

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17c. The graph of f is transformed by a vertical stretch with scale factor $\frac{1}{\ln 3}$. The image of P under this transformation has coordinates (a, b) .

Find the value of a and of b , where $a, b \in \mathbb{N}$.

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18c. Let A_T be the area of the triangle OPQ. Given that $A_T = kA_R$, find the value of k . [4 marks]

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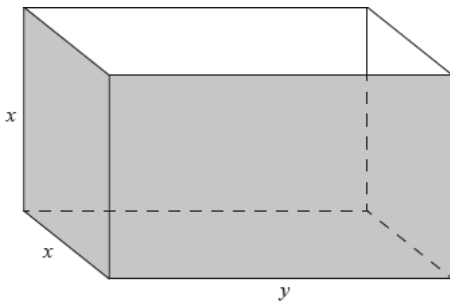
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Fred makes an open metal container in the shape of a cuboid, as shown in the following diagram.



The container has height x m, width x m and length y m. The volume is 36 m^3 .

Let $A(x)$ be the outside surface area of the container.

19a. Show that $A(x) = \frac{108}{x} + 2x^2$. [4 marks]

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19b. Find $A'(x)$.

[2 marks]

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19c. Given that the outside surface area is a minimum, find the height of the container.

[5 marks]

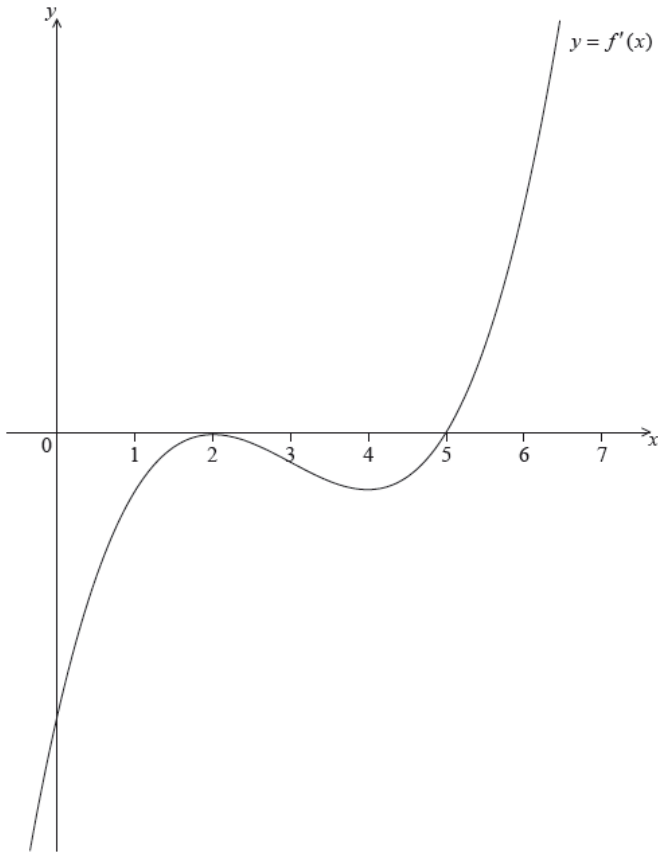
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19d. Fred paints the outside of the container. A tin of paint covers a surface area of 10 m^2 and costs \$20. Find the total cost of the tins needed to paint the container.

[5 marks]

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Let $y = f(x)$, for $-0.5 \leq x \leq 6.5$. The following diagram shows the graph of f' , the derivative of f .



The graph of f' has a local maximum when $x = 2$, a local minimum when $x = 4$, and it crosses the x -axis at the point $(5, 0)$.

20a. Explain why the graph of f has a local minimum when $x = 5$.

[2 marks]

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20b. Find the set of values of x for which the graph of f is concave down.

[2 marks]

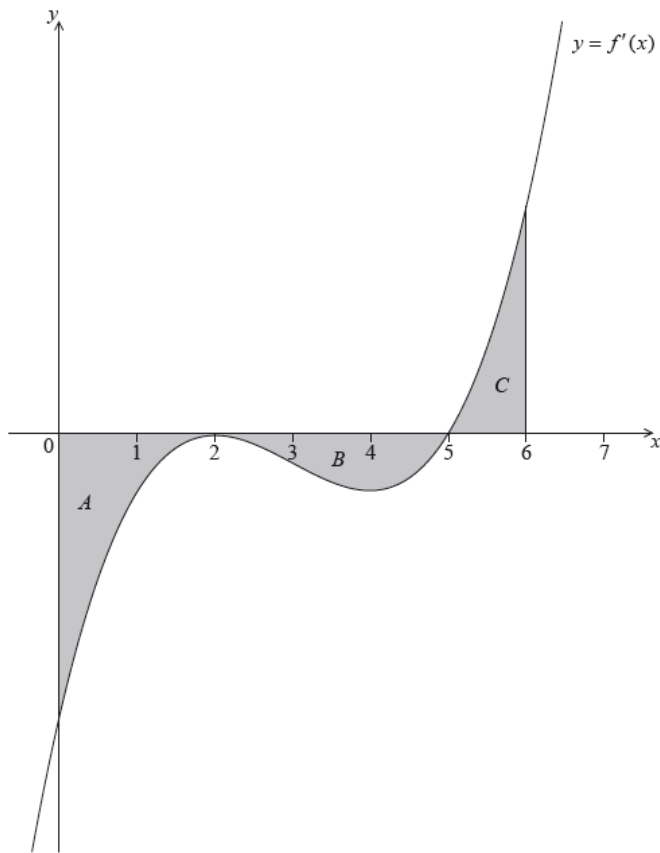
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20c. The following diagram shows the shaded regions A , B and C .

[5 marks]



The regions are enclosed by the graph of f' , the x -axis, the y -axis, and the line $x = 6$.

The area of region A is 12, the area of region B is 6.75 and the area of region C is 6.75.

Given that $f(0) = 14$, find $f(6)$.

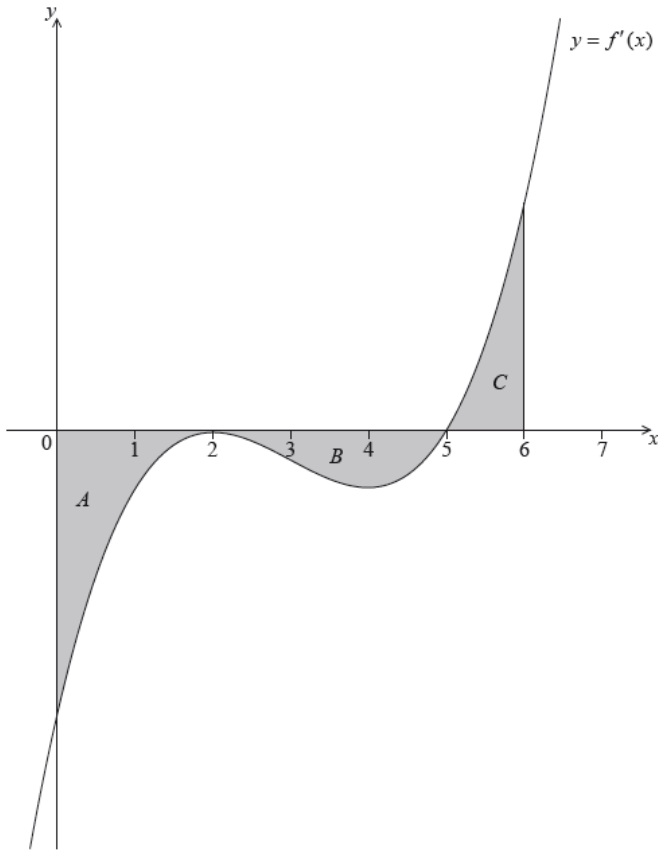
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20d. The following diagram shows the shaded regions A , B and C .

[6 marks]



The regions are enclosed by the graph of f' , the x -axis, the y -axis, and the line $x = 6$.

The area of region A is 12, the area of region B is 6.75 and the area of region C is 6.75.

Let $g(x) = (f(x))^2$. Given that $f'(6) = 16$, find the equation of the tangent to the graph of g at the point where $x = 6$.

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A function f has its derivative given by $f'(x) = 3x^2 - 2kx - 9$, where k is a constant.

21a. Find $f''(x)$.

[2 marks]

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21b. The graph of f has a point of inflexion when $x = 1$.

[3 marks]

Show that $k = 3$.

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21c. Find $f'(-2)$.

[2 marks]

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21d. Find the equation of the tangent to the curve of f at $(-2, 1)$, giving your answer in the form $y = ax + b$. [4 marks]

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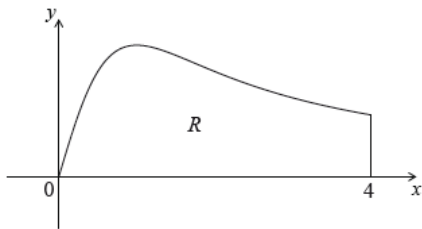
21e. Given that $f'(-1) = 0$, explain why the graph of f has a local maximum when $x = -1$. [3 marks]

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22. The following diagram shows the graph of $f(x) = \frac{x}{x^2+1}$, for $0 \leq x \leq 4$, and the line $x = 4$. [6 marks]



Let R be the region enclosed by the graph of f , the x -axis and the line $x = 4$. Find the area of R .

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Let

$$f(x) = px^3 + px^2 + qx.$$

23a. Find $f'(x)$.

[2 marks]

23b. Given that $f'(x) \geq 0$, show that $p^2 \leq 3pq$.

[5 marks]

Let

$$f(x) = \frac{2x}{x^2+5}.$$

24a. Use the quotient rule to show that $f'(x) = \frac{10-2x^2}{(x^2+5)^2}$.

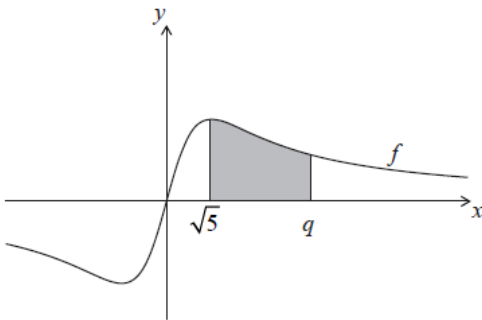
[4 marks]

24b. Find $\int \frac{2x}{x^2+5} dx$.

[4 marks]

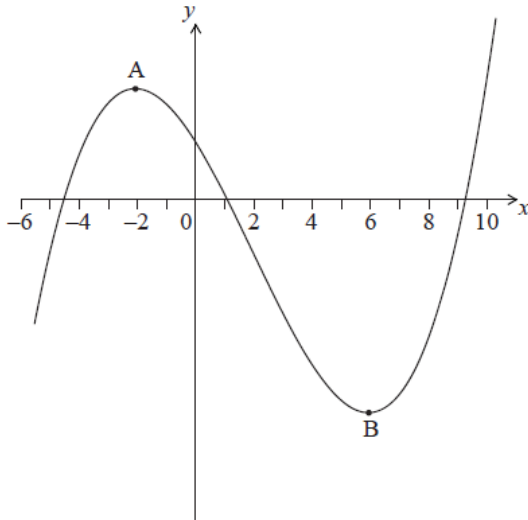
24c. The following diagram shows part of the graph of f .

[7 marks]



The shaded region is enclosed by the graph of f , the x -axis, and the lines $x = \sqrt{5}$ and $x = q$. This region has an area of $\ln 7$. Find the value of q .

The following diagram shows part of the graph of $y = f(x)$.



The graph has a local maximum at A , where $x = -2$, and a local minimum at B , where $x = 6$.

25a. On the following axes, sketch the graph of $y = f'(x)$.

[4 marks]

25b. Write down the following in order from least to greatest: $f(0)$, $f'(6)$, $f''(-2)$.

[2 marks]

Consider the functions

$f(x)$,

$g(x)$ and

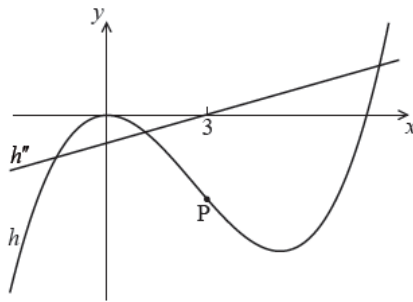
$h(x)$. The following table gives some values associated with these functions.

x	2	3
$f(x)$	2	3
$g(x)$	-14	-18
$f'(x)$	1	1
$g'(x)$	-5	-3
$h''(x)$	-6	0

26a. Write down the value of $g(3)$, of $f'(3)$, and of $h''(2)$.

[3 marks]

The following diagram shows parts of the graphs of h and h'' .



There is a point of inflexion on the graph of h at P, when $x = 3$.

26b. Explain why P is a point of inflexion.

[2 marks]

Given that
 $h(x) = f(x) \times g(x)$,

26c. find the y -coordinate of P.

[2 marks]

26d. find the equation of the normal to the graph of h at P.

[7 marks]

27. A rocket moving in a straight line has velocity v km s⁻¹ and displacement s km at time t seconds. The velocity v is given by $v(t) = 6e^{2t} + t$. When $t = 0$, $s = 10$.

Find an expression for the displacement of the rocket in terms of t .

Let
 $f(x) = \sin x + \frac{1}{2}x^2 - 2x$, for
 $0 \leq x \leq \pi$.

28a. Find $f'(x)$.

[3 marks]

Let
 g be a quadratic function such that
 $g(0) = 5$. The line
 $x = 2$ is the axis of symmetry of the graph of
 g .

28b. Find $g(4)$.

[3 marks]

The function
 g can be expressed in the form
 $g(x) = a(x - h)^2 + 3$.

- 28c. (i) Write down the value of h . [4 marks]
(ii) Find the value of a .

28d. Find the value of x for which the tangent to the graph of f is parallel to the tangent to [6 marks]
the graph of g .