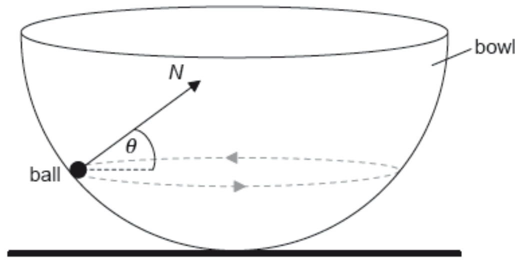
6 *[143 marks]*

A small ball of mass m is moving in a horizontal circle on the inside surface of a frictionless hemispherical bowl.



The normal reaction force N makes an angle θ to the horizontal.

1a. State the direction of the resultant force on the ball. *[1 mark]*

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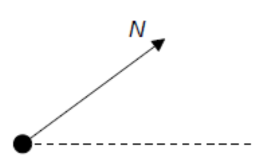
1b. On the diagram, construct an arrow of the correct length to represent the weight of the ball. *[2 marks]*



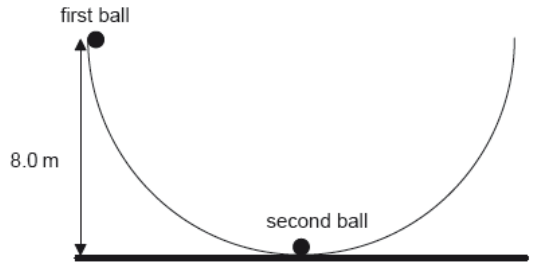
1c. Show that the magnitude of the net force F on the ball is given by the following equation. *[3 marks]*

$$F = \frac{mg}{\tan\theta}$$

1d. The radius of the bowl is 8.0 m and $\theta = 22°$. Determine the speed of the ball. *[4 marks]*

1e. Outline whether this ball can move on a horizontal circular path of radius equal to the radius of the bowl. *[2 marks]*

1f. A second identical ball is placed at the bottom of the bowl and the first ball is displaced so that its height from the horizontal is equal to 8.0 m. *[3 marks]*



The first ball is released and eventually strikes the second ball. The two balls remain in contact. Determine, in m, the maximum height reached by the two balls.

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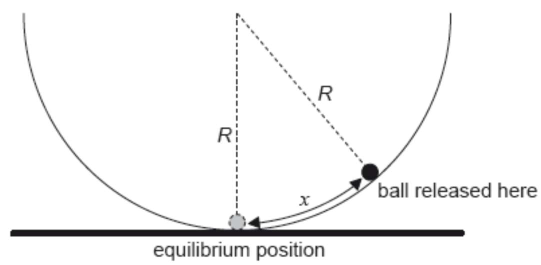
The ball is now displaced through a small distance x from the bottom of the bowl and is then released from rest.



The magnitude of the force on the ball towards the equilibrium position is given by

$$\frac{mgx}{R}$$

where R is the radius of the bowl.

1g. Outline why the ball will perform simple harmonic oscillations about the equilibrium position. *[1 mark]*

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1h. Show that the period of oscillation of the ball is about 6 s. *[2 marks]*

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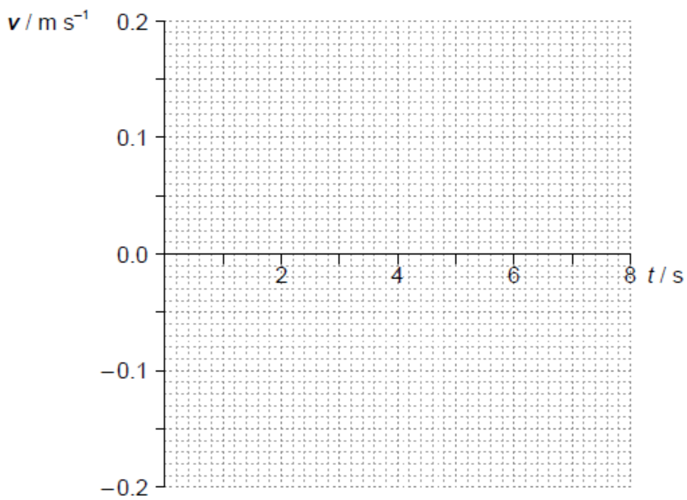
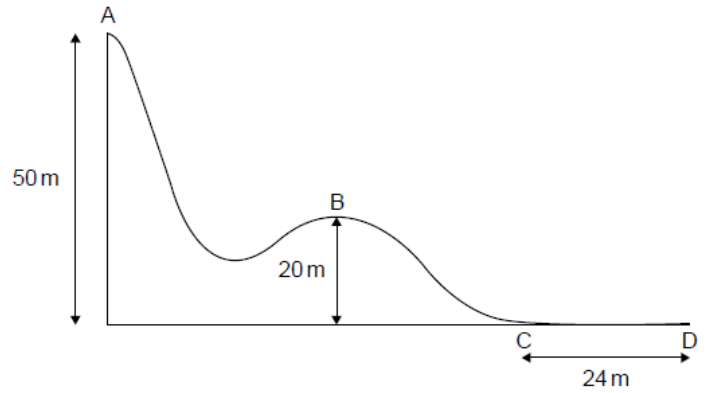
1i. The amplitude of oscillation is 0.12 m. On the axes, draw a graph to show the variation with time t of the velocity $\boldsymbol{v}$ of the ball during one period. *[3 marks]*



The diagram below shows part of a downhill ski course which starts at point A, 50 m above level ground. Point B is 20 m above level ground.



A skier of mass 65 kg starts from rest at point A and during the ski course some of the gravitational potential energy transferred to kinetic energy.

2a. From A to B, 24 % of the gravitational potential energy transferred to kinetic energy. Show that the velocity at B is 12 m s $^{-1}$. *[2 marks]*

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2b. Some of the gravitational potential energy transferred into internal energy of the skis, slightly increasing their temperature. Distinguish between internal energy and temperature. *[2 marks]*

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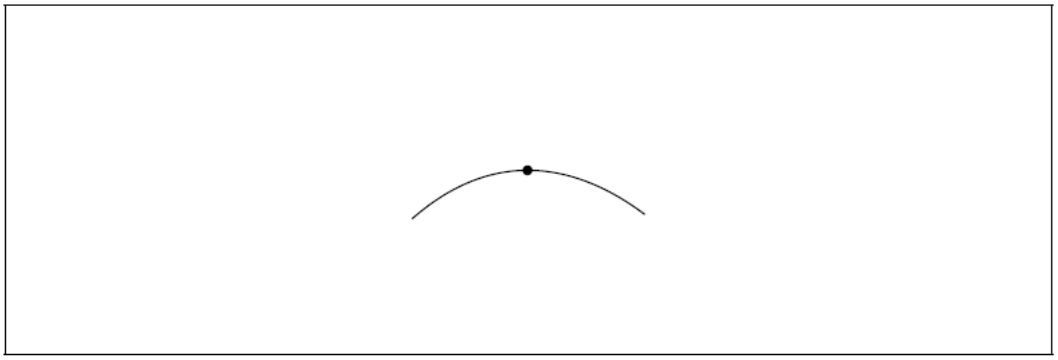
2c. The dot on the following diagram represents the skier as she passes point B. *[2 marks]*
Draw and label the vertical forces acting on the skier.



2d. The hill at point B has a circular shape with a radius of 20 m. Determine whether the skier will lose contact with the ground at point B. *[3 marks]*

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2e. The skier reaches point C with a speed of 8.2 m s^{-1}. She stops after a distance of 24 m at point D. *[3 marks]*

Determine the coefficient of dynamic friction between the base of the skis and the snow. Assume that the frictional force is constant and that air resistance can be neglected.

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At the side of the course flexible safety nets are used. Another skier of mass 76 kg falls normally into the safety net with speed 9.6 m s^{-1}.

2f. Calculate the impulse required from the net to stop the skier and state an appropriate unit for your answer. *[2 marks]*

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2g. Explain, with reference to change in momentum, why a flexible safety net is less likely to harm the skier than a rigid barrier. *[2 marks]*

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The gravitational potential due to the Sun at its surface is -1.9×10^{11} J kg^{-1}. The following data are available.

Mass of Earth	$= 6.0 \times 10^{24}$ kg
Distance from Earth to Sun	$= 1.5 \times 10^{11}$ m
Radius of Sun	$= 7.0 \times 10^{8}$ m

3a. Outline why the gravitational potential is negative. *[2 marks]*

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3b. The gravitational potential due to the Sun at a distance r from its centre is V_S. Show that rV_S = constant. *[1 mark]*

3c. Calculate the gravitational potential energy of the Earth in its orbit around the Sun. Give your answer to an appropriate number of significant figures. *[2 marks]*

3d. Calculate the total energy of the Earth in its orbit. *[2 marks]*

3e. An asteroid strikes the Earth and causes the orbital speed of the Earth to suddenly decrease. Suggest the ways in which the orbit of the Earth will change. *[2 marks]*

3f. Outline, in terms of the force acting on it, why the Earth remains in a circular orbit around the Sun. *[2 marks]*

4a. (i) Define *gravitational field strength.* *[2 marks]*

(ii) State the SI unit for gravitational field strength.

4b. A planet orbits the Sun in a circular orbit with orbital period *T* and orbital radius *R*. The mass of the Sun is *M*. *[4 marks]*

(i) Show that $T=\sqrt{\frac{4\pi^2R^3}{GM}}$.

(ii) The Earth's orbit around the Sun is almost circular with radius 1.5×10^{11} m. Estimate the mass of the Sun.

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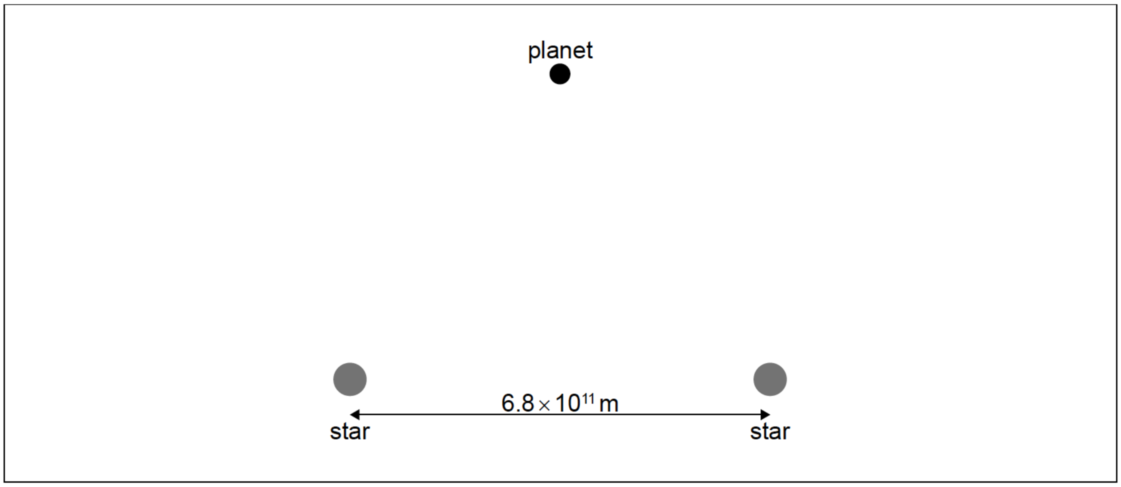
The diagram shows a planet near two stars of equal mass *M*.



Each star has mass $M=2.0\times10^{30}$kg. Their centres are separated by a distance of 6.8×10^{11}m. The planet is at a distance of 6.0×10^{11}m from each star.

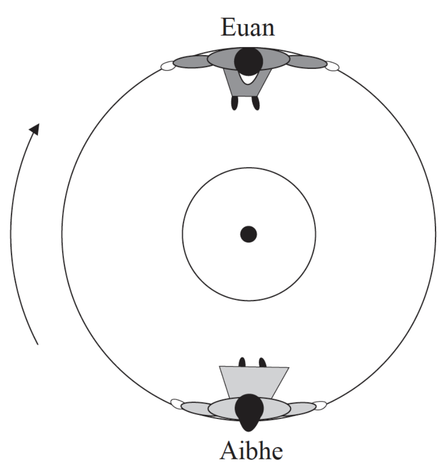
5a. On the diagram above, draw **two** arrows to show the gravitational field strength at the position of the planet due to each of the stars. *[2 marks]*

5b. Calculate the magnitude and state the direction of the resultant gravitational field strength at the position of the planet. *[3 marks]*

This question is in **two** parts. **Part 1** is about two children on a merry-go-round. **Part 2** is about electric circuits.

Part 1 Two children on a merry-go-round

Aibhe and Euan are sitting on opposite sides of a merry-go-round, which is rotating at constant speed around a fixed centre. The diagram below shows the view from above.



Aibhe is moving at speed $1.0 ms^{-1}$ relative to the ground.

6a. Determine the magnitude of the velocity of Aibhe relative to *[2 marks]*

(i) Euan.

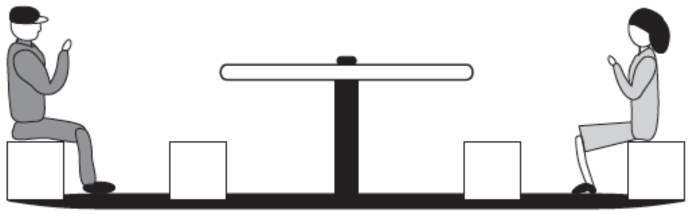
(ii) the centre of the merry-go-round.

6b. (i) Outline why Aibhe is accelerating even though she is moving at constant speed. *[6 marks]*

(ii) Draw an arrow on the diagram on page 22 to show the direction in which Aibhe is accelerating.

(iii) Identify the force that is causing Aibhe to move in a circle.

(iv) The diagram below shows a side view of Aibhe and Euan on the merry-go-round.



Explain why Aibhe feels as if her upper body is being “thrown outwards”, away from the centre of the merry-go-round.

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6c. Euan is rotating on a merry-go-round and drags his foot along the ground to act as a brake. The merry-go-round comes to a stop after 4.0 rotations. The radius of the merry-go-round is 1.5 m. The average frictional force between his foot and the ground is 45 N. Calculate the work done. *[2 marks]*

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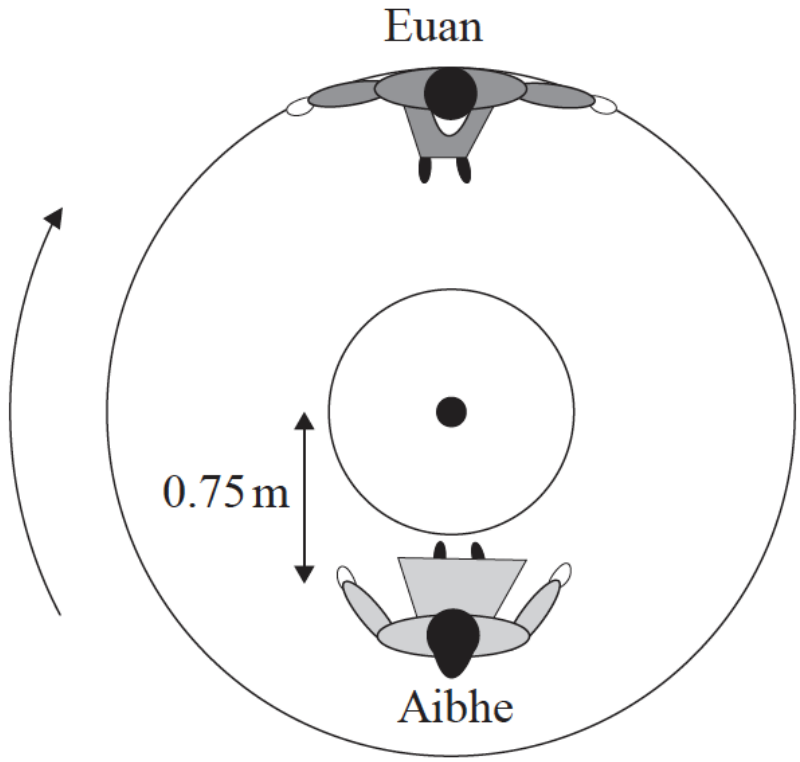
6d. Aibhe moves so that she is sitting at a distance of 0.75 m from the centre of the merry-go-round, as shown below. *[5 marks]*



Euan pushes the merry-go-round so that he is again moving at 1.0 ms^{-1} relative to the ground.

(i) Determine Aibhe's speed relative to the ground.

(ii) Calculate the magnitude of Aibhe's acceleration.

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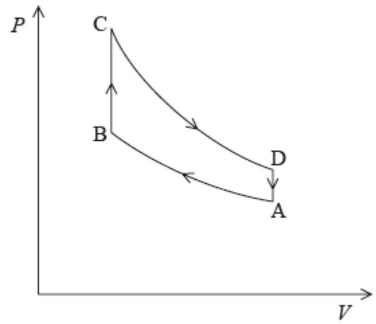
This question is about the thermodynamics of a car engine and the dynamics of the car.

A car engine consists of four cylinders. In each of the cylinders, a fuel-air mixture explodes to supply power at the appropriate moment in the cycle.

The diagram models the variation of pressure *P* with volume *V* for one cycle of the gas, ABCDA, in one of the cylinders of the engine. The gas in the cylinder has a fixed mass and can be assumed to be ideal.



7a. At point A in the cycle, the fuel-air mixture is at 18 °C. During process AB, the gas is compressed to 0.046 of its original volume and the pressure increases by a factor of 40. Calculate the temperature of the gas at point B. *[1 mark]*

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7b. State the nature of the change in the gas that takes place during process BC in the cycle. *[1 mark]*

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7c. Process CD is an adiabatic change. Discuss, with reference to the first law of thermodynamics, the change in temperature of the gas in the cylinder during process CD. *[3 marks]*

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7d. Explain how the diagram can be used to calculate the net work done during one cycle. *[2 marks]*

The car is travelling at its maximum speed of $56\ \mathrm{m\,s^{-1}}$. At this speed, the energy provided by the fuel injected into one cylinder in each cycle is 9200 J. One litre of fuel provides 56 MJ of energy.

7e. (i) Calculate the volume of fuel injected into one cylinder during one cycle. *[3 marks]*

(ii) Each of the four cylinders completes a cycle 18 times every second. Calculate the distance the car can travel on one litre of fuel at a speed of $56\ \mathrm{m\,s^{-1}}$.

7f. A car accelerates uniformly along a straight horizontal road from an initial speed of $12\ \mathrm{m\,s^{-1}}$ to a final speed of $28\ \mathrm{m\,s^{-1}}$ in a distance of 250 m. The mass of the car is 1200 kg. Determine the rate at which the engine is supplying kinetic energy to the car as it accelerates. *[4 marks]*

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A car is travelling along a straight horizontal road at its maximum speed of $56\ \mathrm{m\,s^{-1}}$. The power output required at the wheels is 0.13 MW.

7g. (i) Calculate the total resistive force acting on the car when it is travelling at a constant speed of $56\ \mathrm{m\,s^{-1}}$. *[5 marks]*

(ii) The mass of the car is 1200 kg. The resistive force *F* is related to the speed *v* by $F \propto v^2$. Using your answer to (g)(i), determine the maximum theoretical acceleration of the car at a speed of $28\ \mathrm{m\,s^{-1}}$.

A driver moves a car in a horizontal circular path of radius 200 m. Each of the four tyres will not grip the road if the frictional force between a tyre and the road becomes less than 1500 N.

7h. (i) Calculate the maximum speed of the car at which it can continue to move in the circular path. Assume that the radius of the path is the same for each tyre. *[6 marks]*

(ii) While the car is travelling around the circle, the people in the car have the sensation that they are being thrown outwards. Outline how Newton's first law of motion accounts for this sensation.

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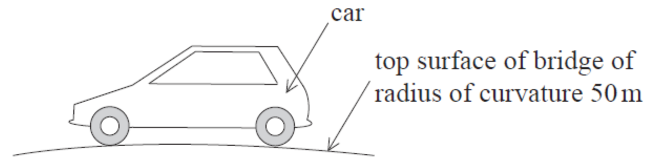
This question is about circular motion.

The diagram shows a car moving at a constant speed over a curved bridge. At the position shown, the top surface of the bridge has a radius of curvature of 50 m.



8a. Explain why the car is accelerating even though it is moving with a constant speed. *[2 marks]*

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8b. On the diagram, draw and label the vertical forces acting on the car in the position shown. *[2 marks]*

8c. Calculate the maximum speed at which the car will stay in contact with the bridge. *[3 marks]*

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This question is in **two** parts. **Part 1** is about gravitational force fields. **Part 2** is about properties of a gas.

Part 1 Gravitational force fields

9a. State Newton's universal law of gravitation. *[2 marks]*

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9b. A satellite of mass *m* orbits a planet of mass *M*. Derive the following relationship between the period of the satellite *T* and the radius of its orbit *R* (Kepler's third law). *[3 marks]*

$T^2 = \frac{4\pi^2 R^3}{GM}$

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9c. A polar orbiting satellite has an orbit which passes above both of the Earth's poles. One polar orbiting satellite used for Earth observation has an orbital period of 6.00×10^3 s. *[8 marks]*

Mass of Earth = 5.97×10^{24} kg
Average radius of Earth = 6.37×10^{6} m

(i) Using the relationship in (b), show that the average height above the surface of the Earth for this satellite is about 800 km.

(ii) The satellite moves from an orbit of radius 1200 km above the Earth to one of radius 2500 km. The mass of the satellite is 45 kg.

Calculate the change in the gravitational potential energy of the satellite.

(iii) Explain whether the gravitational potential energy has increased, decreased or stayed the same when the orbit changes, as in (c)(ii).

Part 2 Gravitational fields

10a. State Newton's universal law of gravitation. *[3 marks]*

10b. Deduce that the gravitational field strength g at the surface of a spherical planet of uniform density is given by *[2 marks]*

$$g = \frac{GM}{R^2}$$

where M is the mass of the planet, R is its radius and G is the gravitational constant. You can assume that spherical objects of uniform density act as point masses.

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10c. The gravitational field strength at the surface of Mars g_M is related to the gravitational field strength at the surface of the Earth g_E by *[2 marks]*

$g_M = 0.38 \times g_E$.

The radius of Mars R_M is related to the radius of the Earth R_E by

$R_M = 0.53 \times R_E$.

Determine the mass of Mars M_M in terms of the mass of the Earth M_E.

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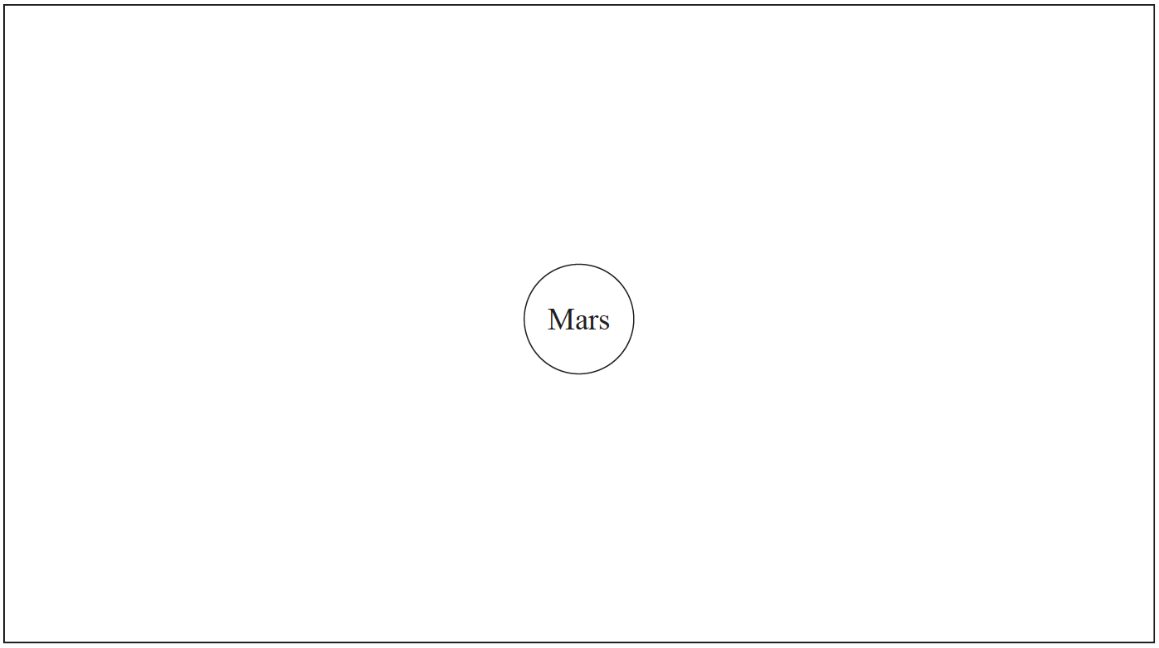
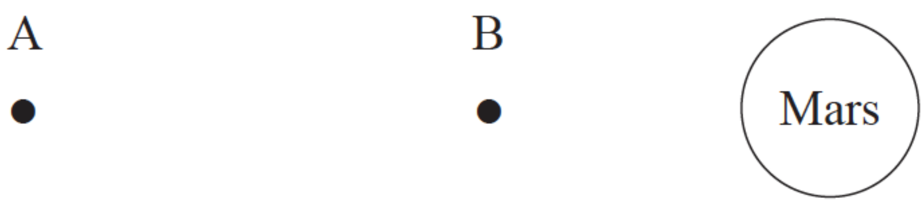
10d. (i) On the diagram below, draw lines to represent the gravitational field around the planet Mars. *[3 marks]*

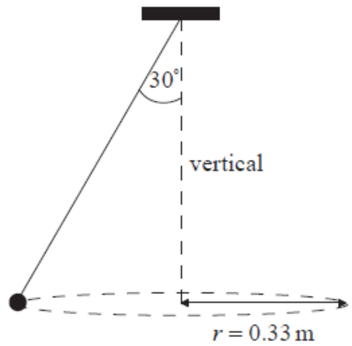


(ii) An object falls freely in a straight line from point A to point B in time t. The speed of the object at A is u and the speed at B is v. A student suggests using the equation $v = u + g_M t$ to calculate v. Suggest **two** reasons why it is not appropriate to use this equation.



This question is about circular motion.

A ball of mass 0.25 kg is attached to a string and is made to rotate with constant speed v along a horizontal circle of radius $r = 0.33$m. The string is attached to the ceiling and makes an angle of 30° with the vertical.



11a. (i) On the diagram above, draw and label arrows to represent the forces on the ball in the position shown. *[4 marks]*

(ii) State and explain whether the ball is in equilibrium.

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11b. Determine the speed of rotation of the ball. *[3 marks]*

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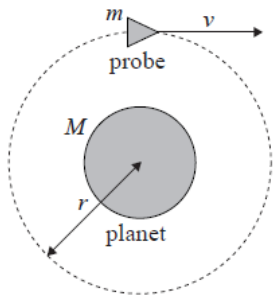
This question is about a probe in orbit.

A probe of mass *m* is in a circular orbit of radius *r* around a spherical planet of mass *M*.



(diagram not to scale)

12a. State why the work done by the gravitational force during one full revolution of the probe is zero. *[1 mark]*

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12b. Deduce for the probe in orbit that its *[4 marks]*

(i) speed is $v = \sqrt{\frac{GM}{r}}$.

(ii) total energy is $E = -\frac{GMm}{2r}$.

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12c. It is now required to place the probe in another circular orbit further away from the planet. To do this, the probe's engines will be fired for a very short time. *[2 marks]*

State and explain whether the work done on the probe by the engines is positive, negative **or** zero.

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