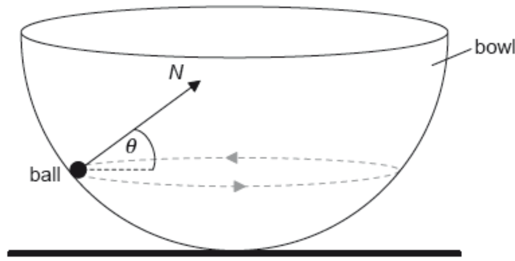
6 *[143 marks]*

A small ball of mass m is moving in a horizontal circle on the inside surface of a frictionless hemispherical bowl.



The normal reaction force N makes an angle θ to the horizontal.

1a. State the direction of the resultant force on the ball. *[1 mark]*

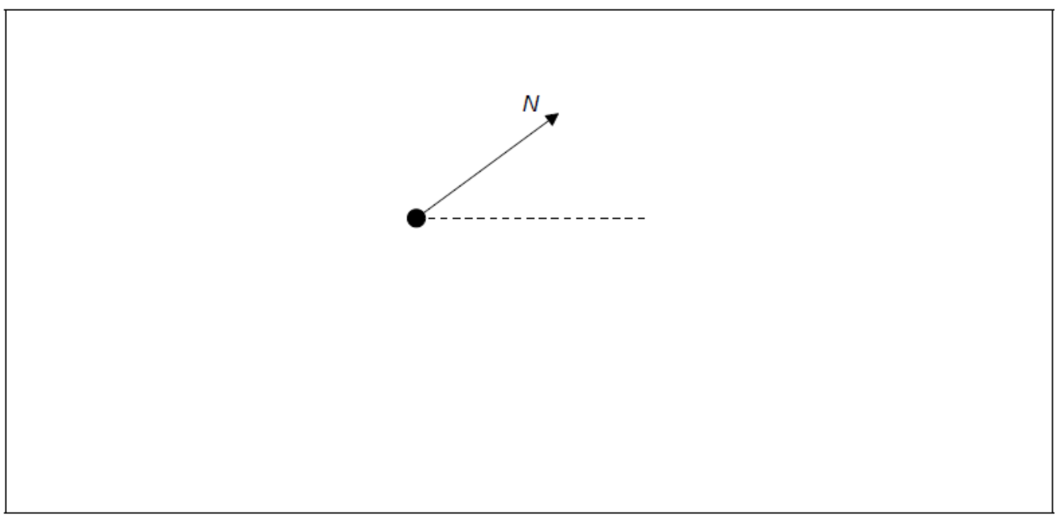
Markscheme

towards the centre «of the circle» / horizontally to the right

Do not accept towards the centre of the bowl

[1 mark]

1b. On the diagram, construct an arrow of the correct length to represent the weight of the ball. *[2 marks]*



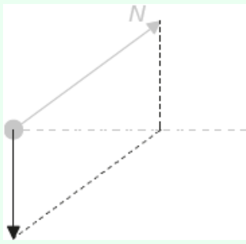
Markscheme

downward vertical arrow of any length

arrow of correct length

Judge the length of the vertical arrow by eye. The construction lines are not required. A label is not required

eg:



[2 marks]

1c. Show that the magnitude of the net force *F* on the ball is given by the following equation. *[3 marks]*

$$F = \frac{mg}{\tan\theta}$$

Markscheme

ALTERNATIVE 1

$F = N\cos\theta$

$mg = N\sin\theta$

dividing/substituting to get result

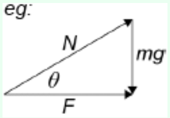
ALTERNATIVE 2

right angle triangle drawn with *F*, *N* and *W/mg* labelled

angle correctly labelled and arrows on forces in correct directions

correct use of trigonometry leading to the required relationship

eg:



$\tan\theta = \frac{O}{A} = \frac{mg}{F}$

[3 marks]

1d. The radius of the bowl is 8.0 m and $\theta = 22°$. Determine the speed of the ball. *[4 marks]*

Markscheme

$\frac{mg}{\tan\theta} = m\frac{v^2}{r}$

$r = R\cos\theta$

$v = \sqrt{\frac{gR\cos^2\theta}{\sin\theta}} / \sqrt{\frac{gR\cos\theta}{\tan\theta}} / \sqrt{\frac{9.81\times8.0\cos22}{\tan22}}$

v = 13.4/13 «ms^{-1}»

*Award **[4]** for a bald correct answer*

*Award **[3]** for an answer of 13.9/14 «ms^{-1}». MP2 omitted*

[4 marks]

1e. Outline whether this ball can move on a horizontal circular path of radius equal to the radius of the bowl. *[2 marks]*

Markscheme

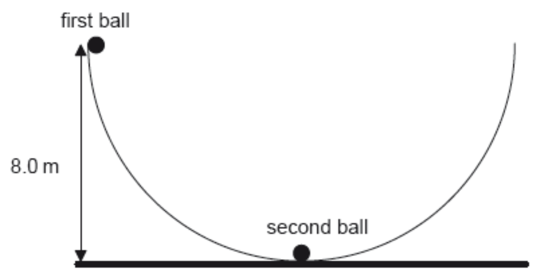
there is no force to balance the weight/N is horizontal

so no / it is not possible

Must see correct justification to award MP2

[2 marks]

1f. A second identical ball is placed at the bottom of the bowl and the first ball is displaced so that its height from the horizontal is equal to 8.0 m. *[3 marks]*



The first ball is released and eventually strikes the second ball. The two balls remain in contact. Determine, in m, the maximum height reached by the two balls.

Markscheme

speed before collision v = «$\sqrt{2gR}$ =» 12.5 «ms^{-1}»

«from conservation of momentum» common speed after collision is $\frac{1}{2}$ initial speed «$v_c = \frac{12.5}{2}$ = 6.25 ms^{-1}»

h = «$\frac{v_c^2}{2g} = \frac{6.25^2}{2\times9.81}$» 2.0 «m»

Allow 12.5 from incorrect use of kinematics equations

*Award **[3]** for a bald correct answer*

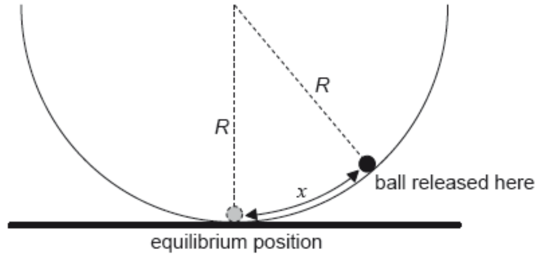
*Award **[0]** for mg(8) = 2mgh leading to h = 4 m if done in one step.*

Allow ECF from MP1

Allow ECF from MP2

[3 marks]

The ball is now displaced through a small distance x from the bottom of the bowl and is then released from rest.



The magnitude of the force on the ball towards the equilibrium position is given by

$$\frac{mgx}{R}$$

where R is the radius of the bowl.

1g. Outline why the ball will perform simple harmonic oscillations about the equilibrium position. *[1 mark]*

Markscheme

the «restoring» force/acceleration is proportional to displacement

Direction is not required

[1 mark]

1h. Show that the period of oscillation of the ball is about 6 s. *[2 marks]*

Markscheme

ω = «$\sqrt{\frac{g}{R}}$» = $\sqrt{\frac{9.81}{8.0}}$ «= 1.107 s^{-1}»

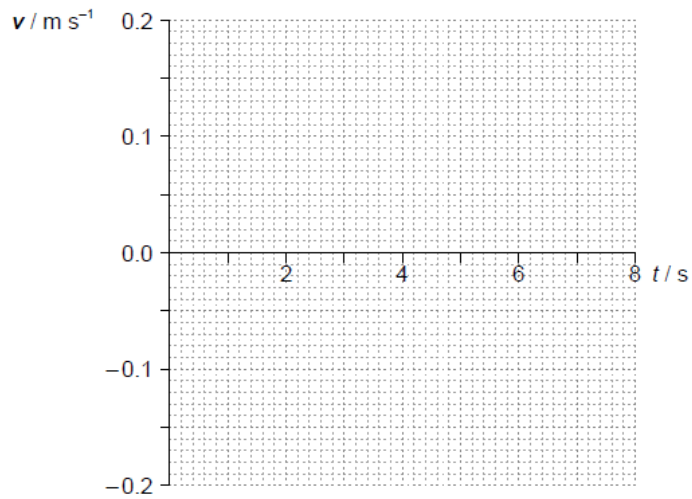
T = «$\frac{2\pi}{\omega}$ = $\frac{2\pi}{1.107}$ =» 5.7 «s»

Allow use of or *g = 9.8 or 10*

*Award **[0]** for a substitution into* $T = 2\pi\sqrt{\frac{l}{g}}$

[2 marks]

1i. The amplitude of oscillation is 0.12 m. On the axes, draw a graph to show the variation with time t of the velocity v of the ball during one period. *[3 marks]*



Markscheme

sine graph

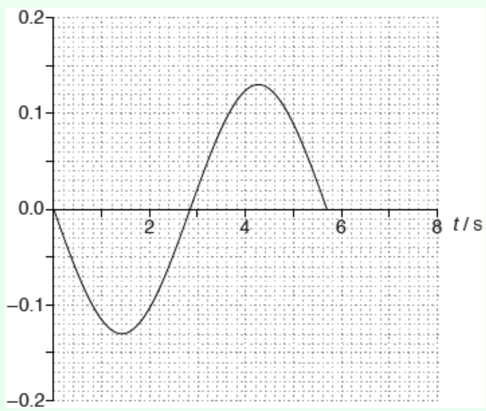
correct amplitude «0.13 m s^{-1}»

correct period and only 1 period shown

Accept ± sine for shape of the graph. Accept 5.7 s or 6.0 s for the correct period.

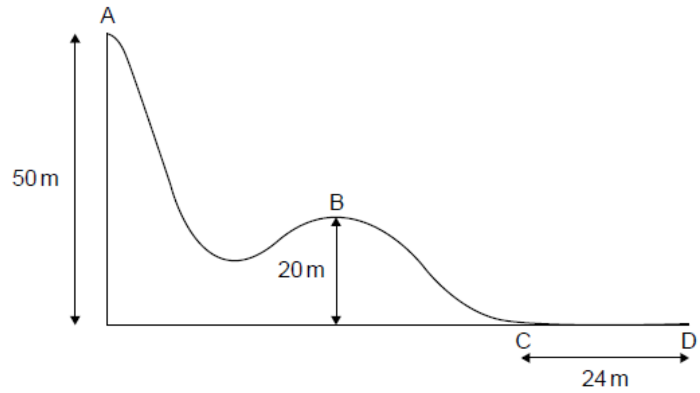
Amplitude should be correct to $\pm\frac{1}{2}$ square for MP2

eg: v /m s^{-1}



[3 marks]

The diagram below shows part of a downhill ski course which starts at point A, 50 m above level ground. Point B is 20 m above level ground.



A skier of mass 65 kg starts from rest at point A and during the ski course some of the gravitational potential energy transferred to kinetic energy.

2a. From A to B, 24 % of the gravitational potential energy transferred to kinetic energy. Show that the velocity at B is 12 m s $^{-1}$. *[2 marks]*

Markscheme

$\frac{1}{2}v^2 = 0.24\text{gh}$

$v = 11.9$ «m s^{-1}»

Award GPE lost = 65 × 9.81 × 30 = «19130 J»

Must see the 11.9 value for MP2, not simply 12.

Allow g = 9.8 ms^{-2}.

2b. Some of the gravitational potential energy transferred into internal energy of the skis, slightly increasing their temperature. Distinguish between internal energy and temperature. *[2 marks]*

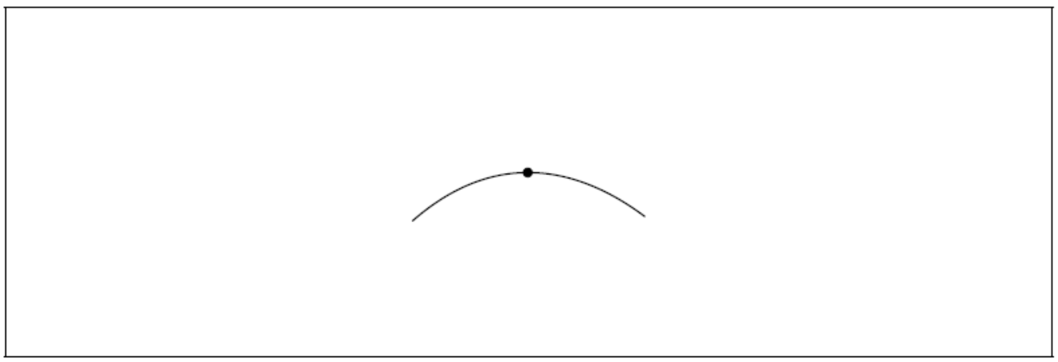
Markscheme

internal energy is the total KE «and PE» of the molecules/particles/atoms in an object

temperature is a measure of the average KE of the molecules/particles/atoms

*Award **[1 max]** if there is no mention of molecules/particles/atoms.*

2c. The dot on the following diagram represents the skier as she passes point B.
Draw and label the vertical forces acting on the skier. *[2 marks]*



Markscheme

arrow vertically downwards from dot labelled weight/W/mg/gravitational force/F_g/$F_{gravitational}$ ***AND*** arrow vertically upwards from dot labelled reaction force/R/normal contact force/N/F_N

W > R

Do not allow gravity.
Do not award MP1 if additional 'centripetal' force arrow is added.
Arrows must connect to dot.
Ignore any horizontal arrow labelled friction.
Judge by eye for MP2. Arrows do not have to be correctly labelled or connect to dot for MP2.

2d. The hill at point B has a circular shape with a radius of 20 m. Determine whether the skier will lose contact with the ground at point B. *[3 marks]*

Markscheme

ALTERNATIVE 1

recognition that centripetal force is required / $\frac{mv^2}{r}$ seen

= 468 «N»

W/640 N (weight) is larger than the centripetal force required, so the skier does not lose contact with the ground

ALTERNATIVE 2

recognition that centripetal acceleration is required / $\frac{v^2}{r}$ seen

a = 7.2 «ms^{-2}»

g is larger than the centripetal acceleration required, so the skier does not lose contact with the ground

ALTERNATIVE 3

recognition that to lose contact with the ground centripetal force ≥ weight

calculation that v ≥ 14 «ms^{-1}»

comment that 12 «ms^{-1}» is less than 14 «ms^{-1}» so the skier does not lose contact with the ground

ALTERNATIVE 4

recognition that centripetal force is required / $\frac{mv^2}{r}$ seen

calculation that reaction force = 172 «N»

reaction force > 0 so the skier does not lose contact with the ground

Do not award a mark for the bald statement that the skier does not lose contact with the ground.

2e. The skier reaches point C with a speed of 8.2 m s^{-1}. She stops after a distance of 24 m at point D. *[3 marks]*

Determine the coefficient of dynamic friction between the base of the skis and the snow. Assume that the frictional force is constant and that air resistance can be neglected.

Markscheme

ALTERNATIVE 1

$0 = 8.2^2 + 2 \times a \times 24$ therefore a = «–»1.40 «m s^{-2}»

friction force = ma = 65 × 1.4 = 91 «N»

coefficient of friction = $\frac{91}{65 \times 9.81}$ = 0.14

ALTERNATIVE 2

$KE = \frac{1}{2}mv^2$ = 0.5 x 65 x 8.2^2 = 2185 «J»

friction force = KE/distance = 2185/24 = 91 «N»

coefficient of friction = $\frac{91}{65 \times 9.81}$ = 0.14

Allow ECF from MP1.

At the side of the course flexible safety nets are used. Another skier of mass 76 kg falls normally into the safety net with speed 9.6 m s^{-1}.

2f. Calculate the impulse required from the net to stop the skier and state an appropriate unit for your answer. *[2 marks]*

Markscheme

«76 × 9.6»= 730
Ns ***OR*** kg ms^{-1}

2g. Explain, with reference to change in momentum, why a flexible safety net is less likely to harm the skier than a rigid barrier. *[2 marks]*

Markscheme

safety net extends stopping time

$F = \frac{\Delta p}{\Delta t}$ therefore F is smaller «with safety net»

OR

force is proportional to rate of change of momentum therefore F is smaller «with safety net»

Accept reverse argument.

The gravitational potential due to the Sun at its surface is -1.9×10^{11} J kg^{-1}. The following data are available.

Mass of Earth	= 6.0×10^{24} kg
Distance from Earth to Sun	= 1.5×10^{11} m
Radius of Sun	= 7.0×10^{8} m

3a. Outline why the gravitational potential is negative. *[2 marks]*

Markscheme

potential is defined to be zero at infinity

so a positive amount of work needs to be supplied for a mass to reach infinity

3b. The gravitational potential due to the Sun at a distance r from its centre is V_S. Show that rV_S = constant. *[1 mark]*

Markscheme

$V_S = -\frac{GM}{r}$ so $r \times V_S$ «$= -GM$» = constant because G and M are constants

3c. Calculate the gravitational potential energy of the Earth in its orbit around the Sun. Give your answer to an appropriate number of significant figures. *[2 marks]*

Markscheme

$GM = 1.33 \times 10^{20}$ «J m kg^{-1}»

GPE at Earth orbit «$= -\frac{1.33\times10^{20}\times6.0\times10^{24}}{1.5\times10^{11}}$» = «–» 5.3×10^{33} «J»

*Award **[1 max]** unless answer is to 2 sf.*

Ignore addition of Sun radius to radius of Earth orbit.

3d. Calculate the total energy of the Earth in its orbit. *[2 marks]*

Markscheme

ALTERNATIVE 1
work leading to statement that kinetic energy $\frac{GMm}{2r}$, ***AND*** kinetic energy evaluated to be «+» 2.7×10^{33} «J»

energy «= PE + KE = answer to (b)(ii) + 2.7×10^{33}» = «–» 2.7×10^{33} «J»

ALTERNATIVE 2
statement that kinetic energy is $= -\frac{1}{2}$ gravitational potential energy in orbit

so energy «$= \frac{\text{answer to (b)(ii)}}{2}$» = «–» 2.7×10^{33} «J»

Various approaches possible.

3e. An asteroid strikes the Earth and causes the orbital speed of the Earth to suddenly decrease. Suggest the ways in which the orbit of the Earth will change. *[2 marks]*

Markscheme

«KE will initially decrease so» total energy decreases
OR
«KE will initially decrease so» total energy becomes more negative

Earth moves closer to Sun

new orbit with greater speed «but lower total energy»

changes ellipticity of orbit

3f. Outline, in terms of the force acting on it, why the Earth remains in a circular orbit around the Sun. *[2 marks]*

Markscheme

centripetal force is required

and is provided by gravitational force between Earth and Sun

*Award **[1 max]** for statement that there is a "centripetal force of gravity" without further qualification.*

4a. (i) Define *gravitational field strength.*

(ii) State the SI unit for gravitational field strength. *[2 marks]*

Markscheme

(i) «gravitational» force per unit mass on a «small **or** test» mass

(ii) N kg^{-1}

Award mark if N kg^{-1} is seen, treating any further work as neutral.
Do not accept bald m s^{-2}

4b. A planet orbits the Sun in a circular orbit with orbital period *T* and orbital radius *R*. The mass of the Sun is *M*. *[4 marks]*

(i) Show that $T = \sqrt{\frac{4\pi^2 R^3}{GM}}$.

(ii) The Earth's orbit around the Sun is almost circular with radius 1.5×10^{11} m. Estimate the mass of the Sun.

Markscheme

i

clear evidence that *v* in $v^2 = \frac{4\pi^2 R^2}{T^2}$ is equated to orbital speed $\sqrt{\frac{GM}{R}}$

OR

clear evidence that centripetal force is equated to gravitational force

OR

clear evidence that *a* in $a = \frac{v^2}{R}$ etc is equated to *g* in $g = \frac{GM}{R^2}$ with consistent use of symbols

Minimum is a statement that
$\sqrt{\frac{GM}{R}}$ *is the orbital speed which is then used in*
$v = \frac{2\pi R}{T}$
Minimum is $F_c = F_g$ ignore any signs.
Minimum is g = a.

substitutes and re-arranges to obtain result
Allow any legitimate method not identified here.
Do not allow spurious methods involving equations of shm etc

$$\ll T = \sqrt{\frac{4\pi^2 R}{\left(\frac{GM}{R^2}\right)}} = \sqrt{\frac{4\pi^2 R^3}{GM}} \gg$$

ii

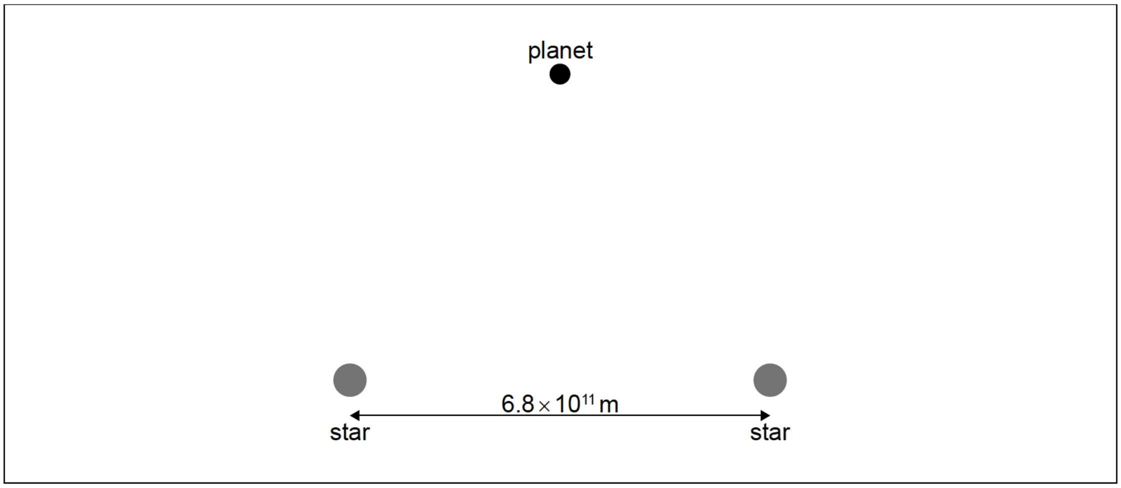
«T = 365 × 24 × 60 × 60 = 3.15 × 10^7 s»

$$M = \ll \frac{4\pi^2 R^3}{GT^2} = \gg = \frac{4 \times 3.14^2 \times \left(1.5 \times 10^{11}\right)^3}{6.67 \times 10^{-11} \times \left(3.15 \times 10^7\right)^2}$$

2×10^{30}«kg»

Allow use of 3.16 x 10^7 s for year length (quoted elsewhere in paper).
Condone error in power of ten in MP1.
*Award **[1 max]** if incorrect time used (24 h is sometimes seen, leading to 2.66 x 10^{35} kg).*
Units are not required, but if not given assume kg and mark POT accordingly if power wrong.
*Award **[2]** for a bald correct answer.*
No sf penalty here.

The diagram shows a planet near two stars of equal mass *M*.



Each star has mass M=2.0×10^{30}kg. Their centres are separated by a distance of 6.8×10^{11}m. The planet is at a distance of 6.0×10^{11}m from each star.

5a. On the diagram above, draw **two** arrows to show the gravitational field strength at the position of the planet due to each of the stars. *[2 marks]*

Markscheme

two arrows each along the line connecting the planet to its star ***AND*** directed towards each star

arrow lines straight and of equal length

Do not allow kinked, fuzzy curved lines.

5b. Calculate the magnitude and state the direction of the resultant gravitational field strength at the position of the planet. *[3 marks]*

Markscheme

$g = \ll \frac{GM}{r^2} = \frac{6.67\times10^{-11}\times2.0\times10^{30}}{(6.0\times10^{11})^2} \gg$ ***OR*** 3.7×$10^{-4}$$\text{Nkg}^{-1}$

$g_{\text{net}} = \ll 2g\cos\theta = 2\times3.7\times10^{-4}\times\frac{\sqrt{6.0^2-3.4^2}}{6.0} = \gg 6.1\times10^{-4}\text{Nkg}^{-1}$

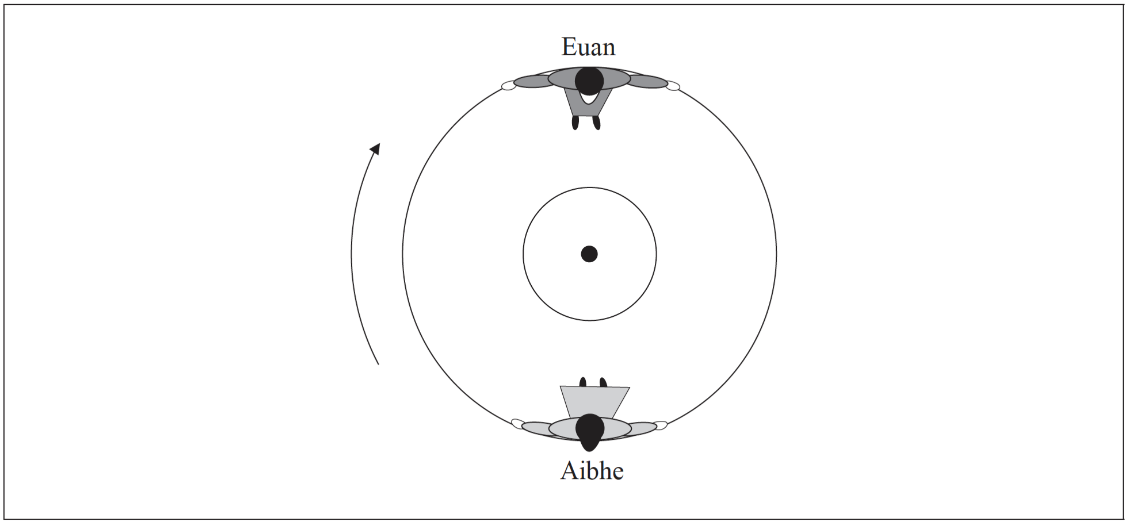
directed vertically down «page» ***OR*** towards midpoint between two stars ***OR*** south

Allow rounding errors.

This question is in **two** parts. **Part 1** is about two children on a merry-go-round. **Part 2** is about electric circuits.

Part 1 Two children on a merry-go-round

Aibhe and Euan are sitting on opposite sides of a merry-go-round, which is rotating at constant speed around a fixed centre. The diagram below shows the view from above.



Aibhe is moving at speed 1.0ms^{-1} relative to the ground.

6a. Determine the magnitude of the velocity of Aibhe relative to *[2 marks]*

(i) Euan.

(ii) the centre of the merry-go-round.

Markscheme

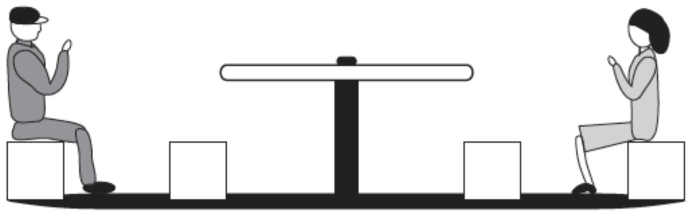
(i) 2.0 ***or*** 0(ms^{-1});
(ii) 1.0 ***or*** 0(ms^{-1});

6b. (i) Outline why Aibhe is accelerating even though she is moving at constant speed. *[6 marks]*

(ii) Draw an arrow on the diagram on page 22 to show the direction in which Aibhe is accelerating.

(iii) Identify the force that is causing Aibhe to move in a circle.

(iv) The diagram below shows a side view of Aibhe and Euan on the merry-go-round.



Explain why Aibhe feels as if her upper body is being “thrown outwards”, away from the centre of the merry-go-round.

Markscheme

(i) her direction is changing;
hence her velocity is changing;
or
since her direction/velocity is changing;
a resultant/unbalanced/net force must be acting on her (hence she is accelerating);

(ii) arrow from Aibhe towards centre of merry-go-round;
Ignore length of arrow.

(iii) the force of the merry-go-round on Aibhe/her;

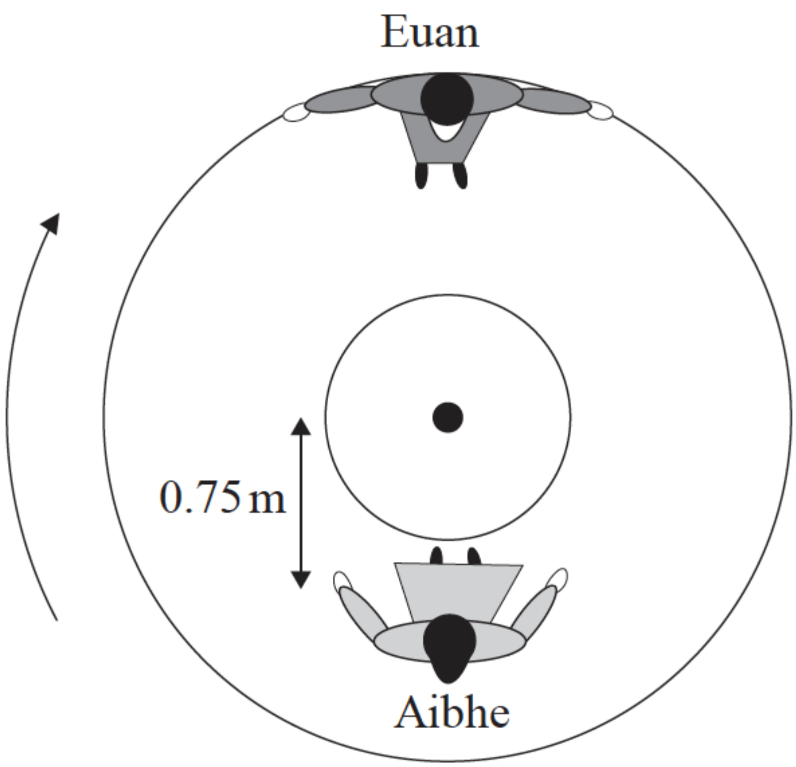
(iv) no force is acting on the upper body towards the centre of the circle / no centripetal force acting on the upper body (to maintain circular motion);
upper body (initially) continues to move in a straight line at constant speed/ velocity is tangential to circle;

6c. Euan is rotating on a merry-go-round and drags his foot along the ground to act as a brake. The merry-go-round comes to a stop after 4.0 rotations. The radius of the merry-go-round is 1.5 m. The average frictional force between his foot and the ground is 45 N. Calculate the work done. *[2 marks]*

Markscheme

distance travelled by Euan=$4.0 \times 2\pi \times 1.5\,(= 37.70\text{m})$;
$W\,(= F_{av}d = 45 \times 37.70) = 1700\,(\text{J})$;

6d. Aibhe moves so that she is sitting at a distance of 0.75 m from the centre of the merry-go-round, as shown below. *[5 marks]*



Euan pushes the merry-go-round so that he is again moving at 1.0 ms^{-1} relative to the ground.

(i) Determine Aibhe's speed relative to the ground.

(ii) Calculate the magnitude of Aibhe's acceleration.

Markscheme

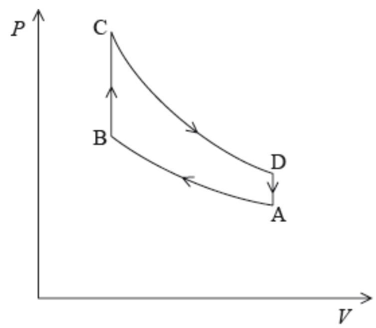
(i) Aibhe's period of revolution is the same as before;
from $v = \frac{2\pi r}{T}$, since *r* is halved, *v* is halved;
v=0.5(ms^{-1});
*Award **[3]** for a bald correct answer.*

(ii) $a\left(=\frac{v^2}{r}\right) = \frac{0.5^2}{0.75}$;
a=0.33(ms^{-2});
Allow ECF from (d)(i).
*Award **[2]** for a bald correct answer.*

This question is about the thermodynamics of a car engine and the dynamics of the car.

A car engine consists of four cylinders. In each of the cylinders, a fuel-air mixture explodes to supply power at the appropriate moment in the cycle.

The diagram models the variation of pressure *P* with volume *V* for one cycle of the gas, ABCDA, in one of the cylinders of the engine. The gas in the cylinder has a fixed mass and can be assumed to be ideal.



7a. At point A in the cycle, the fuel-air mixture is at 18 °C. During process AB, the gas is compressed to 0.046 of its original volume and the pressure increases by a factor of 40. Calculate the temperature of the gas at point B. *[1 mark]*

Markscheme

535 (K) / 262 (°C);

7b. State the nature of the change in the gas that takes place during process BC in the cycle. *[1 mark]*

Markscheme

constant volume change / isochoric / isovolumetric / *OWTTE*;

7c. Process CD is an adiabatic change. Discuss, with reference to the first law of thermodynamics, the change in temperature of the gas in the cylinder during process CD. *[3 marks]*

Markscheme

Q/thermal energy transfer is zero;

$\Delta U = -W$;

as work is done by gas internal energy falls;

temperature falls as temperature is measure of average kinetic energy;

7d. Explain how the diagram can be used to calculate the net work done during one cycle. *[2 marks]*

Markscheme

work done is estimated by evaluating area;

inside the loop / *OWTTE*;

The car is travelling at its maximum speed of $56\ \mathrm{m\,s^{-1}}$. At this speed, the energy provided by the fuel injected into one cylinder in each cycle is 9200 J. One litre of fuel provides 56 MJ of energy.

7e. (i) Calculate the volume of fuel injected into one cylinder during one cycle. *[3 marks]*

(ii) Each of the four cylinders completes a cycle 18 times every second. Calculate the distance the car can travel on one litre of fuel at a speed of $56\ \mathrm{m\,s^{-1}}$.

Markscheme

(i) $1.6 \times 10^{-4}\ (\text{litre})$;

(ii) one litre $= \left(\frac{1}{4\times18\times1.64\times10^{-4}} =\right)$ $87\ \text{s of travel}$;

$(87 \times 56) = 4.7\ (\text{km})$;

Allow rounded 1.6 value to be used, giving 4.9 (km).

7f. A car accelerates uniformly along a straight horizontal road from an initial speed of $12\ \mathrm{m\,s^{-1}}$ to a final speed of $28\ \mathrm{m\,s^{-1}}$ in a distance of 250 m. The mass of the car is 1200 kg. Determine the rate at which the engine is supplying kinetic energy to the car as it accelerates. *[4 marks]*

Markscheme

use of a kinematic equation to determine motion time $(= 12.5\ \text{s})$;

change in kinetic energy $= \frac{1}{2} \times 1200 \times [28^2 - 12^2]\ (= 384\ \text{kJ})$;

rate of change in kinetic energy $= \frac{384000}{12.5}$; } *(allow ECF of 16^2 from (28 − 12)^2 for this mark)*

31 (kW);

or

use of a kinematic equation to determine motion time $(= 12.5\ \text{s})$;

use of a kinematic equation to determine acceleration $(= 1.28\ \mathrm{m\,s^{-2}})$;

work done $\frac{F\times s}{\text{time}} = \frac{1536\times250}{12.5}$;

31 (kW);

A car is travelling along a straight horizontal road at its maximum speed of $56\ \mathrm{m\,s^{-1}}$. The power output required at the wheels is 0.13 MW.

7g. (i) Calculate the total resistive force acting on the car when it is travelling at a constant speed of $56\ \mathrm{m\,s^{-1}}$. *[5 marks]*

(ii) The mass of the car is 1200 kg. The resistive force *F* is related to the speed *v* by $F \propto v^2$. Using your answer to (g)(i), determine the maximum theoretical acceleration of the car at a speed of $28\ \mathrm{m\,s^{-1}}$.

Markscheme

(i) force $= \frac{\text{power}}{\text{speed}}$;

2300 ***or*** 2.3k (N);

*Award **[2]** for a bald correct answer.*

(ii) resistive force $= \frac{2300}{4}$ ***or*** $\frac{2321}{4}$ $(= 575)$; *(allow ECF)*

so accelerating force $(2300 - 580 =)\ 1725\ (\text{N})$ ***or*** 1741 (N);

$a = \frac{1725}{1200} = 1.44\ (\text{m s}^{-2})$ ***or*** $a = \frac{1741}{1200} = 1.45\ (\text{m s}^{-2})$;

*Award **[2 max]** for an answer of 0.49 (m s^{-2} (omits 2300 N).*

A driver moves a car in a horizontal circular path of radius 200 m. Each of the four tyres will not grip the road if the frictional force between a tyre and the road becomes less than 1500 N.

7h. (i) Calculate the maximum speed of the car at which it can continue to move in the circular path. Assume that the radius of the path is the same for each tyre. *[6 marks]*

(ii) While the car is travelling around the circle, the people in the car have the sensation that they are being thrown outwards. Outline how Newton's first law of motion accounts for this sensation.

Markscheme

(i) centripetal force must be $< 6000\ (\text{N})$; *(allow force 6000 N)*

$v^2 = F \times \frac{r}{m}$;

$31.6\ (\text{m s}^{-1})$;

*Allow **[3]** for a bald correct answer.*

*Allow **[2 max]** if 4$\times$ is omitted, giving 15.8 (m s^{-1}).*

(ii) statement of Newton's first law;

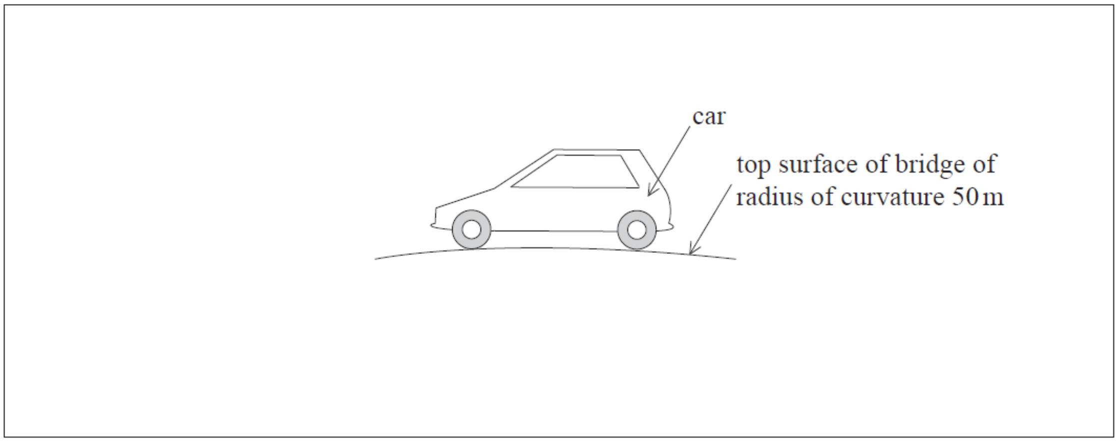
(hence) without car wall/restraint/friction at seat, the people in the car would move in a straight line/at a tangent to circle;

(hence) seat/seat belt/door exerts centripetal force;

(in frame of reference of the people) straight ahead movement is interpreted as "outwards";

This question is about circular motion.

The diagram shows a car moving at a constant speed over a curved bridge. At the position shown, the top surface of the bridge has a radius of curvature of 50 m.



8a. Explain why the car is accelerating even though it is moving with a constant speed. *[2 marks]*

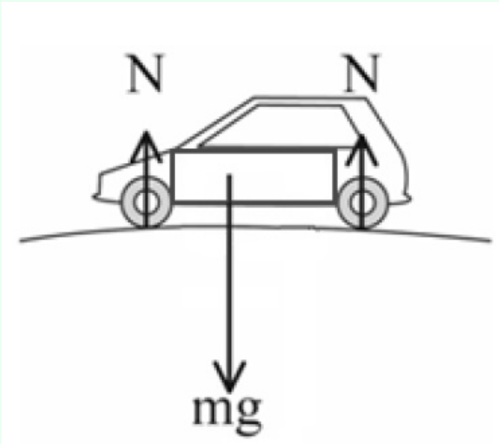
Markscheme

direction changing;

velocity changing so accelerating;

8b. On the diagram, draw and label the vertical forces acting on the car in the position shown. *[2 marks]*

Markscheme



weight/gravitational force/mg/w/F_w/F_g and reaction/normal reaction/perpendicular contact force/N/R/F_N/F_R both labelled; *(do not allow "gravity" for "weight".)*

weight between wheels (in box) from centre of mass and reactions at both wheels / single reaction acting along same line of action as the weight;

Judge by eye. Look for reasonably vertical lines with weight force longer than (sum of) reaction(s). Extra forces (eg centripetal force) loses the second mark.

8c. Calculate the maximum speed at which the car will stay in contact with the bridge. *[3 marks]*

Markscheme

$g = \frac{v^2}{r}$;
$v = \sqrt{50 \times 9.8}$;

22(ms^{-1});

*Allow **[3]** for a bald correct answer.*

This question is in **two** parts. **Part 1** is about gravitational force fields. **Part 2** is about properties of a gas.

Part 1 Gravitational force fields

9a. State Newton's universal law of gravitation. *[2 marks]*

Markscheme

the (attractive) force between two (point) masses is directly proportional to the product of the masses;
and inversely proportional to the square of the distance (between their centres of mass);
Use of equation is acceptable:
*Award **[2]** if all five quantities defined. Award **[1]** if four quantities defined.*

9b. A satellite of mass *m* orbits a planet of mass *M*. Derive the following relationship between the period of the satellite *T* and the radius of its orbit *R* (Kepler's third law). *[3 marks]*

$T^2 = \frac{4\pi^2 R^3}{GM}$

Markscheme

$G\frac{Mm}{R^2} = \frac{mv^2}{R}$ so $v^2 = \frac{Gm}{R}$;
$v = \frac{2\pi R}{T}$;
$v^2 = \frac{4\pi^2 R^2}{T^2} = \frac{Gm}{R}$;

or

$G\frac{Mm}{R^2} = m\omega^2 R;$
$\omega^2 = \frac{4\pi^2}{T^2}$;
$\frac{4\pi^2}{T^2} = \frac{GM}{R^3}$;
*Award **[3]** to a clear response with a missing step.*

9c. A polar orbiting satellite has an orbit which passes above both of the Earth's poles. One polar orbiting satellite used for Earth observation has an orbital period of 6.00 × 10^3s. *[8 marks]*

Mass of Earth = 5.97 × 10^{24} kg
Average radius of Earth = 6.37 × 10^{6} m

(i) Using the relationship in (b), show that the average height above the surface of the Earth for this satellite is about 800 km.

(ii) The satellite moves from an orbit of radius 1200 km above the Earth to one of radius 2500 km. The mass of the satellite is 45 kg.

Calculate the change in the gravitational potential energy of the satellite.

(iii) Explain whether the gravitational potential energy has increased, decreased or stayed the same when the orbit changes, as in (c)(ii).

Markscheme

(i) $R^3 = \frac{6.67\times10^{-11}\times5.97\times10^{24}\times6000^2}{4\times\pi^2}$;
R=7.13x10^6(m);
h=(7.13x10^6-6.37x10^6)=760(km);
*Award **[3]** for an answer of 740 with π taken as 3.14.*

(ii) clear use of $\Delta V = \frac{\Delta E}{m}$ and $V = -\frac{Gm}{r}$ **or** $\Delta E = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$;
one value of potential energy calculated (2.37×10^9 ***or*** 2.02×10^9);
3.5×10^8 (J);
*Award **[3]** for a bald correct answer.*
*Award **[2]** for 7.7x10^9. Award **[1]** for 7.7x10^{12}.*
*Award **[0]** for answers using mgΔh.*

(iii) increased;
further from Earth / closer to infinity / smaller negative value;
*Award **[0]** for a bald correct answer.*

Part 2 Gravitational fields

10a. State Newton's universal law of gravitation. *[3 marks]*

Markscheme

there is an attractive force;
between any two point/small masses;
proportional to the product of their masses;
and inversely proportional to the square of their separation;
Accept formula with all terms defined.

10b. Deduce that the gravitational field strength g at the surface of a spherical planet of uniform density is given by *[2 marks]*

$$g = \frac{GM}{R^2}$$

where M is the mass of the planet, R is its radius and G is the gravitational constant. You can assume that spherical objects of uniform density act as point masses.

Markscheme

use of $g = \frac{F}{m}$ and $F = \frac{GmM}{R^2}$;
evidence of substitution/manipulation;
to get $g = \frac{GM}{R^2}$

10c. The gravitational field strength at the surface of Mars g_M is related to the gravitational field strength at the surface of the Earth g_E by *[2 marks]*

$g_M = 0.38 \times g_E$.

The radius of Mars R_M is related to the radius of the Earth R_E by

$R_M = 0.53 \times R_E$.

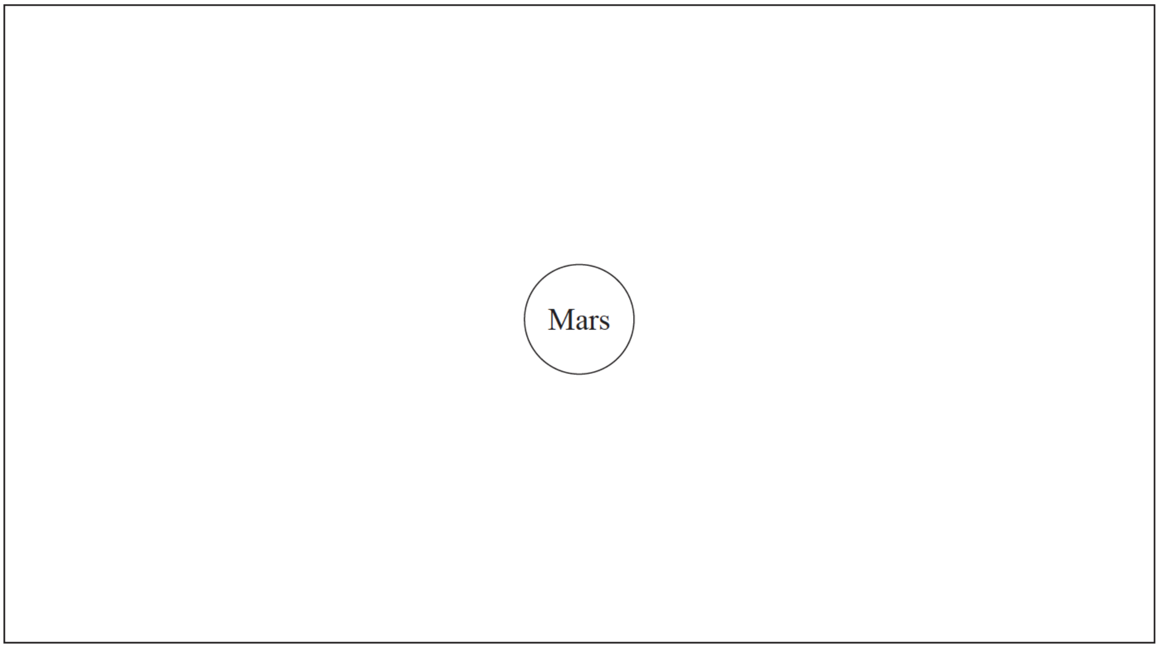
Determine the mass of Mars M_M in terms of the mass of the Earth M_E.

Markscheme

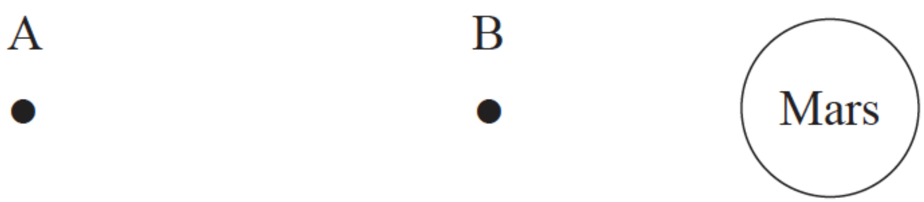
$$\frac{g_M}{g_E} = \frac{\frac{M_M}{R_M^2}}{\frac{M_E}{R_E^2}} \Rightarrow \frac{M_M}{M_E} = \frac{g_M}{g_E} \times \left[\frac{R_M}{R_E}\right]^2;$$

$$M_{\mathrm{M}} \left(= 0.38 \times 0.53^2 M_{\mathrm{E}}\right) = 0.11 M_{\mathrm{E}};$$

10d. (i) On the diagram below, draw lines to represent the gravitational field around the planet Mars. *[3 marks]*

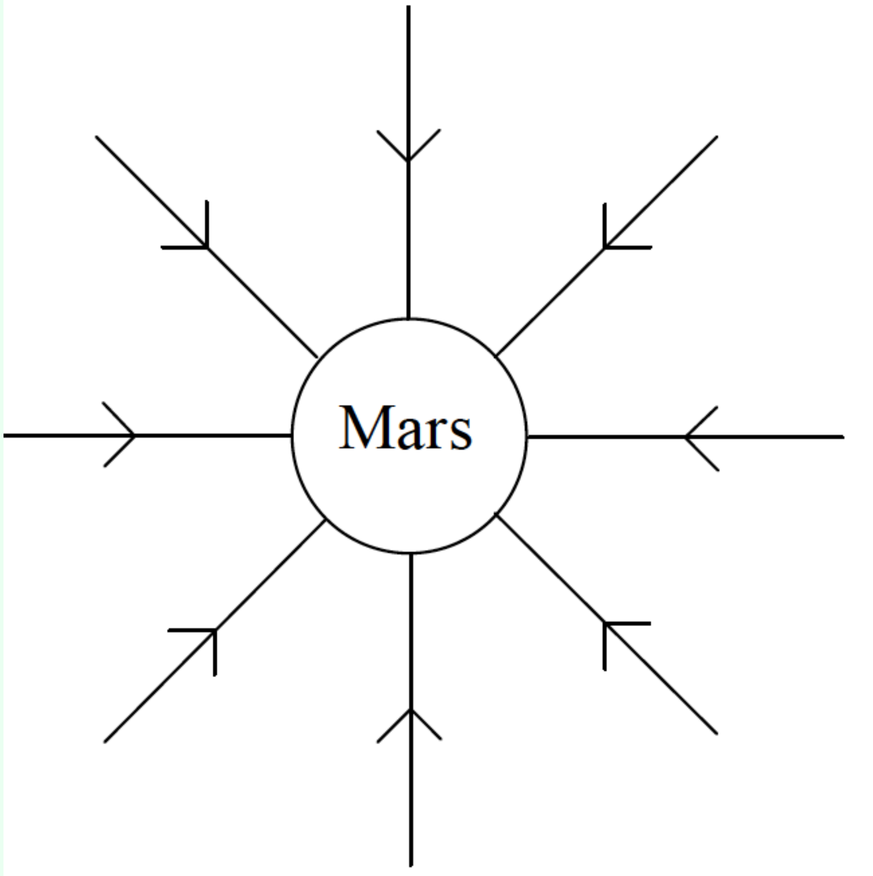


(ii) An object falls freely in a straight line from point A to point B in time t. The speed of the object at A is u and the speed at B is v. A student suggests using the equation $v=u+g_{M}t$ to calculate v. Suggest **two** reasons why it is not appropriate to use this equation.



Markscheme

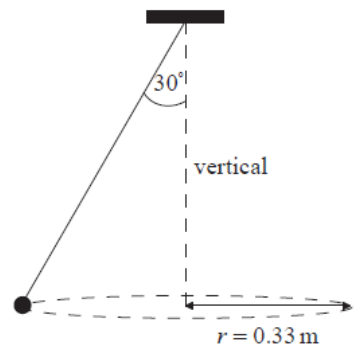
(i) radial field with arrows pointing inwards;



(ii) field between A and B is not equal to field at surface;
acceleration is not constant between these two points;

This question is about circular motion.

A ball of mass 0.25 kg is attached to a string and is made to rotate with constant speed v along a horizontal circle of radius r = 0.33m. The string is attached to the ceiling and makes an angle of 30° with the vertical.

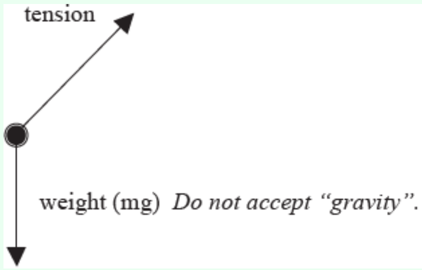


11a. (i) On the diagram above, draw and label arrows to represent the forces on the ball in the position shown. *[4 marks]*

(ii) State and explain whether the ball is in equilibrium.

Markscheme

(i) ***[1]*** each for correct arrow and (any reasonable) labelling;



*Award **[1 max]** for arrows in correct direction but not starting at the ball.*

(ii) no;
because the two forces on the ball can never cancel out / there is a net force on the ball / the ball moves in a circle / the ball has acceleration/it is changing direction;
*Award **[0]** for correct answer with no or wrong argument.*

11b. Determine the speed of rotation of the ball. *[3 marks]*

Markscheme

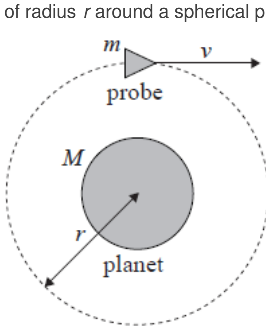
$T\left(=\frac{mg}{\cos 30^\circ}\right) = 2.832\text{N};$
$\frac{mv^2}{r} = T\sin 30^\circ;$
$v = \left(\sqrt{\frac{Tr\sin 30^\circ}{m}} = \sqrt{\frac{2.832\times 0.33\times \sin 30^\circ}{0.25}}\right) = 1.4\text{ms}^{-1};$

or

$T\cos 30^\circ = mg;$
$T\sin 30^\circ = \frac{mv^2}{r};$
$v = \left(\sqrt{gr\tan 30^\circ} = \sqrt{9.81\times 0.33\times \tan 30^\circ}\right) = 1.4\text{ms}^{-1};$

This question is about a probe in orbit.

A probe of mass *m* is in a circular orbit of radius *r* around a spherical planet of mass *M*.



(diagram not to scale)

12a. State why the work done by the gravitational force during one full revolution of the probe is zero. *[1 mark]*

Markscheme

because the force is always at right angles to the velocity / motion/orbit is an equipotential surface;
Do not accept answers based on the displacement being zero for a full revolution.

12b. Deduce for the probe in orbit that its *[4 marks]*

(i) speed is $v = \sqrt{\frac{GM}{r}}$.

(ii) total energy is $E = -\frac{GMm}{2r}$.

Markscheme

(i) equating gravitational force $\frac{GMm}{r^2}$;

to centripetal force $\frac{mv^2}{r}$ to get result;

(ii) kinetic energy is $\frac{GMm}{2r}$;

addition to potential energy $-\frac{GMm}{r}$ to get result;

12c. It is now required to place the probe in another circular orbit further away from the planet. To do this, the probe's engines will be fired for a very short time. *[2 marks]*

State and explain whether the work done on the probe by the engines is positive, negative **or** zero.

Markscheme

the total energy (at the new orbit) will be greater than before/is less negative;

hence probe engines must be fired to produce force in the direction of motion / positive work must be done (on the probe);

*Award **[1]** for mention of only potential energy increasing.*