10.2 *[172 marks]*

A planet has radius *R*. At a distance *h* above the surface of the planet the gravitational field strength is *g* and the gravitational potential is *V*.

1a. State what is meant by gravitational field strength. *[1 mark]*

Markscheme

the «gravitational» force per unit mass exerted on a point/small/test mass

[1 mark]

1b. Show that $V = -g(R + h)$. *[2 marks]*

Markscheme

at height *h* potential is $V = -\frac{GM}{(R+h)}$

field is $g = \frac{GM}{(R+h)^2}$

«dividing gives answer»

Do not allow an answer that starts with $g = -\frac{\Delta V}{\Delta r}$ *and then cancels the deltas and substitutes R + h*

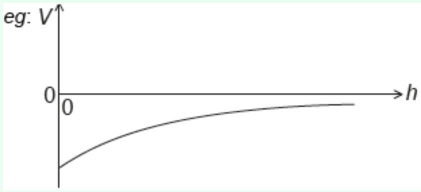
[2 marks]

1c. Draw a graph, on the axes, to show the variation of the gravitational potential *V* of the planet with height *h* above the surface of the planet. *[2 marks]*

Markscheme

correct shape and sign

non-zero negative vertical intercept



[2 marks]

1d. A planet has a radius of 3.1×10^6 m. At a point P a distance 2.4×10^7 m above the surface of the planet the gravitational field strength is 2.2 N kg^{-1}. Calculate the gravitational potential at point P, include an appropriate unit for your answer. *[1 mark]*

Markscheme

V = «–2.2 × ($3.1 \times 10^6 + 2.4 \times 10^7$) =» «–» 6.0×10^7 J kg^{-1}

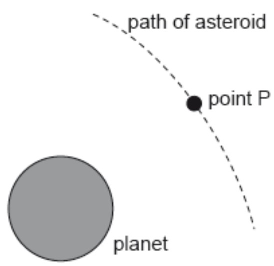
Unit is essential

Allow eg MJ kg^{-1} if power of 10 is correct

Allow other correct SI units eg m^2s^{-2}, N m kg^{-1}

[1 mark]

1e. The diagram shows the path of an asteroid as it moves past the planet. *[3 marks]*



When the asteroid was far away from the planet it had negligible speed. Estimate the speed of the asteroid at point P as defined in (b).

Markscheme

total energy at P = 0 / KE gained = GPE lost

« $\frac{1}{2}mv^2 + mV = 0$ ⇒» $v = \sqrt{-2V}$

v = « $\sqrt{2 \times 6.0 \times 10^7}$ =» 1.1×10^4 «ms^{-1}»

Award ***[3]*** *for a bald correct answer*

Ignore negative sign errors in the workings

Allow ECF from 6(b)

[3 marks]

1f. The mass of the asteroid is 6.2 × 10^{12} kg. Calculate the gravitational force experienced by the **planet** when the asteroid is at point P. *[2 marks]*

Markscheme

ALTERNATIVE 1

force on asteroid is «6.2 × 10^{12} × 2.2 =» 1.4 × 10^{13} «N»

«by Newton's third law» this is also the force on the planet

ALTERNATIVE 2

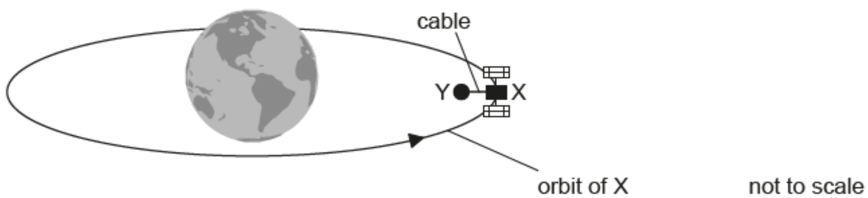
mass of planet = 2.4 x 10^{25} «kg» «from $V = -\frac{GM}{(R+h)}$»

force on planet «$\frac{GMm}{(R+h)^2}$» = 1.4 × 10^{13} «N»

MP2 must be explicit

[2 marks]

There is a proposal to power a space satellite X as it orbits the Earth. In this model, X is connected by an electronically-conducting cable to another smaller satellite Y.



2a. Satellite X orbits 6600 km from the centre of the Earth. *[2 marks]*

Mass of the Earth = 6.0 x 10^{24} kg

Show that the orbital speed of satellite X is about 8 km s^{-1}.

Markscheme

«$v = \sqrt{\frac{GM_E}{r}}$» = $\sqrt{\frac{6.67\times10^{-11}\times6.0\times10^{24}}{6600\times10^{3}}}$

7800 «m s^{-1}»

Full substitution required

Must see 2+ significant figures.

Satellite Y orbits closer to the centre of Earth than satellite X. Outline why

2b. the orbital times for X and Y are different. *[1 mark]*

Markscheme

Y has smaller orbit/orbital speed is greater so time period is less

Allow answer from appropriate equation

Allow converse argument for X

2c. satellite Y requires a propulsion system. *[2 marks]*

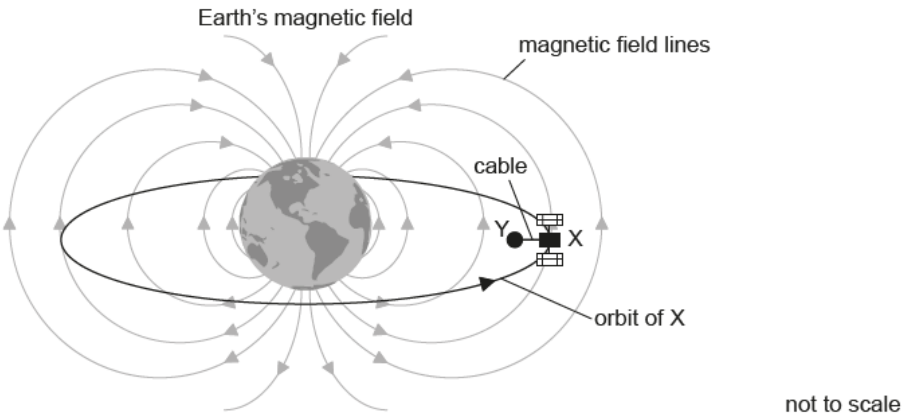
Markscheme

to stop Y from getting ahead

to remain stationary with respect to X

otherwise will add tension to cable/damage satellite/pull X out of its orbit

2d. The cable between the satellites cuts the magnetic field lines of the Earth at right angles. *[3 marks]*



Explain why satellite X becomes positively charged.

Markscheme

cable is a conductor and contains electrons

electrons/charges experience a force when moving in a magnetic field

use of a suitable hand rule to show that satellite Y becomes negative «so X becomes positive»

Alternative 2

cable is a conductor

so current will flow by induction flow when it moves through a B field

use of a suitable hand rule to show current to right so «X becomes positive»

Marks should be awarded from either one alternative or the other.

Do not allow discussion of positive charges moving towards X

2e. Satellite X must release ions into the space between the satellites. Explain why the current in the cable will become zero unless there is a method for transferring charge from X to Y. *[3 marks]*

Markscheme

electrons would build up at satellite Y/positive charge at X

preventing further charge flow

by electrostatic repulsion

unless a complete circuit exists

2f. The magnetic field strength of the Earth is 31 μT at the orbital radius of the satellites. The cable is 15 km in length. Calculate the emf induced in the cable. *[2 marks]*

Markscheme

«$\varepsilon = Blv$ =» 31 x 10^{-6} x 7990 x 15000

3600 «V»

Allow 3700 «V» from v = 8000 m s^{-1}.

The cable acts as a spring. Satellite Y has a mass m of 3.5 x 10^2 kg. Under certain circumstances, satellite Y will perform simple harmonic motion (SHM) with a period T of 5.2 s.

2g. Estimate the value of k in the following expression. *[3 marks]*

$T = 2\pi\sqrt{\frac{m}{k}}$

Give an appropriate unit for your answer. Ignore the mass of the cable and any oscillation of satellite X.

Markscheme

use of k = «$\frac{4\pi^2 m}{T^2}$ =» $\frac{4\times\pi^2\times 350}{5.2^2}$

510

N m^{-1} ***or*** kg s^{-2}

Allow MP1 and MP2 for a bald correct answer

Allow 500

Allow N/m etc.

2h. Describe the energy changes in the satellite Y-cable system during one cycle of the oscillation. *[2 marks]*

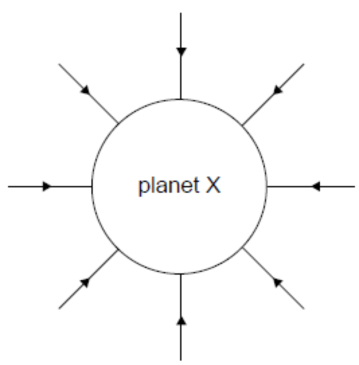
Markscheme

E_p in the cable/system transfers to E_k of Y

and back again twice in each cycle

Exclusive use of gravitational potential energy negates MP1

The diagram shows the gravitational field lines of planet X.

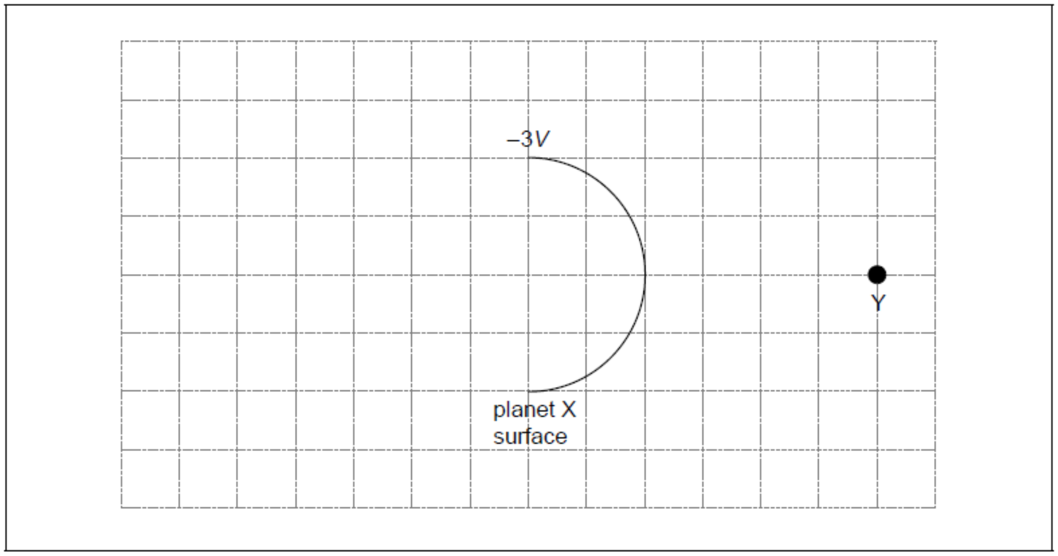


3a. Outline how this diagram shows that the gravitational field strength of planet X decreases with distance from the surface. *[1 mark]*

Markscheme

the field lines/arrows are further apart at greater distances from the surface

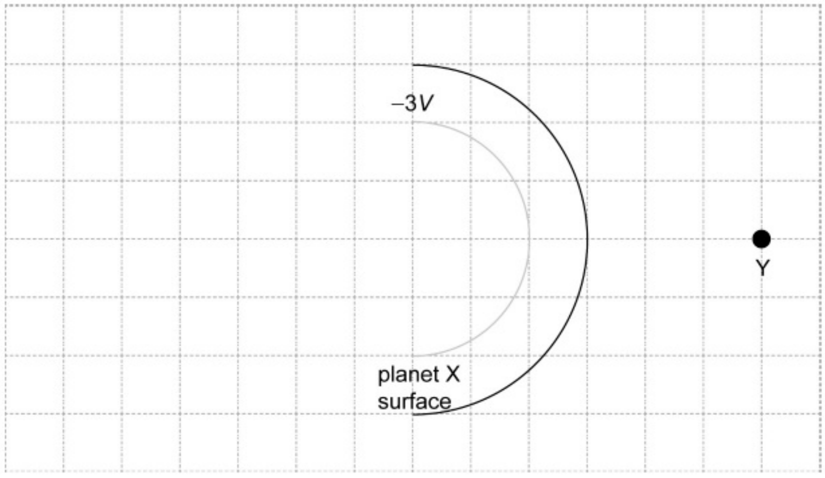
3b. The diagram shows part of the surface of planet X. The gravitational potential at the surface of planet X is –3 *V* and the gravitational potential at point Y is – *V*. *[2 marks]*



Sketch on the grid the equipotential surface corresponding to a gravitational potential of –2 *V*.

Markscheme

circle centred on Planet X
three units from Planet X centre



3c. A meteorite, very far from planet X begins to fall to the surface with a negligibly small initial speed. The mass of planet X is 3.1 × 10^{21} kg and its radius is 1.2 × 10^{6} m. The planet has no atmosphere. Calculate the speed at which the meteorite will hit the surface. *[3 marks]*

Markscheme

loss in gravitational potential = $\frac{6.67\times10^{-11}\times3.1\times10^{21}}{1.2\times10^{6}}$

«= 1.72 × 10^{5} JKg^{-1}»

equate to $\frac{1}{2}v^2$

v = 590 «m s^{-1}»

Allow ECF from MP1.

3d. At the instant of impact the meteorite which is made of ice has a temperature of 0 °C. Assume that all the kinetic energy at impact gets transferred into internal energy in the meteorite. Calculate the percentage of the meteorite's mass that melts. The specific latent heat of fusion of ice is 3.3×10^5 J kg^{-1}. *[2 marks]*

Markscheme

available energy to melt one kg 1.72×10^5 «J»

fraction that melts is $\frac{1.72\times10^5}{3.3\times10^5}$ = 0.52 ***OR*** 52%

Allow ECF from MP1.

Allow 53% from use of 590 ms^{-1}.

The gravitational potential due to the Sun at its surface is -1.9×10^{11} J kg^{-1}. The following data are available.

Mass of Earth	= 6.0×10^{24} kg
Distance from Earth to Sun	= 1.5×10^{11} m
Radius of Sun	= 7.0×10^8 m

4a. Outline why the gravitational potential is negative. *[2 marks]*

Markscheme

potential is defined to be zero at infinity

so a positive amount of work needs to be supplied for a mass to reach infinity

4b. The gravitational potential due to the Sun at a distance *r* from its centre is V_S. Show that *[1 mark]*

rV_S = constant.

Markscheme

$V_S = -\frac{GM}{r}$ so $r \times V_S$ «$= -GM$» = constant because *G* and *M* are constants

4c. Calculate the gravitational potential energy of the Earth in its orbit around the Sun. Give your answer to an appropriate number of significant figures. *[2 marks]*

Markscheme

$GM = 1.33 \times 10^{20}$ «J m kg^{-1}»

GPE at Earth orbit «$= -\frac{1.33\times10^{20}\times6.0\times10^{24}}{1.5\times10^{11}}$» = «–» 5.3×10^{33} «J»

*Award **[1 max]** unless answer is to 2 sf.*

Ignore addition of Sun radius to radius of Earth orbit.

4d. Calculate the total energy of the Earth in its orbit. *[2 marks]*

Markscheme

ALTERNATIVE 1

work leading to statement that kinetic energy $\frac{GMm}{2r}$, ***AND*** kinetic energy evaluated to be «+» 2.7 x 10^{33} «J»

energy «= PE + KE = answer to (b)(ii) + 2.7 x 10^{33}» = «–» 2.7 x 10^{33} «J»

ALTERNATIVE 2

statement that kinetic energy is $= -\frac{1}{2}$ gravitational potential energy in orbit

so energy «$= \frac{\text{answer to (b)(ii)}}{2}$» = «–» 2.7 x 10^{33} «J»

Various approaches possible.

4e. An asteroid strikes the Earth and causes the orbital speed of the Earth to suddenly decrease. Suggest the ways in which the orbit of the Earth will change. *[2 marks]*

Markscheme

«KE will initially decrease so» total energy decreases

OR

«KE will initially decrease so» total energy becomes more negative

Earth moves closer to Sun

new orbit with greater speed «but lower total energy»

changes ellipticity of orbit

4f. Outline, in terms of the force acting on it, why the Earth remains in a circular orbit around the Sun. *[2 marks]*

Markscheme

centripetal force is required

and is provided by gravitational force between Earth and Sun

*Award **[1 max]** for statement that there is a "centripetal force of gravity" without further qualification.*

5a. Explain what is meant by the gravitational potential at the surface of a planet. *[2 marks]*

Markscheme

the «gravitational» work done «by an external agent» per/on unit mass/kg

Allow definition in terms of reverse process of moving mass to infinity eg "work done on external agent by…".
Allow "energy" as equivalent to "work done"

in moving a «small» mass from infinity to the «surface of» planet / to a point

***N.B.**: on SL paper Q5(a)(i) and (ii) is about "gravitational field".*

5b. An unpowered projectile is fired vertically upwards into deep space from the surface of planet Venus. Assume that the gravitational effects of the Sun and the other planets are negligible. *[5 marks]*

The following data are available.

Mass of Venus = 4.87×10^{24} kg Radius of Venus = 6.05×10^{6} m Mass of projectile = 3.50×10^{3} kg Initial speed of projectile = 1.10×escape speed

(i) Determine the initial kinetic energy of the projectile.

(ii) Describe the subsequent motion of the projectile until it is effectively beyond the gravitational field of Venus.

Markscheme

i
escape speed
Care with ECF from MP1.

v = «$\sqrt{\left(\frac{2GM}{R}\right)}=$»

$\sqrt{\left(\frac{2\times6.67\times10^{-11}\times4.87\times10^{24}}{6.05\times10^{6}}\right)}$ ***or*** 1.04×10^{4}«$m\ s^{-1}$»

or «$1.1\times1.04\times10^{4}$ m s^{-1}»= 1.14×10^{4} «m s^{-1}»

KE = «$0.5\times3500\times(1.1\times1.04\times10^{4}$ m $s^{-1})^{2}$ =» 2.27×10^{11} «J»

*Award **[1 max]** for omission of 1.1 – leads to 1.88×10^{11} m s^{-1}.*
*Award **[2]** for a bald correct answer.*

ii
Velocity/speed decreases / projectile slows down «at decreasing rate»

«magnitude of» deceleration decreases «at decreasing rate»
Mention of deceleration scores MP1 automatically.

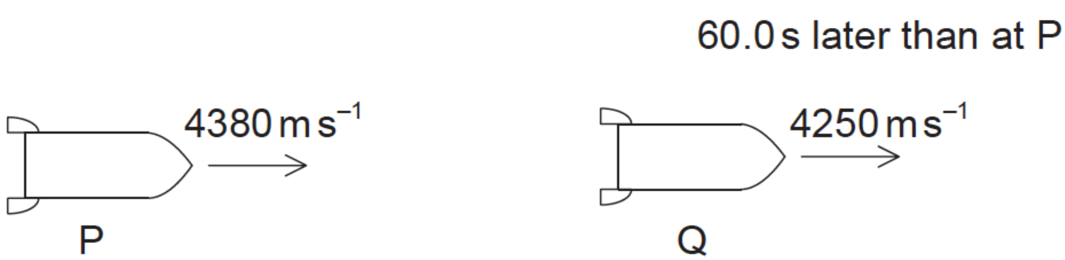
velocity becomes constant/non-zero
OR
deceleration tends to zero

Accept "negative acceleration" for "deceleration".

Must see "velocity" not "speed" for MP3.

Part 2 Motion of a rocket

A rocket is moving away from a planet within the gravitational field of the planet. When the rocket is at position P a distance of 1.30×10^{7}m from the centre of the planet, the engine is switched off. At P, the speed of the rocket is 4.38×10^{3}ms^{-1}.



At a time of 60.0 s later, the rocket has reached position Q. The speed of the rocket at Q is 4.25×10^{3}ms^{-1}. Air resistance is negligible.

6a. Outline, with reference to the energy of the rocket, why the speed of the rocket is changing between P and Q. *[2 marks]*

Markscheme

gravitational potential energy is being gained;
this is at the expense of kinetic energy (and speed falls);

6b. Estimate the average gravitational field strength of the planet between P and Q. *[2 marks]*

Markscheme

$\left(\text{acceleration} = \frac{(v-u)}{t} = \frac{4.25\times10^3 - 4.38\times10^3}{60} =\right) (-)\,2.17\,(\text{ms}^{-2})$;

gravitational field strength = acceleration of rocket (=2.17 N kg–1); } *(allow g = a in symbols)*

or

computes potential difference from KE per unit mass change (5.61
×105),
computes distance travelled (0.259 Mm), uses
$g = \frac{(-)\Delta V}{\Delta r}$;
g=(
−)2.17(ms^{-2});

6c. A space station is in orbit at a distance *r* from the centre of the planet in (e)(i). A satellite is launched from the space station so as just to escape from the gravitational field of the planet. The launch takes place in the same direction as the velocity of the space station. Outline why the launch velocity relative to the space station can be less than your answer to (e)(i). *[1 mark]*

Markscheme

the satellite has velocity/kinetic energy as it is orbiting with the space station;

This question is about the energy of an orbiting satellite.

A space shuttle of mass *m* is launched in the direction of the Earth's South Pole.

7a. The kinetic energy $E_{\rm K}$ given to the shuttle at its launch is given by the expression *[2 marks]*

$$E_{\rm K} = \frac{7GMm}{8R_{\rm E}}$$

where G is the gravitational constant, M is mass of the Earth and $R_{\rm E}$ is the radius of the Earth. Deduce that the shuttle cannot escape the gravitational field of the Earth.

Markscheme

KE needs to be $\geq$ (magnitude of) GPE at surface $\left(-\frac{GMm}{R_E}\right)$;

But KE is $\frac{7GMm}{8R_{\rm E}} < \frac{GMm}{R_{\rm E}}$ / *OWTTE*;

or

shows that total energy at launch $= -\frac{GMm}{8R_{\rm E}}$; *(appropriate working required)*

this is < 0, so escape impossible;

or

states that escape velocity needed is $\sqrt{\frac{2GM}{R_{\rm E}}}$;

shows launch velocity is only $\sqrt{\frac{7GM}{4R_{\rm E}}}$; *(appropriate working required)*

The shuttle enters a circular orbit of radius R around the Earth.

7b. Show that the total energy of the shuttle in its orbit is given by $-\frac{GMm}{2R}$. Air resistance is negligible. *[3 marks]*

Markscheme

$E_{\text{tot}} = \text{PE} + \text{KE}$;

shows that kinetic energy $= (\frac{1}{2}mv^2 =) \frac{GMm}{2R}$; *(appropriate working required)*

adds PE $\left(-\frac{GMm}{R}\right)$ and KE to get given answer; *(appropriate working required)*

7c. Using the expression for E_{K} in (a) and your answer to (b)(i), determine R in terms of R_{E}. *[3 marks]*

Markscheme

$-\frac{GMm}{R_{\text{E}}} + \frac{7GMm}{8R_{\text{E}}} = -\frac{GMm}{2R}$; *(equating total energy at launch and in orbit)*

$\frac{1}{8R_{\text{E}}} = \frac{1}{2R}$;

$R = 4R_{\text{E}}$;

*Award **[0]** for an answer such as* $R = \frac{4R_{\text{E}}}{7}$.

7d. In practice, the total energy of the shuttle decreases as it collides with air molecules in the upper atmosphere. Outline what happens to the speed of the shuttle when this occurs. *[2 marks]*

Markscheme

total energy decreases/becomes a greater negative value, so *R* decreases;

as R decreases kinetic energy increases;

speed increases;

Allow third marking point even if reasoning is incorrect.

This question is in **two** parts. **Part 1** is about the use of renewable energy sources. **Part 2** is about the gravitational potential of the Earth.

Part 2 Gravitational potential of the Earth

8. The table gives the gravitational potential *V* for various distances *r* from the **surface** of Earth. The radius of Earth is 6.4×10^3 km. *[3 marks]*

V / 10^7 J kg^{-1}	r / 10^3 km
−6.24	0
−3.84	4.0
−1.04	32

Show that the data are consistent with Earth acting as a point mass with its mass concentrated at its centre.

Markscheme

recognize that $V \times$ a distance is constant / $\frac{V_1}{V_2} = \frac{R_2}{R_1}$ $(= 1.625)$;

use of $R_E + r$;

evaluates at least two data points 4.0×10^{14}; } *(allow $V \times R$ to be expressed in any consistent unit)*

*Award **[2 max]** if answer fails to use radius of Earth but infers wrong conclusion from correct evaluation of two or more data points.*

*Award **[3 max]** if answer evaluates mass of Earth and shows that it is the same for two or more data points.*

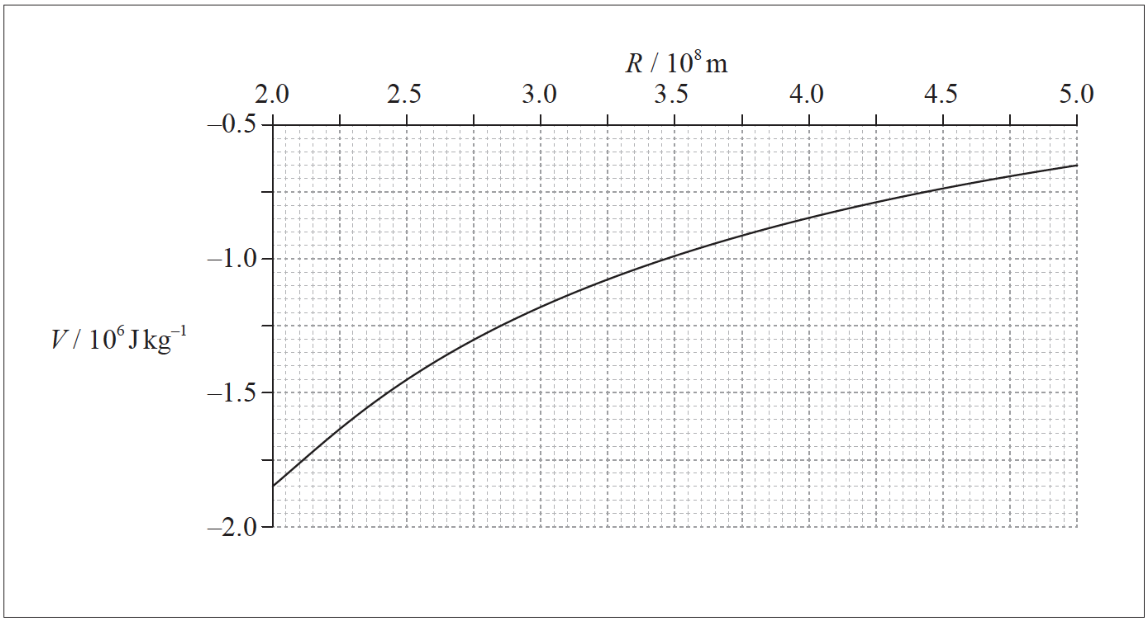
Part 2 Gravitational potential

9a. Define *gravitational potential* at a point in a gravitational field. *[3 marks]*

Markscheme

work done per unit mass;
in bringing (test) mass from infinity to point;
reference to small/point (test) mass;

9b. The graph shows how the gravitational potential V of Earth varies with distance R from the centre of Earth in the range R=2.0×10^8 m to R=5.0×10^8 m. *[6 marks]*



The Moon is at a distance of 4.0×10^8 m from the centre of Earth. At some time in the past it was at a distance of 2.7×10^8 m from the centre of Earth.

Use the graph opposite to determine

(i) the present day magnitude of the acceleration of the Moon.

(ii) by how much the potential energy of the Moon has changed as a result of moving from R=2.7×10^8 m to R=4.0×10^8 m. The mass of the Moon is 7.4×10^{22} kg.

Markscheme

(i) tangent construction attempted at R=4.0×10^8 m;
triangle/pair of coordinates used in calculation;
attempt to calculate gradient;
2.5×10^{-3} ms^{-2}; *(accept answers in the range of 2.2 to 2.7)*
*Award **[1 max]** for*
$\frac{V}{R}$ *to give (–)2.1×10^{-3}.*

(ii) change in V=0.45×10^6-0.50×10^6 Jkg^{-1};
change in PE = (0.5×10^6×7.4×10^{22}=)3.3-3.7×10^{28} J;

9c. State why the change of potential energy in (f)(ii) is an increase. *[1 mark]*

Markscheme

work is done against the gravitational field of Earth / Moon is now closer to infinity/further from Earth / $\frac{-GMm}{R}$ means that as R increases potential increases/becomes less negative;

This question is in two parts. **Part 1** is about electric charge and electric circuits. **Part 2** is about momentum.

Part 1 Electric charge and electric circuits

10a. State Coulomb's law. *[2 marks]*

Markscheme

the force between two (point) charges;
is inversely proportional to the square of their separation and (directly) proportional to (the product of) their magnitudes;

*Allow **[2]** for equation with F, Q and r defined.*

10b. In a simple model of the hydrogen atom, the electron can be regarded as being in a circular orbit about the proton. The radius of the orbit is 2.0×10^{-10} m. *[7 marks]*

(i) Determine the magnitude of the electric force between the proton and the electron.

(ii) Calculate the magnitude of the electric field strength *E* and state the direction of the electric field due to the proton at a distance of 2.0×10^{-10} m from the proton.

(iii) The magnitude of the gravitational field due to the proton at a distance of 2.0×10^{-10} m from the proton is *H*.
Show that the ratio $\frac{H}{E}$ is of the order 10^{-28}C kg^{-1}.

(iv) The orbital electron is transferred from its orbit to a point where the potential is zero. The gain in potential energy of the electron is 5.4×10^{-19}J. Calculate the value of the potential difference through which the electron is moved.

Markscheme

(i) $F=\left(k\frac{q_1q_2}{r^2}=\right)\frac{9\times10^9\times\left[1.6\times10^{-19}\right]^2}{4\times10^{-20}}$;
$=5.8\times10^{-9}$(N);
*Award **[0]** for use of masses in place of charges.*

(ii) ($\frac{(b)(i)}{1.6\times10^{-19}}$ **or** 3.6×10^{10} (NC^{-1}) **or** (Vm^{-1});
(directed) away from the proton;
Allow ECF from (b)(i).
Do not penalize use of masses in both (b)(i) and (b)(ii) – allow ECF.

(iii) $H=\left(G\frac{m}{r^2}=\right)\frac{6.67\times10^{-11}\times1.673\times10^{-27}}{4\times10^{-20}}=2.8\times10^{-18}\left(\mathrm{Nkg}^{-1}\right)$;

$\frac{H}{E}=\frac{2.8\times10^{-18}}{3.6\times10^{10}}$ **or** 7.8×10^{-29}(Ckg^{-1});
($\approx10^{28}Ckg^{-1}$)
Allow ECF from (b)(i).

(iv) 3.4(V);

10c. An electric cell is a device that is used to transfer energy to electrons in a circuit. A particular circuit consists of a cell of emf *ε* and internal resistance *r* connected in series with a resistor of resistance 5.0 Ω. *[6 marks]*

(i) Define *emf of a cell.*

(ii) The energy supplied by the cell to one electron in transferring it around the circuit is 5.1×10^{-19}J. Show that the emf of the cell is 3.2V.

(iii) Each electron in the circuit transfers an energy of 4.0×10^{-19} J to the 5.0 Ω resistor. Determine the value of the internal resistance *r*.

Markscheme

(i) power supplied per unit current / energy supplied per unit charge / work done per unit charge;

(ii) energy supplied per coulomb=$\frac{5.1\times10^{-19}}{1.6\times10^{-19}}$ ***or*** 3.19(V);
(≈3.2V)

(iii) pd across 5.0Ω resistor=$\left(\frac{4.0\times10^{-19}}{1.6\times10^{-19}}=\right)2.5\,(\mathrm{V})$;
pd across *r*=(3.2-2.5=)0.70(V);

and

either

current in circuit=$\left(\frac{2.5}{5.0}=\right)0.5\,(\mathrm{A})$;
resistance of *r*=$\left(\frac{0.70}{0.50}=\right)1.4\,(\Omega)$;

or

resistance of *r*=$\frac{0.70}{2.5}\times 5.0$;
=1.4(Ω);

or

3.2=0.5(R+*r*);
resistance of *r*=1.4(Ω);
*Award **[4]** for alternative working leading to correct answer.*
*Award **[4]** for a bald correct answer.*

This question is in **two** parts. **Part 1** is about gravitational force fields. **Part 2** is about properties of a gas.

Part 1 Gravitational force fields

11a. State Newton's universal law of gravitation. *[2 marks]*

Markscheme

the (attractive) force between two (point) masses is directly proportional to the product of the masses;
and inversely proportional to the square of the distance (between their centres of mass);
Use of equation is acceptable:
*Award **[2]** if all five quantities defined. Award **[1]** if four quantities defined.*

11b. A satellite of mass *m* orbits a planet of mass *M*. Derive the following relationship between the period of the satellite *T* and the radius of its orbit *R* (Kepler's third law). *[3 marks]*

$T^2=\frac{4\pi^2R^3}{GM}$

Markscheme

$G\frac{Mm}{R^2}=\frac{mv^2}{R}$ so $v^2=\frac{Gm}{R}$;
$v=\frac{2\pi R}{T}$;
$v^2=\frac{4\pi^2R^2}{T^2}=\frac{Gm}{R}$;

or

$G\frac{Mm}{R^2}=m\omega^2R$;
$\omega^2=\frac{4\pi^2}{T^2}$;
$\frac{4\pi^2}{T^2}=\frac{GM}{R^3}$;
*Award **[3]** to a clear response with a missing step.*

11c. A polar orbiting satellite has an orbit which passes above both of the Earth's poles. One polar orbiting satellite used for Earth observation has an orbital period of 6.00 × 10^3s. *[8 marks]*

Mass of Earth = 5.97 × 10^{24} kg
Average radius of Earth = 6.37 × 10^{6} m

(i) Using the relationship in (b), show that the average height above the surface of the Earth for this satellite is about 800 km.

(ii) The satellite moves from an orbit of radius 1200 km above the Earth to one of radius 2500 km. The mass of the satellite is 45 kg.

Calculate the change in the gravitational potential energy of the satellite.

(iii) Explain whether the gravitational potential energy has increased, decreased or stayed the same when the orbit changes, as in (c)(ii).

Markscheme

(i) $R^3 = \frac{6.67\times10^{-11}\times5.97\times10^{24}\times6000^2}{4\times\pi^2}$;
R=7.13x10^6(m);
h=(7.13x10^6-6.37x10^6)=760(km);
*Award **[3]** for an answer of 740 with π taken as 3.14.*

(ii) clear use of $\Delta V = \frac{\Delta E}{m}$ and $V = -\frac{Gm}{r}$ **or** $\Delta E = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$;
one value of potential energy calculated (2.37×10^9 ***or*** 2.02×10^9);
3.5×10^8 (J);
*Award **[3]** for a bald correct answer.*
*Award **[2]** for 7.7x10^9. Award **[1]** for 7.7x10^{12}.*
*Award **[0]** for answers using mgΔh.*

(iii) increased;
further from Earth / closer to infinity / smaller negative value;
*Award **[0]** for a bald correct answer.*

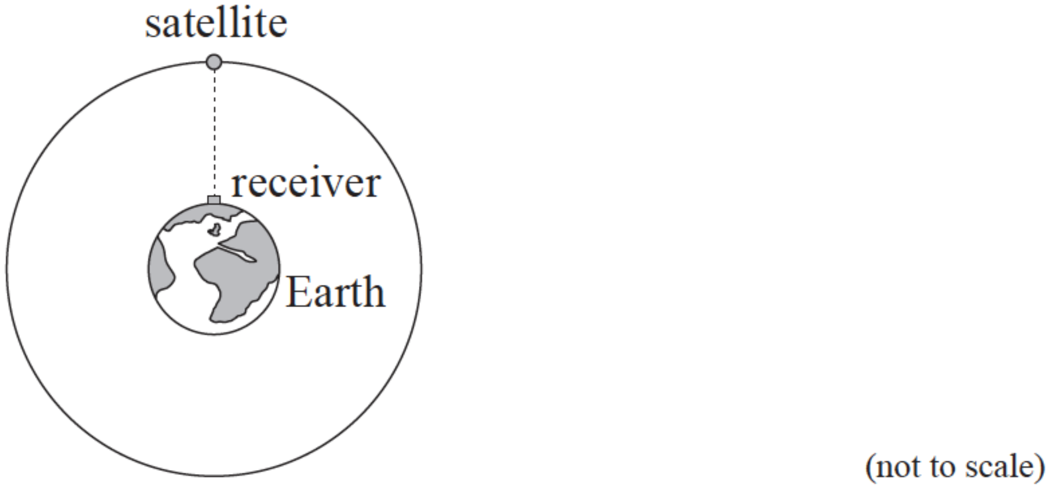
Part 2 Satellite

12a. State, in words, Newton's universal law of gravitation. *[2 marks]*

Markscheme

force is proportional to product of masses and inversely proportional to square of distance apart;
reference to point masses;

12b. The diagram shows a satellite orbiting the Earth. The satellite is part of the network of global-positioning satellites (GPS) that transmit radio signals used to locate the position of receivers that are located on the Earth. *[3 marks]*



When the satellite is directly overhead, the microwave signal reaches the receiver 67ms after it leaves the satellite.

(i) State the order of magnitude of the wavelength of microwaves.

(ii) Calculate the height of the satellite above the surface of the Earth

Markscheme

(i) order of 1 cm;

(ii) $3\times10^8\times67\times10^{-3}$;
2.0×10^7m;

12c. (i) Explain why the satellite is accelerating towards the centre of the Earth even though its orbital speed is constant. *[8 marks]*

(ii) Calculate the gravitational field strength due to the Earth at the position of the satellite.

Mass of Earth = 6.0×10^{24}kg
Radius of Earth = 6.4×10^6m

(iii) Determine the orbital speed of the satellite.

(iv) Determine, in hours, the orbital period of the satellite.

Markscheme

(i) force required towards centre of Earth to maintain orbit;
force means that there is an acceleration / *OWTTE*;

or

direction changes;
a change in velocity therefore acceleration;

(ii) uses$=\frac{GM}{r^2}$ ***or*** $\frac{6.7\times10^{-11}\times6.0\times10^{24}}{\left[2.6\times10^7\right]^2}$;
0.57Nkg^{-1}; (allow ms^{-2})

(iii) $v=\sqrt{0.57\times\left(2.0\times10^7+6.4\times10^6\right)}$ by equating $\frac{v^2}{r}$ and *g*;
3900ms^{-1};

(iv) $T=2\pi\frac{2.6\times10^7}{3900}$;
11.9 hours;

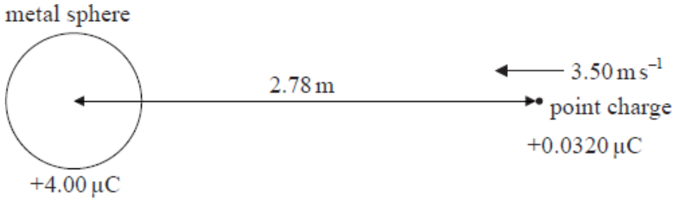
This question is about electric potential.

13a. Define *electric potential* at a point in an electric field. *[3 marks]*

Markscheme

the work done per unit charge;
when a small/test/point positive charge; *(charge sign is essential)*
is moved from infinity to the point;

13b. A positive point charge is moving towards a small, charged metal sphere along a radial path. *[6 marks]*



At the position shown in the diagram, the point charge has a speed of 3.50 m s^{-1} and is at a distance of 2.78 m from the centre of the metal sphere. The charge on the sphere is +4.00 μC.

(i) State the direction of the velocity of the point charge with respect to an equipotential surface due to the metal sphere.

(ii) Show that the electric potential *V* due to the charged sphere at a distance of 2.78 m from its centre is 1.29×10^4 V.

(iii) The electric potential at the surface of the sphere is 7.20×10^4 V. The point charge has a charge of +0.0320 μC and its mass is 1.20×10^{-4} kg. Determine if the point charge will collide with the metal sphere.

Markscheme

(i) perpendicular / at right angles / at 90° / normal;

(ii) $V = \frac{8.99 \times 10^{9} \times 4.00 \times 10^{-6}}{2.78}$ ***or*** $1.2935 \times 10^{4}\mathrm{V}$; (*use of* $\frac{1}{4\pi\varepsilon_0}$ *gives 1.29378×10*^{4})
(≈1.29×10^{4}V)

(iii) difference in potential = (7.20×10^{4}-1.29×10^{4}=)5.91×10^{4};
required loss in kinetic energy/minimum kinetic energy to reach sphere = ($0.032\times10^{-6}\times5.91\times10^{4}$=)$1.89\times10^{-3}$J ;
available kinetic energy=(½×$1.20\times10^{-4}\times3.50^{2}$=)$7.35\times10^{-4}$J; not enough (initial) kinetic energy to reach sphere;
Response needs some statement of conclusion, e.g. so it does not reach sphere.
Allow answer in terms of minimum speed to reach sphere 5.61ms^{-1}.

This question is about escape speed and gravitational effects.

14a. Explain what is meant by escape speed. *[2 marks]*

Markscheme

(minimum) speed of object to escape gravitational field of a planet/travel to infinity;
at surface of planet;
without (further) energy input;

14b. Titania is a moon that orbits the planet Uranus. The mass of Titania is 3.5×10^{21}kg. The radius of Titania is 800 km. *[5 marks]*

(i) Use the data to calculate the gravitational potential at the surface of Titania.

(ii) Use your answer to (b)(i) to determine the escape speed for Titania.

Markscheme

(i) $-\frac{6.67\times10^{-11}\times3.5\times10^{21}}{8.0\times10^{5}}$;
-2.9×10^{5}Jkg^{-1}; *(allow Nmkg*$^{-1}$*)*
Award ***[1 max]*** *if negative sign omitted.*

(ii) $\frac{1}{2}mv^{2} = mV$;
speed== $\sqrt{2\times2.9\times10^{5}}$; *(allow ECF from (b)(i))*
7.6 ×10^{2}ms^{-1};
Ignore sign.
Award ***[3]*** *for a bald correct answer.*

14c. An astronaut visiting Titania throws an object away from him with an initial horizontal velocity of 1.8 m s $^{-1}$. The object is 1.5 m above the moon's surface when it is thrown. The gravitational field strength at the surface of Titania is 0.37 N kg^{-1}. *[3 marks]*

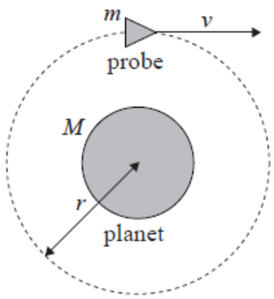
Calculate the distance from the astronaut at which the object first strikes the surface.

Markscheme

time to hit surface $= \sqrt{\frac{2.0\times1.5}{0.37}}(=2.85\mathrm{s})$;
distance to impact = 2.85×1.8;
5.1m;

This question is about a probe in orbit.

A probe of mass *m* is in a circular orbit of radius *r* around a spherical planet of mass *M*.



(diagram not to scale)

15a. State why the work done by the gravitational force during one full revolution of the probe is zero. *[1 mark]*

Markscheme

because the force is always at right angles to the velocity / motion/orbit is an equipotential surface;
Do not accept answers based on the displacement being zero for a full revolution.

15b. Deduce for the probe in orbit that its *[4 marks]*

(i) speed is $v = \sqrt{\frac{GM}{r}}$.

(ii) total energy is $E = -\frac{GMm}{2r}$.

Markscheme

(i) equating gravitational force $\frac{GMm}{r^2}$;
to centripetal force $\frac{mv^2}{r}$ to get result;

(ii) kinetic energy is $\frac{GMm}{2r}$;
addition to potential energy $-\frac{GMm}{r}$ to get result;

15c. It is now required to place the probe in another circular orbit further away from the planet. To do this, the probe's engines will be fired for a very short time. *[2 marks]*

State and explain whether the work done on the probe by the engines is positive, negative **or** zero.

Markscheme

the total energy (at the new orbit) will be greater than before/is less negative;
hence probe engines must be fired to produce force in the direction of motion / positive work must be done (on the probe);
*Award **[1]** for mention of only potential energy increasing.*

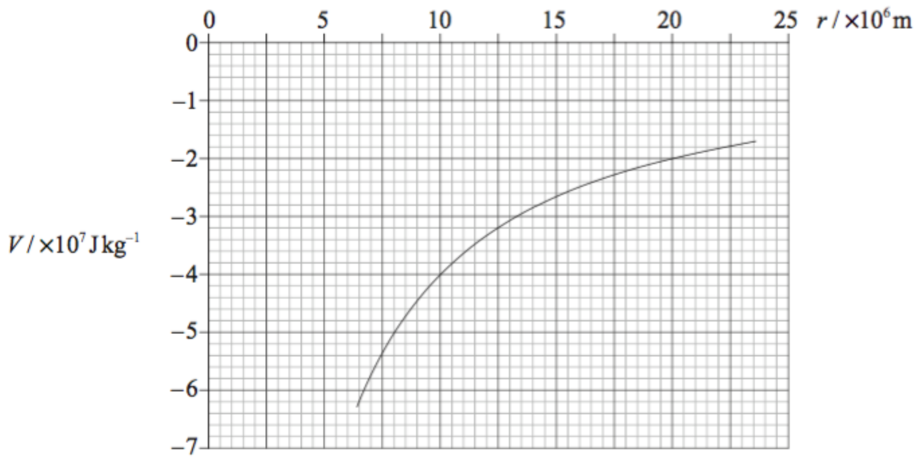
Part 2 Orbital motion

16a. A satellite, of mass *m*, is in orbit about Earth at a distance *r* from the centre of Earth. Deduce that the kinetic energy E_K of the satellite is equal to half the magnitude of the potential energy E_P of the satellite. *[3 marks]*

Markscheme

$\frac{mv^2}{r} = \frac{GMm}{r^2}$;
$E_K = \frac{1}{2}mv^2 = \frac{GMm}{2r}$;
$E_P = -\frac{GMm}{r}$ (hence magnitude of E_K=½ magnitude of E_P);

16b. The graph shows the variation with distance r of the Earth's gravitational potential V. Values of V for $r<R$, where R is the radius of Earth, are not shown. *[6 marks]*



The satellite in (a) has a mass of 8.2×10^2kg and it is in orbit at a distance of 1.0×10^7m from the centre of Earth. Using data from the graph and your answer to (a), calculate for the satellite

(i) its total energy.

(ii) its orbital speed.

(iii) the energy it must gain to move to an orbit a distance 2.0×10^7 m from the centre of the Earth.

Markscheme

(i) total energy=(KE+PE=)$-\frac{Vm}{2}$;
$= \left(-\frac{4.0\times10^7\times8.2\times10^2}{2} =\right) - 1.6\times10^{10}\text{J}$;

(ii) $v=\sqrt{V}$; (or use of $E_K=½mv^2$)
$=6.3\times10^3\text{ms}^{-1}$;

(iii) total energy in new orbit=$\left(-\frac{2.0\times10^7\times8.2\times10^2}{2} =\right) - 0.82\times10^{10}\,(\text{J})$;
energy required=$(1.6\times10^{10}-0.82\times10^{10}=)7.8\times10^9$J;

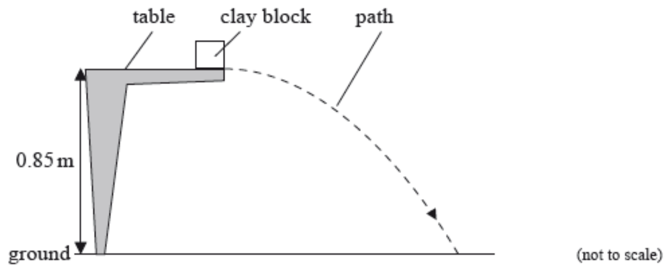
or

total energy is proportional to E_P;
so energy required =–(b)(i)÷2=8 ***or*** 8.2×10^9J; *(allow ECF from (b)(i))*

This question is in **two** parts. **Part 1** is about collisions. **Part 2** is about the gravitational field of Mars.

Part 1 Collisions

The experiment is repeated with the clay block placed at the edge of the table so that it is fired away from the table. The initial speed of the clay block is $4.3\ \mathrm{ms}^{-1}$ horizontally. The table surface is 0.85 m above the ground.



17a. (i) Ignoring air resistance, calculate the horizontal distance travelled by the clay block before it strikes the ground. *[7 marks]*

(ii) The diagram in (c) shows the path of the clay block neglecting air resistance. On the diagram, draw the approximate shape of the path that the clay block will take assuming that air resistance acts on the clay block.

Markscheme

(i) use of kinematic equation to yield time;

$t=\sqrt{\frac{2s}{g}}\ (=0.42\ \mathrm{s});$

$s=\text{horizontal speed}\times\text{time};$

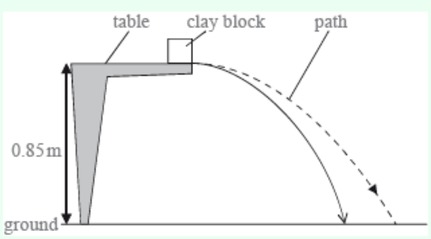
$=1.8\ \mathrm{m};$

Accept g = 10 m s $^{-2}$ *equivalent answers 1.79 from 9.8, 1.77 from 10.*

(ii) initial drawn velocity horizontal; *(judge by eye)*

reasonable shape; *(i.e. quasi-parabolic)*

horizontal distance moved always decreasing when compared to given path / range less than original;



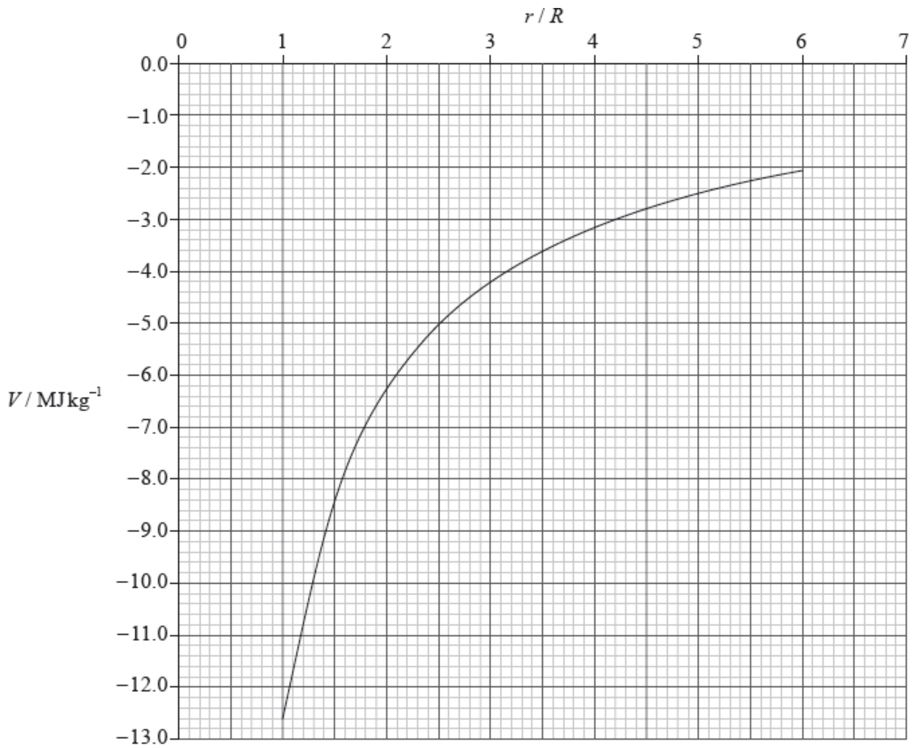
Part 2 Gravitational field of Mars

17b. Define *gravitational potential energy* of a mass at a point. *[1 mark]*

Markscheme

work done in moving mass from infinity to a point;

The graph shows the variation with distance r from the centre of Mars of the gravitational potential V. R is the radius of Mars which is 3.3 Mm. (Values of V for $r < R$ are not shown.)



A rocket of mass 1.2×10^4 kg lifts off from the surface of Mars. Use the graph to

17c. (i) calculate the change in gravitational potential energy of the rocket at a distance 4 R from the centre of Mars. *[5 marks]*

(ii) show that the magnitude of the gravitational field strength at a distance 4 R from the centre of Mars is 0.23 N kg^{-1}.

Markscheme

(i) read offs -12.6 and -3.2;

gain in gpe $1.2 \times 10^4 \times [12.6 - 3.2]$ ***or*** gain in g potential $[12.6 \times 10^6 - 3.2 \times 10^6]$;

$= 1.13 \pm 0.05 \times 10^5$ MJ ***or*** $1.13 \pm 0.05 \times 10^{11}$ J;

(ii) use of gradient of graph to determine g;

values substituted from drawn gradient $\left(\text{typically } \frac{6.7 \times 10^6}{7 \times 3.3 \times 10^6}\right)$;

$= 0.23 \text{ N kg}^{-1}$ *(allow answers in the range of 0.20 to 0.26 N kg–1)*

*Award **[0]** for solutions from $\frac{V}{r}$.*